On Market-Based Coordination of Thermostatically Controlled Loads With User Preference

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Abstract—This paper presents a market-based control framework to coordinate a group of Thermostatically Controlled Loads (TCL) to achieve system-level objectives with price incentives. The problem is formulated as maximizing the social welfare subject to a feeder power constraint. It allows the coordinator to affect the aggregated power of a group of dynamical systems, and creates an interactive market where the users and the coordinator cooperatively determine the optimal energy allocation and energy price. The optimal pricing strategy is derived, which maximizes social welfare while respecting the feeder power constraint. The bidding strategy is also designed for the coordinator to compute the optimal price based on local device information. Numerical simulations based on realistic price and model data are performed. The simulation results demonstrate that the proposed approach can effectively maximize the social welfare and reduce power congestion at key times.

I. INTRODUCTION

As demand response is becoming increasingly important, various control schemes have been developed to engage responsive loads in demand response programs to maintain the reliability of the power grid. Among these control frameworks, market-based coordination has attracted considerable research attention. It borrows ideas from microeconomics to solve various engineering problems, such as communication networks [1], multi-commodity flow problems [2], energy management problems [3], [4], etc. Inheriting from economics, the market-based approach is amenable to problems where self-interested users are coordinated to achieve social efficiency. For example, a market-based approach is proposed in [5], [6] to fairly allocate thermal resources among offices within a building, ensuring that all the offices are at the same comfort level. Using this thermal resource allocation problem as an example, [7] performed a comparative study between the market-based coordination and centralized control, which yields insights about the benefits and limitations of the market-based approach. Among existing literature, the most relevant work to our paper is reported in [8] and [9], where a distributed algorithm is presented for the utility company and users to jointly determine optimal prices and demand schedules.

This paper presents a market-based coordination framework, where a group of TCLs are coordinated to achieve system-level objectives with price incentives. The proposed framework is different from most existing literature with the following key features. First, our framework incorporates realistic internal dynamics of TCLs. Such load dynamics affect the actual load response to price and thus complicate the market clearing process. Second, it allows for the users to indicate their preferences regarding how the TCL temperature setpoints respond to the market clearing price. Third, for many existing works, such as [8] and [9], an optimal price can be found to align individual optimality with social optimality. This property does not hold in general when the feeder power constraint is imposed on the system. Fourth, the coordination algorithms in the literature ([8], [9]) often require multiple iterations between the devices and market coordinator for each market clearing cycle, which demands considerable communication resources, while in our framework each device only needs to bid once during each market clearing cycle.

In our framework, an optimal pricing strategy is derived, which maximizes the social welfare subject to a feeder power constraint. A device bidding strategy is also proposed to enable the coordinator to compute the optimal price only based on online bidding information. The effectiveness of the proposed approach is demonstrated via a number of simulations based on realistic models of residential air conditioning loads obtained from GridLAB-D. Once properly implemented, the proposed framework can effectively cap the aggregated power below the feeder capacity and maximize the social welfare.

The rest of the paper proceeds as follows. An example is introduced in Section II, which motivates the coordination framework proposed in Section III. In Section IV the optimal pricing strategy and the corresponding bidding strategy is proposed. Simulation results are shown in Section V to demonstrate the efficacy of the proposed framework. Some concluding remarks are given Section VI.

II. MOTIVATING EXAMPLE

The problem considered in this paper is largely motivated by the Pacific Northwest GridWise™ demonstration project [10], where a group of TCLs are managed by a coordinator to cap peak aggregated power and maximize social welfare. Before each market clearing cycle, each device needs to measure its current room temperature $T_c$ and submit a bid to the coordinator. Each bid consists of a price $P_{bid}$ and the power of the most recent period during which the load is on. The bidding price is determined according to the bidding curve, as shown in Fig. 1, where $P_{avg}$ is the average price of the past 24 hours, $\sigma$ is the standard variation.
of the price of the past 24 hours, and $T_{\text{min}}$, $T_{\text{max}}$ and $T_{\text{desired}}$ are user-specified minimum, maximum and desired temperature, respectively. In addition, the user can specify the user preferences $K$ (Fig. 1) through a smart thermostat interface, which determines the slope of the bidding curve.

