**Ionospheric effects on Earth-space links**

1. Attenuation
2. Faraday rotation
3. Group delay
4. Scintillations
5. Refraction

**Tropospheric scatter links**

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VII. Ionospheric effects on trans-ionospheric links

- **Attenuation** due to absorption (this was seen in the last lecture).
- **Faraday rotation**, i.e. linear polarized waves undergo a rotation on the polarization plane.
- **Group delay**, caused by a decrease on the group velocity of propagation.
- **Scintillations**, i.e. small signal variation due to small-scale ionospheric irregularities.
- **Refraction**, which cause a change on the apparent angle of signal arrival.
2. Faraday rotation

- We have discussed the physical mechanism behind Faraday rotation in a prior section.

- Assume a plane wave propagating along the $z$ (vertical) direction and linearly polarized along a particular direction, say $\hat{x}$. Upon traversing the ionosphere along a small distance $dz$, the wave incident will emerge linear polarized still, but with a new direction of polarization given by

$$\hat{x} \cos(d\Omega) + \hat{y} \sin(d\Omega)$$  \hspace{1cm} (1)$$

with a rotation angle $d\Omega = k_0 dz (n_+ - n_-)/2$, where $n\pm$ are the refraction indices associated with the two (circular polarized) characteristics waves seen before.
Next, by using the $Q_L$ approximation for the $n\pm$, and assuming frequencies sufficient high so that $Y_L \ll 1$ and $X \ll 1$ hold (also necessary for a trans-ionospheric link to exist, to begin with), one can obtain the following approximation

$$d\Omega \approx \frac{\omega}{2c}XY_L dz$$ (2)

Using the expressions for $X$, $Y_L$ etc., the above expression can be more conveniently expressed as

$$|d\Omega| \approx \frac{1}{8\pi^2} \frac{e^3}{\varepsilon_0 m_e^2 c} \frac{\mu_0 |H_L|}{f^2} N(z)dz$$ (3)
Upon integrating the above expression along $z$, we arrive at

$$|\Omega| \approx \frac{1}{8\pi^2} \frac{e^3}{\epsilon_0 m_e^2 c} \frac{\mu_0 |H_L|}{f^2} T_{ec}$$

where

$$T_{ec} = \int_S N(z) dz$$

is the total electron content (TEC) in the ionosphere along the propagation path. Substituting the numerical values,

$$|\Omega| \approx 2.34 \times 10^4 \frac{\mu_0 |H_L|}{f^2} T_{ec}$$

for $\Omega$ in radians, $f$ is Hertz, $\mu_0 H_L$ in Tesla, and $T_{ec}$ in electrons/m$^2$. 
2. Faraday rotation

The variability of $T_{ec}$ makes exact prediction of $\Omega$ difficult and linear polarization unsuited for trans-ionospheric links below about 8 GHz. Circular polarization is typically used instead (C-band satellites).

Figure: Faraday rotation versus frequency and TEC along path.
3. Group delay and dispersion

- Charged particles in the ionosphere slow down electromagnetic signal propagation. The infinitesimal time delay $dt$ experience by a signal propagating through an infinitesimal path $ds$ is given by

$$dt = \frac{ds}{v_g} = \frac{ds}{cnR_{\pm}} \quad (7)$$

- Using the appropriate approximations for the refractive index at VHF and SHF bands, we have that the excess delay experience by the wave w.r.t. to the vacuum is given by

$$dt_\pm^e = \left( \frac{ds}{cnR_{\pm}} - \frac{ds}{c} \right) \approx \frac{X}{c} ds \quad (8)$$

- Substituting the appropriate expressions for small $X$, we get

$$dt_\pm^e \approx dt^e \approx \frac{1}{8\pi^2} \frac{e^2}{\epsilon_0 m_e c} \frac{N(s)}{f^2} ds \quad (9)$$
3. Group delay and dispersion

Integrating along the propagation path, we get

\[ \delta t^e \approx \frac{1}{8\pi^2} \frac{e^2}{\epsilon_0 m_e c} \frac{T_{ec}}{f^2} \quad (10) \]

and substituting the values of the physical constants

\[ \delta t^e \approx 1.34 \times 10^{-7} \frac{T_{ec}}{f^2} \quad (11) \]

The figure in the next page shows the percentage of yearly average daytime hours when \( \delta t^e > 20 \) ns for a 1.6 GHz signal vertically incident on the ionosphere during a period of relatively high solar activity.
Figure: Percentage of yearly average daytime hours when $\delta t^e > 20$ ns for a 1.6 GHz signal vertically incident on the ionosphere, with sunspot number $= 140$. 
3. Group delay and dispersion

