

Ambiguity and Regularization in Parallel MRI

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Abstract—In this paper, we formulate the parallel magnetic resonance imaging (pMRI) as a multichannel blind deconvolution problem with subsampling. First, the model allows formal characterization of image solutions consistent with data obtained by uniform subsampling of k -space. Second, the model allows analysis of the minimum set of required calibration data. Third, the filter bank formulation provides analysis of the sufficient sizes of interpolation kernels in widely used reconstruction techniques. Fourth, the model suggests principled development of regularization terms to fight ambiguity and ill-conditioning; specifically, subspace regularization is adapted from the blind image super-resolution work of Sroubek et al. [11]. Finally, characterization of the consistent set of image solutions leads to a cautionary comment on $L1$ regularization for the peculiar class of piece-wise constant images. Thus, it is proposed that the analysis of the subsampled blind deconvolution task provides insight into both the multiply determined nature of the pMRI task and possible design strategies for sampling and reconstruction.

problem [6], albeit with no subsampling. Similarly [7] also invoked EVAM results for the estimation of coil sensitivities when autocalibration lines provide a reference image.

In this paper, we first characterize the solutions consistent with the k -space data, revealing the multiply determined nature of the task. We assume that both the sensitivities and the image are deterministic but unknown, and that k -space data are uniformly downsampled in the phase encoding direction. In addition, the smoothness of coil sensitivities in the image domain is exploited by modeling their k -space representations as having small support (FIR filters). A polyphase decomposition is used to describe the non-uniqueness of images consistent with all data in the ideally noiseless case. Knowledge of the set of all solutions motivates selection of regularization terms to augment the non-convex least-square data fitting, thereby combating both local minima and non-unique global minima.