Before each market clearing cycle, the coordinator collects all the bids from the devices, and orders the bidding price in a decreasing sequence $P_{\text{bid}}^1, \ldots, P_{\text{bid}}^N$, where $N$ denotes the number of users. With the price sequence and the associated bidding power sequence $Q_{\text{bid}}^1, \ldots, Q_{\text{bid}}^N$, the coordinator can construct the demand curve that maps the market energy price to the aggregated power. Fig. 2 shows how the demand curve is constructed. Using the demand curve, the coordinator can clear the market and determine the energy price to ensure that the aggregated power does not exceed the feeder capacity: if the total power demand is less than the feeder power constraint, then the clearing price is equal to the base price $P_{\text{base}}$ (Fig. 3), which is the wholesale market energy price; otherwise, the clearing price $P_e$ is determined by the intersection of demand curve and the feeder power constraint (as shown in Fig. 4).

After the market is cleared, each device receives the energy price $P_e$ and determines the control actions according to a response curve, which maps the market price $P_e$ to the TCL temperature setpoint $T_{\text{set}}$, as illustrated in Fig. 5. At the end of the market clearing cycle, the device measures the room temperature, submits the bids for the next cycle, and repeats this whole process. Notice that both the response and bidding processes are automatically executed by a programmable controller. The user only needs to specify the user preference via a smart thermostat interface. In addition, to initiate the framework, the user needs to specify $T_{\text{min}}$, $T_{\text{max}}$, $T_{\text{desired}}$ and $K$, the device needs to measure the temperature, on/off state and the power of the most recent cycle during which the load is on, while the coordinator needs to collect all the bids, compute the power of unresponsive loads $Q_{\text{uc}}$ and the power feeder constraint $Q_{\text{lim}}$.

Aside from the GridWise project, another demonstration project is implemented in AEP, Ohio [11], which makes a few modifications but follows the same overall principle. These projects provide a real-world demonstration for the market-based coordination of TCL systems from the practical point of view. In this paper we address the problem from a different angle, and develop a general framework to systematically design and analyze this kind of market-based coordination of demand response assets.

III. A MARKET-BASED COORDINATION FRAMEWORK

This section considers a double auction market that is cleared every $T$ units of time, and provides the formal mathematical description of the coordination problem.

A. Individual Load Dynamics

Let $f_{\text{on}}^i$ and $f_{\text{off}}^i : \mathbb{R}^n \times \mathbb{R}^{n_{\text{on}}^i} \rightarrow \mathbb{R}^n$ denote the dynamics of the $i$th TCL for on and off state, respectively. Let $z_i(t)$ be the continuous state of load $i$. Denote $q_i(t) \in \mathbb{R}$ as the on/off state: $q_i(t) = 0$ when the TCL is off, and $q_i(t) = 1$ when it is on. The system dynamics are given as follows:

$$
\dot{z}_i(t) = \begin{cases} 
 f_{\text{on}}^i(z_i(t); \theta_{m}^i) & \text{if } q_i(t) = 1 \\
 f_{\text{off}}^i(z_i(t); \theta_{t}^i) & \text{if } q_i(t) = 0 
\end{cases}
$$

(1)
where $\theta^m_i \in \mathbb{R}^{n_m}$ is the model parameter of the $i$th TCL. In the example of the demonstration project, the thermal dynamics can be captured by the second-order Equivalent Thermal Parameter (ETP) model [12]:

$$z_i(t) = A_i z_i(t) + B^i_{on}(B^i_{off})$$

where $z_i(t)$ consists of the room temperature $T_i^c(t)$ and the inner mass temperature, and the model parameters include $A_i, B^i_{on}$ and $B^i_{off}$, i.e., $\theta^m_i = [A_i, B^i_{on}, B^i_{off}]^T$.