- Group delays can be a source of error for global navigation satellite systems such as GPS. The range error in this case is $\delta \rho = c \Delta t^e$.
- Because the behavior of the group delay with frequency is known to be $1/f^2$, it can be compensated using dual-frequency receivers.
- The figure in the next page shows the difference in the group delay between the lower and upper frequency of the spectrum of a pulse with temporal width $\tau$ transmitted through the ionosphere.
**Figure:** Difference in the group delay between the lower and upper frequency of the spectrum of a pulse with temporal width $\tau$. 
4. Ionospheric scintillations

- Time-varying, small-scale changes in the ionization density that cause fluctuations on the angle-of-arrival, amplitude, and phase of the signal.
- Analogous to similar effects seen in the troposphere
- We will focus on amplitude effects
- The instantaneous variation of the signal intensity is modeled by a Nakagami-\( m \) probability density function
- The fading rate varies from about 0.01 Hz to about 1 Hz.
- The figure in the next page illustrates a typical power spectral density of the scintillation in a geostationary satellite link operating at 4 GHz.
Figure: Typical power spectral density of the scintillation in a geostationary satellite link operating at 4 GHz, for a sequence of measurements taken 30 min or 1 hour apart. Note the $1/f^3$ behavior in the 0.1 Hz to 1.0 Hz frequency range.
5. Ionospheric absorption for trans-ionospheric links

- For trans-ionospheric links at VHF and above, the frequency of the signal is much greater than the collision frequency.
- The attenuation factor $\alpha$ can be approximated as
  \[
  \alpha \approx \left( \frac{e^2}{2\epsilon_0 m_e c} \right) \frac{\nu N}{\omega^2}
  \]  
  under the condition of weak refraction.
- Note the $1/f^2$ dependency here.
- For vertical links at about 30 MHz, the typical one-way attenuation at mid-latitude is already somewhat weak, on the order of 0.2 to 0.5 dB.
- At 1 GHz and above, the attenuation os below 0.01 dB and hence typically neglected.
5. Ionospheric refraction for trans-ionospheric links

- The sky-wave mode studied before at MF and HF frequencies is an extreme example of refraction as it bends the ray back to Earth.
- At higher frequencies, refraction becomes progressively weaker and hence trans-ionospheric links established.
- Refraction nevertheless causes a change on the apparent angle of arrival from the satellite signal to the Earth station.
- The precise amount of refraction depends on the electron density distribution along the ray path.
- Under identical physical condition on the ionosphere, the error in the apparent angle-of-arrival behaves as $1/f^2$ in the VHF and SHF, similarly to the behavior of the Faraday rotation and group delay seen before.
- For example, the error in the angle-of-arrival for a signal with elevation angle of $30^0$ is 250 $\mu$rad at 500 MHz, 60 $\mu$rad at 1 GHz, and less that 1 $\mu$rad at 10 GHz.
Form the previous discussion, it is clear that TEC is very important in determining the effect of ionospheric perturbation in trans-ionospheric links.

For prediction purposes, TEC estimates can be obtained from computer models such as the International Reference Ionosphere (IRI) models or the NeQuick model mentioned in Chapter 10.

For monitoring, a network of dual-frequency ground receivers or low-orbit satellites can be utilized, among other techniques.

The next page shows the vertical TEC measured over North America on two different days, under ordinary conditions and under ionospheric storm conditions.
Vertical TEC measured over North America on two different days, under ordinary conditions (left) and under ionospheric storm conditions (right), where TEC values above 200 are seen in parts of central Mexico.
We have now finished our study of the ionosphere; we’ve learned the basics of how an ionized layer forms, equations for the permittivity of a magnetized, lossy plasma, and basic techniques for predicting ionospheric propagation.

We have only scratched the surface of this subject; the literature is immense and there are currently numerous ionospheric research groups throughout the world.

Subject is useful not only for HF communications system design and analysis, but also for studying the Earth and Sun.

Amateur radio utilizes HF systems for global communications with only modest equipment.

