# Linear Quadratic Regulator and Observer Design for a Flexible Joint 

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#### Abstract

In this lab you will apply the LQR that you designed during the prelab, and after that you will implement an observer for this plant. The idea is to control the position of a flexible joint using, in the first case, two encoders and the tachometer and in the second case, using only the encoder from the motor.


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## 1 Introduction

We will divide the lab into two main parts. First, you will implement a state feedback controller using the gains that you found in the prelab for the LQR controller. You will see that you will need to tune these values since the model you use for design is not accurate. Therefore, you will tune the values of the matrices $Q$ and $R$, and find resulting gains $k_{1}, k_{2}, k_{3}$, and $k_{4}$. Next, you are ready to develop an observer. For that, you will sense only the input from the motor's encoder, and you will estimate $\dot{\theta}, \alpha$, and $\dot{\alpha}$.

## 2 Laboratory Procedures

### 2.1 Necessary Equipment

- 1 dSPACE software.
- 1 DS1104 interface card.
- 1 Universal Power Module, UPM-2405.
- 1 SRV 02ET servo DC motor.
- 1 Flexible Joint.
- 2 Encoder cables.
- 1 To load Cable.
- 1 From D/A Cable.
- 1 To A/D Cable.


### 2.2 Connections

1. Fix the high gear ratio on the DC servo.
2. Connect the flexible joint as shown in Figures 1 and 2. If you do not understand the figure, follow the next steps.
3. As you can see in the figures, we will use the tachometer from the motor, as well as the two encoders that are available: the one that belongs to the motor, and the one that belongs to the flexible joint.
4. Use channel \# 3 in dSPACE (\# 2 in the DS1104 card), to acquire the data that comes from the tachometer. For that, first connect to the rear part of the motor the cable named "From Analog Sensors" to the bottom part, and the other side of the cable should be connected to the UPM-2405 power module.
5. Connect the cable "From D/A" to the analog output \# 1 (\# 0 in the DS1104 card), and the "To Load cable" should be connected to the motor as in the other experiments.
6. Connect one side of one of the "Encoder Cables" to "ENC1" on the DS1104 card, and the other side to the motor.
7. Connect one side of one of the "Encoder Cables" to "ENC2" on the DS1104 card, and the other side to the flexible joint.
8. Connect spring \# 2 in the position $[A, 3]$, as you can see in Figure 3.


Figure 1: Flexible joint connections, front view.


Figure 2: Flexible joint connections, back view.

### 2.3 Linear Quadratic Regulator

In the pre-lab you designed an LQR. Here, you will see how good your design is for the actual plant. For that, you will use the matrices $Q$ and $R$ that you found previously.

1. Design one block that has two outputs: one that you will name $\theta$, and the other that you will name $\dot{\theta}$. This block will produce the position and the velocity of the motor.
2. Remember that since you are using the tachometer and the encoder you have to use the appropriate blocks to acquire these values. Besides, in this case you are using the high gear ratio, i.e., you are using a gear ratio of $14: 5$ instead of a gear ratio of $14: 1$. Hence, one of the gains of the tachometer has to change from $1 / 14$ to $1 / 70$.


Figure 3: Diagram of the connections of the Flexible Joint.
3. Design another block similar to the one that you designed, but in this case, since you will not be able to sense $\dot{\alpha}$, you have to implement a filter. The filter that we suggest is in the form $\frac{150 s}{s+150}$. Why is this a reasonable choice?
4. Take the values of $\theta$ and $\alpha$ and add them. Name the output of this sum $\gamma$.
5. Construct a block that has the following characteristics:

- The input is the value of $\gamma$.
- The idea is to take the value of $\gamma$ and compare it to a value that you will name "Angle," and if the value of $\gamma$ is greater than the value of "Angle," the experiment should stop.
- Name this block "Safety Stop Angle."

6. Fix the "Safety Stop Angle" at 50 degrees.
7. Use the "Safety Stop Time" block that you constructed in the second lab, and set the stop time at 60 seconds.
8. Use a square wave with an amplitude of 30 and a frequency of 0.05 Hz . This will be the input to the system. The units that you will use in this model are degrees. Name this output $\theta_{d}$.
9. The error that you will force to be zero will be in this case be $\theta-\theta_{d}$.
10. Use a multiplexer, and the gain $K$ that you compute in the pre-lab to close the loop of the system. Remember that the units are in degrees, and pay attention to the sign of the values that you are using.
11. Fix the sampling time as 1 ms , and compile your experiment.
12. Now, create a new experiment in dSPACE. This experiment will have the file that you already compiled, and a layout with a plotter, 4 displays that show the gains, and one display that will show the actual position in degrees. You can put more elements in your experiment if you want.
13. Run the experiment the first time. Is the value of the gain vector $K$ that you computed in the pre-lab a good one? If not, try to tune until you will have a better response.
14. Once you find the correct values for these gains, save the closed-loop response data as a Matlab file (*.mat) using any method that you want (Hint:You can use the Save button that you have been using, or changing the $x$-axes in the plotter, and the length you will be able to save the data using the classical methods. Also you can apply the method described in the tutorial to save the data without seeing the plotter.).

### 2.4 Observer

Now that you saw that the LQR controller worked appropriately, you will implement the observer that was designed in the pre-lab. For that follow the next steps.

1. Use the same blocks for the flexible joint, and the motor that you implemented before (do not disconnect the encoder for $\alpha$ ).
2. Using the state-space block that you have in Simulink, implement the observer. Follow the steps that you had in the pre-lab, using the same matrices $A, B, C$, and $D$ that you used for the observer pole placement problem.
3. The input and the gains for $K$ remain the same, but now the signals that you will input to the multiplexer should be the estimated ones (i.e., the controller input is now $\hat{x}$, not $x$ ).
4. Again, use the "Safety Stop Time", and the "Safety Stop Angle" blocks. Fix the first one at 60 seconds, and the second one at 50 degrees.
5. Fix the sampling time at 1 ms , and compile the model that you saved.
6. Build a new experiment with the file that was compiled, and add two layouts, one that has the same elements as in Section 2.3, and another that plots the difference between the real data for $\theta, \dot{\theta}, \alpha$, and $\dot{\alpha}$, and the estimated ones.
7. Try the observer gains from the theory, tune as necessary, and save the closed-loop data (save the data for both layouts).

## 3 Post-Laboratory Exercises

1. Which elements did you use to build the block to acquire the position and the velocity of the motor?
2. Which elements did you use to build the block to acquire $\alpha$ and $\dot{\alpha}$ ?
3. Show how you created the model. Include the figure for your model, and explain clearly each of the blocks that you used.
4. Were the values of the gain in the pre-lab good enough in practice?
5. How did you tune the gain $K$ in the LQR case? What are the values of the elements of $K$ that you finally used?
6. In the same figure, plot the response of $\theta_{d}$ and the final position of your flexible joint, i.e. $\gamma$. Show your work by adding the commented Matlab file.
7. Show how you created the model for the observer case. Include the figure for your model, and explain clearly each of the blocks that you used.
8. For the observer experiment, what are the matrices $A, B, C$, and $D$ mentioned above? Explain.
9. In the same figure, plot the 4 errors that you found in Section 2.4. Each error should be on a different plot.
10. In the same figure plot the result for $\theta_{d}$, and the final position.
11. Compare the results that you obtained in the LQR casew with "full state feedback," and in the observer case when the LQR uses the estimate of the state. Show your work by adding the commented Matlab file.