The power state of the TCL is typically regulated by a hysteretic controller based on the control deadband $[u_i(t) - \delta/2, u_i(t) + \delta/2]$, where $u_i(t)$ is the temperature setpoint of the $i$th TCL and $\delta$ is the deadband. In air conditioning mode, the controller turns off the system when $T_i^c(t) \leq u_i(t) - \delta/2$, turns on the system when $T_i^c(t) \geq u_i(t) + \delta/2$, and remains in the same power state otherwise. This control policy can be represented as follows:

$$q_i(t^+) = \begin{cases} 
1 & \text{if } T_i^c(t) \geq u_i(t) + \delta/2 \\
0 & \text{if } T_i^c(t) \leq u_i(t) - \delta/2 \\
q_i(t) & \text{otherwise}
\end{cases}$$

For notational convenience, define a hybrid state as $x_i(t) = (q_i(t), z_i(t))^T$ for the $i$th TCL. Let $[t_k, t_k + T]$ be the $k$th market clearing cycle. The energy consumption of the TCL during each market clearing cycle depends on the current state and the control. Given a state $x_i(t_k)$ and the constant setpoint control $u_i(t_k)$, we can easily estimate the portion of time the system is on during a market cycle, and hence derive the energy consumption $e_i(x_i(t_k), u_i(t_k))$ of the $i$th TCL based on the system dynamics (1) and control strategy (3), where $e_i : \mathbb{R}^{n+1} \times \mathbb{R} \rightarrow \mathbb{R}$. An example of the function $e_i$ using model (2) is shown in Fig. 6. For notation convenience, let $a_i$ represent the energy allocation of $i$th user, i.e., $a_i = e_i(x_i(t_k), u_i(t_k))$.

### B. User Preferences and Valuation

Each user responds to the price by adjusting the control setpoint. This can be represented as a response function $g : \mathbb{R} \rightarrow \mathbb{R}^m$ that is parameterized by $\theta^u_i$, i.e., $u_i(t_k) = g(P_i; \theta^u_i)$. The user response curve inherently reflects the trade-off between comfort and cost, e.g. when energy price $P_c$ is relatively high, the device will adjust control $u_i(t_k)$ to reduce power consumption. Therefore, the user response can be viewed as being obtained by solving an optimization problem to maximize the individual utility: user comfort minus energy cost. For this reason, we define a valuation (utility) function $V_i : \mathbb{R} \times \mathbb{R}^{n+1} \rightarrow \mathbb{R}$ to represent the $i$th user’s valuation (comfort) over $a_i$ units of energy allocation. We assume that $V_i$ is concave, continuously differentiable, $V_i(0) = 0$ and $V’(0) > 0$. Let $E^m_i$ be the total energy if the $i$th device is on during the entire cycle, then the user response can be fully captured by the following optimization problem:

$$\max_{a_i} V_i(a_i, x_i(t_k); \theta^u_i) - Pa_i$$

subject to:

$$0 \leq a_i \leq E^m_i$$

where $P$ is the energy price variable. Denote the optimal solution to problem (4) as:

$$h_i(P, x_i(t_k)) = \arg \max_{0 \leq a_i \leq E^m_i} V_i(a_i, x_i(t_k); \theta^u_i) - Pa_i$$

An example of $h_i$ in the demonstration project is shown in Fig. 7. For notation convenience, in the rest of this paper, we drop the dependence of the response curve $g(P; \theta^u_i)$ and the valuation function $V_i(a_i, x_i(t_k); \theta^u_i)$ on the user preference parameter $\theta^u_i$, and denote them as $g_i(P)$ and $V_i(a_i, x_i(t_k))$, respectively.

### C. Bidding and Pricing Strategy

As the valuation functions are private information, the coordinator requires each user to submit a bid for collective decision making. Let $b_i : \mathbb{R}^{n+1} \times \mathbb{R} \times \mathbb{R}^{n_m} \rightarrow \mathbb{R}^n$ be the bidding function that maps the system state $x_i(t)$, user input $\theta^u_i$ and model parameter $\theta^m_i$ to a bidding vector. In the motivating example, the bidding vector includes the bidding price $P^i_{bid}$ and the bidding power $Q^i_{bid}$. These bids can be denoted as $b(x(t_k), \theta^u_i, \theta^m_i) = [b_1(x_1(t_k), \theta^u_1, \theta^m_1), \ldots, b_N(x_N(t_k), \theta^u_N, \theta^m_N)]^T$, and a pricing strategy $w$ can be defined to map the bidding collection to the energy price, i.e. $w : \mathcal{B} \rightarrow \mathbb{R}$, where $\mathcal{B}$ is the set of all the feasible bid collections. In the motivating example, the price is determined according to the demand curve constructed by bidding collection, which is shown in Fig. 3 and Fig. 4.