Many large shortwave broadcasting systems: Voice of America, BBC, Radio Free Europe, etc.
Tropospheric scattering ("troposscatter") links

1. Introduction to troposscatter
2. Physical mechanism
3. Troposscatter empirical path loss models
4. Troposscatter fading models
I. Introduction to troposcatter

- As we progress to larger distances at higher than HF frequencies, path loss falls off rapidly in terrain diffraction mechanism.
- Eventually loss reaches a point where it declines more slowly, approximately 0.1 dB/km.
- Mechanism here is tropospheric scatter: scattering from atmospheric irregularities.
- BLOS (beyond line-of-sight) communications: Enables transmission of narrow band signals over long distances (up to about 400 mi), somewhat wider band over shorter distances (about 200 mi).
- Loss is very large so we need high power, large antennas, sensitive receivers, and usually some form of diversity.
- Antenna size means higher than 200 MHz, increasing attenuation means less than 5 GHz.
- The plot in the next page illustrate the *rough* order-of-magnitude path loss with distance.
Figure: Path loss $A$ in dB rough-order-of-magnitude fall-off with distance $d$. 
Figure: Aerial photo of the “White Alice” troposcatter system in Alaska.
“White Alice” system: a communication system implemented in Alaska and northern Canada during the late 1950’s, before the advent of satellite communications.

It was designed partly for general communications needs, and partly to convey information from the Distant Early Warning (DEW) Line to Command Centers of the U.S. Defense forces.

The establishment and maintenance of communications centers in an inhospitable environment in the Arctic was a difficult and expensive task.

In the high-frequency (HF) band (3 to 30MHz) HF system might seem to have been the best solution in this case. However, as we have seen HF propagation depends strongly on the ionosphere, an ionized atmospheric region that is significantly influenced by the Sun.
At times, the Sun ejects huge streams of charged particles that severely upset the ionosphere and make HF communication in the Arctic and sub-Arctic region particularly difficult. Such unreliability was unacceptable for this application.

The propagation mechanism chosen in this case was tropospheric scatter.

Very large antennas and high-powered transmitters are required as apparent in the last page picture.

The high reliability and relative freedom from interference associated with tropospheric scatter propagation outweighed the cost and other considerations.

The White Alice system was eventually superseded by satellite-based systems.
I. Introduction to troposcatter

- Theoretical predictions require precise knowledge of the atmosphere; this is difficult to come by so theory is of limited use. We will only discuss those briefly here to understand the basic physics behind such propagation mechanism.
- For propagation predictions, empirical models are available; even these are questionable when not applied to same measurement configuration.
- Models usually for annual mean or median losses to reduce uncertainty.
- In typical configuration transmit and receive antennas aim near horizon, scattering from “common volume”, i.e. the volume where the main beams of the transmit and receiver antennas intersect, produces signal.
- Illustration in the next page clarifies the concept of “common volume”.

Illustration in the next page clarifies the concept of “common volume”.

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Figure: Common volume defined by the intersection of the main beams of the transmit and receive antennas.
I. Troposcatter

- Optimum antenna height has been found to be 0.2 to 0.6 beamwidths above the horizon due to atmospheric properties.
- Fading should definitely be expected since scattering occurs from many atmospheric irregularities which are changing in time.
- Mechanism is also dispersive since different frequencies will yield different phases for each scatterer.
- Historically, typical systems have been relatively narrow band: 1 MHz to 10 MHz max but wideband systems are finding increasing usage.
I. Troposscatter

- The basic model we will use for tropospheric scatter propagation is based on ITU report 238-6
- Has empirical estimate models for annual transmission loss exceeded less than 50% of the time
- Note this includes aperture to medium coupling loss, $L_{coup}$, and the estimates change depending on the climate
- Two methods are given for calculating transmission loss in the “worst month”
- Diversity is very useful in troposcatter systems for avoiding fading problems; space, frequency, and angle diversity are possible
- Frequency of fading and transmissible bandwidths are also discussed
I. Troposcatter

- Tropospheric scatter provides the only ground based means of long distance point-to-point communications when skywave links are very weak, i.e. at high frequencies, or unreliable, such as near the poles.
- However, satellite systems can also be used for the same purpose and are more common than troposcatter systems.
- Advantages of troposcatter systems: small number of stations needed, reduces support and maintenance costs, sometimes easier to cover impenetrable or unfriendly terrain, jamming and interception are difficult, less capital costs than satellite system.
- Disadvantages: large high power systems required, propagation properties vary, dispersive, fading channels.
Nevertheless, troposcatter systems are still used for long-distance terrestrial links where it is not feasible or cost-effective to employ many repeater stations between the transmitter and receiver, such as in links over the ocean to distant off-shore oil platforms or islands.

Fixed and, more recently, “mobile” troposcatter systems are also of importance in military applications, either to augment/replace existing satellite capabilities or as a back-up mode of operation.