### D. Market-Based Coordination Framework

Based on preceding discussion, the coordinator collects all the user bids and clears the market to maximize the social welfare. To formulate the overall problem, we adopt a step-by-step approach, where we first start with a simple energy allocation problem, and then gradually add key components until it fully captures all the features of our problem.

As a starting point, let us first consider the following constrained optimization problem:
Problem 1: Find the optimal energy allocation to maximize social welfare subject to a feeder power constraint:
\[
\max_a \sum_{i=1}^{N} V_i(a_i, x(t_k)) - C \left( \sum_{i=1}^{N} a_i \right) \quad (7)
\]
subject to
\[
\begin{align*}
\sum_{i=1}^{N} a_i & \leq D \\
0 & \leq a_i \leq E_i^m, \quad \forall i = 1, \ldots, N \quad (8a)
\end{align*}
\]
where \( D = B \cdot T \), \( B \) is the feeder capacity and \( C \) is the cost for the coordinator to purchase the energy from the wholesale market.

Remark 1: This is a simple convex optimization problem, and gives an optimal energy allocation that maximizes the social welfare. However, an important concern is that the energy allocation vector has \( N \) degrees of freedom, while the coordinator can only determine the single-valued price. Therefore, there is no guarantee that the optimal energy allocation can be realized through pricing.

Example 1: As a counterexample, consider two users with \( V_1 = a_1, V_2 = 3a_2 \) and \( C(a_1 + a_2) = 2a_1 + 2a_2 \). Assume that \( D = 1 \) and \( E_i^m = 2 \). The optimal solution to Problem 1 is \( a_1 = 1, a_2 = 1 \). However, according to (4), given any energy price, \( a_i \) is either 0 or 2. Therefore, the optimal energy allocation can not be achieved via pricing.

To address this issue, we introduce the concept of implementable energy allocation:

Definition 1: The energy allocation vector \( a = [a_1, \ldots, a_N] \) is called implementable if there exists a price \( P \), such that \( a_i = h_i(P, x(t_k)) \) for all \( i = 1, \ldots, N \). In this case, we say \( P \) implements the energy allocation \( a \) in the \( k \)th cycle.

With the above definition, the set of the implementable energy allocation can be defined as \( \mathcal{A}_k = \{ a \mid P, s.t. a_i = h_i(P, x(t_k)) \}, \forall i = 1, \ldots, N \}. \) Now we look at the energy allocation problem considering implementable allocation and the feeder power constraint:

Problem 2: Find the optimal implementable energy allocation to maximize social welfare subject to a feeder power constraint:
\[
\max_a \sum_{i=1}^{N} V_i(a_i, x(t_k)) - C \left( \sum_{i=1}^{N} a_i \right) \quad (9)
\]
subject to:
\[
\begin{align*}
\sum_{i=1}^{N} a_i & \leq D \\
0 & \leq a_i \leq E_i^m, \forall i = 1, \ldots, N
\end{align*}
\]

Remark 2: The proposed framework is different from the wholesale energy market, as the internal dynamics of the TCLs are incorporated into the decision making. The energy price triggers the setpoint control, which modifies the system dynamics and affects the power consumption. Notice that this is the first step towards a fully dynamic version of the market-based coordination problem which maximizes the social welfare over multiple periods.

IV. Optimal Pricing and Bidding

In this section, we derive the optimal pricing strategy and the corresponding bidding strategy in two steps. First, we assume that the coordinator has the complete information and derive an analytic expression of the optimal price. Second, the complete information assumption is removed, and a bidding strategy is proposed to implement the optimal pricing strategy in a decentralized manner.

A. Pricing Strategy with Complete Information

Throughout this subsection, we neglect the function \( V_i \) and \( h_i \)'s dependence on the system state \( x(t_k) \), as it does not affect our result. In all other sections, we consider this dependence unless otherwise stated.