Troposcatter can be relatively secure also, since precise antenna alignment is important.
I. Troposcatter

- Recent advances on modem capabilities, coding and modulation schemes, as well as high-power solid-state amplifiers, have resulted in lighter and more mobile/versatile troposcatter systems ("light tropo")

- Improvements on packet-based communications have also resulted in more flexibility for deployment (link distances) and for data capacity according to the (local, variable) atmospheric conditions
II. Physical troposcatter mechanism

- We consider an atmosphere where the relative permittivity consists of an average value $\epsilon_m$ and a small fluctuation (random) part $\epsilon_f$.
- The fluctuations on the permittivity are space and time dependent and typically on the order of only $10^{-6}$ in relative magnitude.
- The fluctuation are also relatively small in volume (compared to the wavelength of the incident radiation). Because of this, we have a scattering mechanism instead of a refraction mechanism.
- Fluctuations are due to local variations temperature and pressure.
- Under the effect of an ambient electric field, the induced polarization produced in the troposphere writes as

$$\overline{P} = (\epsilon - \epsilon_0) \overline{E} = \epsilon_0 (\epsilon_s - 1 + \epsilon_f) \overline{E}$$  \hspace{1cm} (13)

- The fluctuating part that produces scattering is the term $\epsilon_f \overline{E}$. 
II. Physical troposcatter mechanism

- Since the fluctuations are small, the ambient electric field can be approximated by the incident field $\vec{E}_1$ produced by the transmit antenna.
- This approximation ignores multiple scattering effects, i.e. mutual interaction between scattering currents.
- It also corresponds to a *linearization* of the problem or a "Born approximation."
- The induced polarization current in the medium then writes as
  \[ \vec{J}_p = j\omega\vec{P} = j\omega\varepsilon_0\varepsilon_f \vec{E}_1 \]  
  (14)
- We can also write the field radiated by the transmit antenna as
  \[ \vec{E}_1 = \frac{jk_0 \cos \theta_1}{2\pi r_1} l_1 \overline{f}_1(\theta_1, \phi_1) e^{-jk_0r_1} \]  
  (15)

where $\overline{f}_1(\theta_1, \phi_1)$ represents the far-field pattern and the subscript 1 refers to the transmit antenna.
Combining the previous two expressions, we arrive at

\[ J_p = \frac{-\epsilon_f k_0^2 l_1}{2\pi \eta r_1} \cos \theta_1 e^{-jk_0 r_1} \overline{f}_1(\theta_1, \phi_1) \]

The angles \( \theta_1 \) and \( \phi_1 \) are the elevation and azimuth of a point within the common volume referred to the coordinate system attached to the transmit antenna. A similar definition can be made for \( \theta_1 \) and \( \phi_1 \) w.r.t. the receive antenna. See illustration for \( \theta_1 \) and \( \theta_2 \) in the next page.
Figure: Basic troposcatter-link geometric parameters.
Recall that, from the reciprocity theorem, the receive and transmit pattern of any antenna are identical at any given frequency.

This means that the interaction of the induced current $\mathbf{J}_p$ with the field of the receiving antenna (pattern) in the common volume gives the open-circuit voltage $V_{oc}$ at the receiver.

In other words,

$$V_{oc} = \frac{k_0^3 I_1}{4\pi^2 \eta} \int_{V} \frac{\cos \theta_1 \cos \theta_2}{r_1 r_2} \epsilon_f(r_1) \mathbf{f}_1 \cdot \mathbf{f}_2 e^{-jk_0(r_1+r_2)} dV$$

(17)

where the integration domain $V$ refer to the common volume.

We will next make some approximations on this integral to simplify the analysis.
II. Physical troposcatter mechanism

- For high-gain antennas typically used in troposcatter,\[ \cos \theta_1 \approx \cos \theta_2 \approx 1 \]
- Also, \( r_1 \) and \( r_2 \) do not vary much with respect to their average values \( R_1 \) and \( R_2 \) (center points) within the common volume, so they can be well approximated by the latter and treated as constants for the amplitude factor in this integral.
- Further, for antennas with rotational symmetry \( f_1 \) and \( f_2 \) do not depend on \( \phi_1 \) and \( \phi_2 \), but only on \( \theta_1 \) and \( \theta_2 \), resp.
- In this way,

\[
V_{oc} = \frac{k_0^3}{4\pi^2 \eta} \frac{l_1}{R_1 R_2} \int_V \epsilon_f(\bar{r}_1) \bar{f}_1(\theta_1) \cdot \bar{f}_2(\theta_2) e^{-j k_0 (r_1 + r_2)} dV \quad (18)
\]
The average received power is

$$P_r = \frac{V_{oc}^* V_{oc}}{4R_L}$$  \hspace{1cm} (19)$$

which, using the integral expression above for $V_{oc}$, is given by the ensemble average

$$P_r = \frac{k_0^6 |I_1|^2}{64\pi^2 \eta^2 R_1^2 R_2^2 R_L} \int_V \int_V \langle \epsilon_f(\bar{r}_1) \epsilon_f(\bar{r}_1') \rangle 
\times \left[ f_1(\theta_1) \cdot f_2(\theta_2) \right] \left[ f_1(\theta_1') \cdot f_2(\theta_2') \right] ^* 
\times e^{-jk_0(r_1+r_2-r_1'-r_2')} dVdV'$$  \hspace{1cm} (20)$$

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II. Physical troposcatter mechanism

- Luckily, a number of simplifications can be made on this integral
- First, we can assume that $\epsilon_f(\vec{r})$ is a homogeneous, or stationary, random process within $V$ so that the correlation function $\langle \epsilon_f(\vec{r}_1)\epsilon_f(\vec{r}'_1) \rangle$ is a function of $\vec{r}_1 - \vec{r}'_1$ only, which we denote as $C(\vec{r}_1 - \vec{r}'_1)$.
- The corresponding spatial spectrum is $S(\vec{\beta})$ is given by the Fourier transform

$$S(\vec{\beta}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C(\vec{r}_1 - \vec{r}'_1) e^{-j\vec{\beta} \cdot (\vec{r}_1 - \vec{r}'_1)} d(\vec{r}_1 - \vec{r}'_1) \quad (21)$$

- The correlation function $C(\vec{r}_1 - \vec{r}'_1)$ is small for points separated by more than 50 or 100 m in the atmosphere
II. Physical troposcatter mechanism

- By referring to the last figure and using the fact that in practice $r_1, r_2, r'_1, r'_2 \gg \rho, \rho'$, we can make the following simplifications:

  $$
  \bar{r}_1 = \bar{R}_1 + \bar{\rho} \approx R_1 + \bar{u}_1 \cdot \bar{\rho} \quad (22)
  $$

- And, similarly:

  $$
  \bar{r}'_1 \approx R_1 + \bar{u}_1 \cdot \bar{\rho}'
  $$

  $$
  \bar{r}_2 \approx R_2 + \bar{u}_2 \cdot \bar{\rho}
  $$

  $$
  \bar{r}'_2 \approx R_2 + \bar{u}_2 \cdot \bar{\rho} \quad (23)
  $$

- So that,

  $$
  \bar{r}_1 - \bar{r}'_1 \approx \bar{\rho} - \bar{\rho}'
  $$

  $$
  r_1 + r_2 - r'_1 - r'_2 \approx (\bar{\rho} - \bar{\rho}') \cdot (\bar{u}_1 + \bar{u}_2) \quad (24)
  $$

- In the above, $\bar{u}_1$ and $\bar{u}_2$ are unit vectors along $\bar{R}_1$ and $\bar{R}_2$, resp.
II. Physical troposcatter mechanism

Using these approximations in the expression for the average received power, we arrive at

\[ P_r = \frac{k_0^6 |l_1|^2}{64\pi^2 \eta^2 R_1^2 R_2^2 R_L} \int_V \int_V C(\vec{r}_1 - \vec{r}_1') \times \left[ \bar{f}_1(\theta_1) \cdot \bar{f}_2(\theta_2) \right] \left[ \bar{f}_1(\theta_1') \cdot \bar{f}_2(\theta_2') \right]^* \times e^{-jk_0(\rho - \rho')(\vec{u}_1 + \vec{u}_2)} dVdV' \]  

(25)

Since the antenna patterns are almost constant over the small region over which the correlation function has significant values, we can let \( \theta_1 \approx \theta_1 \approx \theta_c = (\theta_1 + \theta_2)/2 \), and similarly for \( \theta_1' \) and \( \theta_2' \).