To derive the optimal price to Problem 3, we first define a price \( P^* \), which is the price that implements the optimal solution of the following energy allocation problem:
\[
\max_a \sum_{i=1}^{N} V_i(a_i) - C \left( \sum_{i=1}^{N} a_i \right) \quad (10)
\]
subject to:
\[
0 \leq a_i \leq E_i^m, \forall i = 1, \ldots, N
\]

Notice that the energy allocation problem (10) is reduced from Problem 1 by dropping the feeder power constraint (8a). According to the welfare theorem [13], the optimal price that implements the optimal energy allocation to (10) should be the marginal cost of energy. We summarize this result as the following proposition:

Proposition 1: Let \( a^* \) be the optimal solution of problem (10), then \( a^* \in \mathcal{A}_k, \) and \( P^* = C' \left( \sum_{i=1}^{N} a_i^* \right) \) implement \( a^* \).

Remark 3: The proof of Proposition 1 is similar with that in [9], where a game-based approach is developed to coordinate users for demand response. The result implies that the optimal energy allocation strategy to (10) is always implementable. However, this property does not hold when the coupled constraint (8a) is taken into consideration.

To find the optimal solution of Problem 2, we impose the constraint \( a \in \mathcal{A}_k \) on Problem 1 and check its cost function.

Problem 3: Design the bidding function \( h_i \), and determine the pricing strategy \( w \) such that \( w(b(x(t_k), \theta^u, \theta^m)) \) implements the optimal solution to Problem 2.

\[
U(P) = \sum_{i=1}^{N} V_i(h_i(P)) - C \left( \sum_{i=1}^{N} h_i(P) \right) \quad (11)
\]
Proposition 2: $U(P)$ is non-increasing with respect to $P$ when $P \geq P^*$. 

Proof: Since $U(P)$ is continuous, we only need to prove that $U'(P^+) \leq 0$, $\forall P \geq P^*$, where $U'(P^+)$ denotes the right derivative of function $U$ at $P$. As $U'(P^+) = \sum_{i=1}^{N}(V_i'(h_i(P)) - C'(\sum_{i=1}^{N}h_i(P))) \cdot h'_i(P^+)$, it suffices to show that $V_i'(h_i(P)) - C'(\sum_{i=1}^{N}h_i(P))) \cdot h'_i(P^+) \leq 0$ for all $i$. For notation convenience, let $\gamma_i(P) = V'_i(h_i(P)) - C'(\sum_{i=1}^{N}h_i(P)))$, we need to prove $\gamma_i(P) + h'_i(P^+) \leq 0$ for $\forall i$ when $P \geq P^*$. Now we divide all the users into two groups. The user in the first group satisfies $\gamma_i(P^+) \geq 0$. As $V_i$ is concave, $C$ is convex, and $h_i$ is non-increasing, $\gamma_i(P)$ is non-decreasing. Therefore, $\gamma_i(P) \geq \gamma_i(P^+) \geq 0$, which indicates that $\gamma_i(P) \cdot h'_i(P^+) \leq 0$. For the second group, we have $\gamma_i(P^+) < 0$. Note that $\gamma_i(P^*)$ is the derivative of (4) with respect to $a_i$ evaluated at the optimal point $h_i(P^*)$ when $P = P^*$. As (4) is concave and differentiable, $\gamma_i(P^*) < 0$ indicates that the optimal solution of (4) is on the boundary of the constraint: $h_i(P^*) = 0$. Moreover, since $h_i(P^*)$ is non-increasing, $P > P^*$, $h_i(P) = 0$, which indicates that $h_i(P^+) = 0$ for $P > P^*$. In addition, $P^* < P$. Therefore, $\gamma_i(P) \cdot h'_i(P^+) = 0$ for $i$ in the second group. This completes the proof. 

Furthermore, we define $\hat{P}$ as the solution to $\sum_{i=1}^{N}h_i(P) = D$. The existence and uniqueness of $\hat{P}$ is guaranteed as long as the function $h_i$ is non-increasing. Then we have the following result: 

Theorem 1: Let $P^*_c$ be the price that implements the optimal solution of Problem 2, then $P^*_c = \max\{P, P^*\}$. 