Further, we define \( \bar{\rho}_c = (\bar{\rho} + \bar{\rho}')/2 \) and \( \bar{\rho}_d = \bar{\rho} - \bar{\rho}' \).
These changes of variables lead to

\[ P_r = \frac{k_0^6 |I_1|^2}{64\pi^2 \eta^2 R_1^2 R_2^2 R_L} \int_{V_d} C(\bar{\rho}_d') e^{-jk_0\bar{\rho}_d \cdot (\bar{u}_1 + \bar{u}_2)} d\bar{\rho}_d \]

\[ \times \int_{V_c} |\bar{f}_1(\theta_{1c}) \cdot \bar{f}_2(\theta_{2c})|^2 d\bar{\rho}_c \]  

(26)

Since \( C(\bar{\rho}_d') \) is very small outside the common volume \( V \), the first integral can be identified as \( S(-k_0\bar{u}_1 - k_0\bar{u}_2) \), or simply \( S(\bar{k}_s - \bar{k}_i) \), where \( \bar{k}_s \) and \( \bar{k}_i \) are the wavenumber vectors of the scattered and incident fields, resp.
II. Physical troposcatter mechanism

- Significance of $S(\bar{k}_s - \bar{k}_i)$: Consider a dielectric medium with

\[ \epsilon_f(\bar{r}) = A e^{-j\beta \cdot \bar{r}} \]  

and incident wave $\bar{E}_o e^{-j\bar{k}_i \cdot \bar{r}}$ will produce a polarization current

\[ \bar{J}_p = j\omega (\epsilon_f(\bar{r}) - 1) A \bar{E}_o e^{-j(\beta + \bar{k}_i) \cdot \bar{r}} \]  

- From the above, $\bar{k}_s = \beta + \bar{k}_i$, so that $S(\beta) = S(\bar{k}_s - \bar{k}_i)$.
- $\bar{k}_s = \beta + \bar{k}_i$ is the so-called Bragg condition.
II. Physical troposcatter mechanism

- Next, assuming that the antennas are linearly polarized in the plane of $R_1$, and replacing $\bar{f}_i(\theta)$, $i = 1, 2$, by their value at $\theta = 0$, the integral

$$\times \int_{V_c} |\bar{f}_1(\theta_{1c}) \cdot \bar{f}_2(\theta_{2c})|^2 d\rho_c$$

(29)

...can be approximated as

$$|\bar{f}_1(0)|^2 |\bar{f}_2(0)|^2 V_c \cos^2 \psi$$

(30)

where $\psi$ is the angle shown in the last figure.

- Further, we can relate $|\bar{f}_i(0)|^2$ with the respective antenna gain $G_i$ as

$$G_i = \frac{k_0^2 |\bar{f}_i(0)|^2}{\pi \eta R_L}$$

(31)
Finally, using the fact that $P_t = R_L |I_1|^2$, we can write

$$P_r = \frac{P_t k_0^2 S(\overline{k}_s - \overline{k}_i)}{4\pi^2 d^4} G_1 G_2 V_c \cos^2 \psi$$  \hspace{1cm} (32)$$

where we have used $R_1 = R_2 = d/2$.

The following estimates are available:

$$S(\overline{\beta}) = (3.3 \times 10^{-2} C_n^2) 32\pi^3 \beta^{11/3}$$  \hspace{1cm} (33)$$
in m^3 and

$$V_c = \frac{d^3 \theta_1^3}{\psi^{1/2}}$$  \hspace{1cm} (34)$$

with $\theta_1/2$ being the antenna half-power beamwidth, $\psi = d/a_e$, where $a_e$ is the equivalent earth radius (5,280 mi) and $C_n$ is the so-called structure constant for the permittivity fluctuations. Typically, $C_n$ ranges from $5 \times 10^{-7}$ to $2 \times 10^{-8}$ m$^{-1/3}$. 
II. Physical troposcatter mechanism

- The magnitude of $\beta$ is $2k_0 \sin \psi / 2$, from the geometry of the problem.
- Since $\psi$ is very small, $2k_0 \sin \psi / 2 \approx k_0 \psi$.
- The final expression for $P_r$ becomes:

$$P_r = (1.8 \times 10^{32}) k_o^{-5/3} P_t G_1 G_2 C_n^2 \cos^2 \psi \frac{\theta_1^3}{d^{17/3}} \theta_1^{3/2} \quad (35)$$

- Note that this is a theoretical estimate that may or may not match well the conditions at a specific link site.
- It provides valuable insight into the expected behavior of the receive power as a function of the link parameters.
III. Troposcatter empirical path loss model

- Due to the difficulty in obtaining very accurate, real-time data about the local structure constant and the spatial spectrum, empirical models are widely used instead.
- ITU-R Recommendation P.617-3 provides an empirical estimate of the annual median path loss for troposcatter links. This estimate is based on data from 200 MHz to 4 GHz, and it is written in decibels as:

  \[ L_p = 30 \log_{10} f + 10 \log_{10} d + 30 \log_{10} \theta + N(H, h) + L_{coup} + M \]  

  where \( f \) is the frequency in MHz.

- The path length \( d \) in kilometers, and the scatter angle \( \theta \) expressed in milliradians (mrad). They are indicated in the Figure of next page.

- Here, the term “path loss” \( L_p \) has the same meaning as used elsewhere in the book, e.g. in Chapter 8.
Figure: Geometric parameters used for troposcatter empirical path loss model in ITU-R Rec. P.617-3
III. Troposcatter empirical path loss model

- The antenna-to-medium coupling loss $L_{\text{coup}}$ is given by
  \[
  L_{\text{coup}} = 0.07 e^{0.055(G_T + G_R)},
  \]
  (37)
  where $G_T$ and $G_R$ are the antenna gains in dBi.

- The term $N(H, h)$ also depends on the scatter angle and is given by
  \[
  N(H, h) = 20 \log_{10} (5 + \gamma H) + 4.34 \gamma h,
  \]
  (38)
  where $H = 2.5 \times 10^{-4} \theta d$ and $h = 1.25 \times 10^{-7} \theta^2 R_e$, with $\theta$ and $d$ in mrad and km, respectively, and where $R_e = \kappa a$ is the effective Earth radius for median conditions (Ch. 6) in km.
This empirical model incorporates two structure parameters. The meteorological structure parameter $M$ depends on the local climate and varies between about 20 and 40 dB. The atmospheric structure parameter $\gamma$ also depends on the climate and assumes values in the range $0.3 \pm 0.03 \text{ km}^{-1}$. A table with values for $M$ and $\gamma$ in different climates can be found in the ITU Rec. P.617-3.

The above empirical model provides an estimate for the annual median path loss. Sometimes, a similar estimate can be found for the “worst month.”

In practice, the path loss in troposcatter links shows both daily and seasonal variations.

Daily variations are caused by changes in tropospheric conditions such as pressure, moisture, and temperature. Typically, but not always, the path loss tends to be larger in the afternoon, and decreases at night and in the morning.
Similarly, seasonal variations occur as these conditions vary according to annual cycles. These seasonal variations depend on the local climate. In temperate climates, the path loss is larger during the winter; in desert climates the path loss is larger during the summer. Path losses in equatorial climates have less seasonal variation.

In hot, dry climates attenuation reaches a maximum in the summer. The annual variations of the monthly medians can exceed 20 dB, while the diurnal variations are very large.

Coastal paths or path over the sea can be further affected by ducting phenomena, and hence have larger variation.
IV. Troposcatter fading models

- Daily and seasonal variations correspond to a **slow fading** of the signal in decibels that can be described statistically in terms of a **Gaussian distribution**, as seen in Chapter 8. Equivalently, for the received signal in Watts, this slow fading is described statistically as a log-normal distribution.

- The variance of the slow fading tends to decrease as the path length increases because the common volume is then attained at higher altitudes, which show less daily variability in scatter properties than at lower altitudes. For a link with $d = 80$ km, the slow fading standard deviation is typically about 30 dB, whereas for $d = 500$ km, it decreases to about 10 dB.
Superimposed on the slow fading are faster variations in the signal that can reach up to 20 fades per second. This fast fading is caused by fluctuations in the number, orientation, and position of the tropospheric fluctuations in the common volume, and is described statistically by a Rayleigh distribution for the received signal strength in V/m or, equivalently, by a exponential distribution for the power in the received signal.

This statistical behavior is the same as that seen in Chapter 8 for the fast fading in non-line-of-sight (NLOS) links in environments with strong multipath (rich scattering).
IV. Troposcatter fading models

- **Slow fading example:** Assume a 2 GHz troposcatter radio system from an oil rig over the sea to a land base. Assuming that the mean path loss is 146 dB, estimate the probability that the hourly mean path loss will exceed 161 dB, assuming that the standard deviation of the distribution is 7.5 dB.

- **Solution:** We are provided the values of path loss in dB, which follow a Gaussian distribution for slow fades. This type of fading is assumed here because we are asked about an hourly mean path loss. This slow fade is equal to two standard deviations (given value), which, for a Gaussian distribution, gives a probability of being exceeded equal to 2.3%.
IV. Troposcatter fading models

- **Fast fading example:** During a short period of time, the mean value of the received power via a troposcatter link is -80 dBW. What is the level of the received power that is exceeded 99.99% of the time?

- **Solution:** The fast fades obey a exponential distribution in terms of power. For an 0.01% occurrence, or a probability occurrence of $10^{-4}$, the power level would be approximately 40 dB below the mean, or -120 dBW.
Because fading in troposcatter links can be severe, **diversity schemes** become necessary.

Spatial, frequency, or angle (where multiple antenna feeds spaced in the vertical direction are used in a common reflector, creating different vertically-spaced common volumes) diversity can be used to combat troposcatter fading.

Mostly applicable to fast fading.

We will consider frequency and space diversity next.
V. Diversity schemes for troposcatter

- For space diversity, the vertical or horizontal separation between antennas should ideally be larger than the (spatial) correlation length.
- The correlation length can be estimated as:

\[ D_h = \frac{3\lambda a_e}{4d} \]  

(39)

where \( \lambda \) is the wavelength, \( a_e \) is the equivalent Earth radius, and \( d \) is the horizontal path distance.
- Many troposcatter systems utilize the simple rule \( D_h \approx 100\lambda \). So, for a 2 GHz link for example, one would have \( D_h = 15 \text{ m} \).
V. Diversity schemes for troposcatter

- For frequency diversity, one has to take into consideration the frequency correlation factor $\rho_f$. The latter can be approximated as

$$\rho_f = e^{-2\pi \sigma_f \Delta_f^2}$$

(40)

where $\Delta_f$ is the frequency separation, and

$$\sigma_f = \frac{2l \sin \frac{\theta}{2}}{c}$$

(41)

where $c$ is the speed of light, $\theta$ is the angle of scatter as defined before, $\theta = \theta_0 + \theta_1 + \theta_2$, and $l$ is the standard deviation of linear dimension of the scatter volume, which also depends on meteorological conditions.

- For the sake of illustration, $\Delta_f = 3$ MHz would typically suffice for a link of about 350 km at 2 GHz to produce $\rho_f < 0.6$, with a 10m transmit dish antenna and a 3m receive dish antenna.
V. Diversity schemes for troposcatter

- Recommendation ITU.R P-617.3 provides some practical guidelines to produce adequate space and frequency diversity.

- For frequencies greater than 1 GHz, the empirical guidelines for the horizontal $\Delta h$ and vertical $\Delta v$ antenna spacing in meters are

$$\Delta_{h,v} = 0.36 \sqrt{D^2 + 4l_{h,v}^2}$$  \hspace{1cm} (42)

where $D$ is the antenna diameter in meters, $l_h = 20 \text{ m}$, and $l_v = 15$ are scale length factors for the horizontal and vertical spacings, resp.

- For frequency diversity, the separation between carriers above 1 GHz can be estimated in MHz as

$$\Delta f = 1.44 \frac{f}{\theta d} \sqrt{D^2 + l_v^2}$$  \hspace{1cm} (43)

where $f$ is the (averaged) carrier frequency in MHz, $\theta$ is again the scatter angle, and $D$ and $l_v$ are defined as above.
V. Diversity schemes for troposcatter

- **Diversity example:** Assume 2-fold diversity by say, space-diversity by using 2 antennas spaced sufficiently apart at the receiver site.

  Assume further, for simplicity, that the received signals at the 2 antenna terminals have zero correlation, and that the diversity scheme selects the maximum signal as the output. Considering the last example, the probability that the output signal will drop below -120 dBW is now the product of the probabilities of the 2 antenna signals dropping as much. This is equal to \(10^{-4} \times 10^{-4} = 10^{-8}\), a much smaller probability. Thus, a much more reliable link can be obtained.

- A difficulty, however, is that (a) independence between received signals typically cannot be assumed in practice and (b) the troposcatter link with diversity has three sets of simultaneous distributions: Gaussian for slow fades, exponential (or Rayleigh) for fast fades, and a third one for the diversity combining system. Such combined statistics are beyond our scope here.
Because the median path loss increases as the scatter angle $\theta$ increases, the antennas in a troposcatter link are generally positioned to minimize the scatter angle and maximize the common volume.

The antenna main beam are directed slightly above the horizon. The optimum elevation angle depends on the atmospheric conditions and path geometry, but it typically lies at about 0.2 to 0.6 beamwidths above the horizon.

Note that tropospheric scattering can be a source of interference in other communication systems. In this case, the common volume can arise even from the intersection of the sidelobes of two antennas, or the sidelobe of one antenna and the mainlobe of a second antenna.

Further information is available in the Recommendation ITU-R F.1106 and ITU-R P.617-3