Proof: First, let us consider the case where $P^* \geq P$. As $\hat{P}(P)$ is non-increasing with respect to $P$, it is clear that $P^* \geq P$ indicates $\sum_{i=1}^{N}h_i(P^*) \leq D$. In addition, according to Proposition 1, $\hat{P}$ implements an $a^*$ such that $a^* \in \mathcal{I}_k$. With above conditions satisfied, Problem 1 and Problem 2 are equivalent, which indicates $P^*_c = P^*$. Second, let us consider the case where $P^* < P$. As $\hat{P}$ is non-increasing, the feeder capacity constraint is satisfied if and only if $P \geq P^*$. Moreover, Proposition 2 guarantees that the utility function of Problem 2 is non-increasing with respect to $P$ when $P \geq P^*$. Therefore, $P^*_c = \hat{P}$. This completes the proof. 

Theorem 1 indicates that the optimal price can be determined by a comparison between $P^*$ and $\hat{P}$. However, the computation of this optimal price requires global information, which is often not available to the coordinator. Therefore, a bidding strategy is presented in the next subsection to collect device information and compute the optimal price. 

B. Bidding Strategy Design

In this subsection we first present a general bidding strategy for the coordination of TCLs, then apply the bidding strategy to the air conditioning systems in the GridWise demonstration project. 

1) General Bidding Design: Ideally, each device can submit all the private information to the coordinator, including valuation function $V_i$, system state $x_i(t_k)$, model parameter $\theta^u_i$, and the user input $\theta^b_i$. Then the coordinator can easily solve Problem 3 and derive the optimal price. However, this bidding strategy is difficult to implement in practice due to computational and communicational limitations. To address this issue, we define a sufficient bidding as follows: 

Definition 2: A bidding vector $b(x(t_k), \theta^u, \theta^m)$ is a sufficient bidding, if the coordinator can compute the optimal price $P_c$ merely based on $b(x(t_k), \theta^u, \theta^m)$. 

It is clear that if the users bid all the private information, then we have a sufficient bidding. To derive a realistic sufficient bidding, we refer to (6), and assume that the energy function $e_i(x(t_k), g_i(\cdot))$ can be characterized by a parameter vector $\theta^b_i$, i.e., the structure of the energy function for each TCL is the same, while their differences can be captured by $\theta^b_i$. This assumption is justified because the TCLs can be captured by the same ETP model and regulated by the same hysteretic controller (3). Therefore, the energy function can be defined as: 

$$h_i(\cdot, x_i(t_k)) = e_i(x_i(t_k), g_i(\cdot)) \triangleq \tilde{c}(\cdot; \theta^b_i)$$ 

where $\tilde{c}$ is the energy versus price function parameterized by $\theta^b_i$. Now we have the following result: 

Theorem 2: Regard $e_i(x_i(t_k), g_i(P))$ as a function of price $P$ and assume this function can be parameterized by $\theta^b_i$, i.e., $e_i(x_i(t_k), g_i(P)) = \tilde{c}(P; \theta^b_i)$. Then $[\theta^b_1, \ldots, \theta^b_N]^T$ is a sufficient bidding. 

Proof: According to Theorem 1, $P_c = \max\{\hat{P}, P^*\}$, where $\sum_{i=1}^{N}h_i(P, x_i(t_k)) = D$ and $P^* = C'(\sum_{i=1}^{N}a^*_i)$. As $P^*$ implements $a^*$, we have $a^*_i = h_i(P^*, x_i(t_k))$ for $i = 1, \ldots, N$ and hence the following: 

$$\begin{cases} 
\sum_{i=1}^{N}h_i(P, x_i(t_k)) = D \\
P^* = C'(\sum_{i=1}^{N}h_i(P^*, x_i(t_k)))
\end{cases} $$ 

(12) 

Based on (12), the coordinator only needs to know the function $h_i(\cdot) \triangleq \sum_{i=1}^{N}h_i(\cdot, x_i(t_k))$ to compute the optimal price numerically. As $e_i(x_i(t_k), g_i(\cdot))$ can be parameterized by $\theta^b_i$, according to (6), $\theta^b_i$ contains all the information of function $h_i(\cdot, x_i(t_k))$. Therefore, $[\theta^b_1, \ldots, \theta^b_N]^T$ is a sufficient bidding. 

2) Application to GridWise Demonstration Project: In the GridWise demonstration project, the coordinator orders all the bids in decreasing order and clears the market based on the demand curve constructed in Fig. 2. This indicates the coordinator expects that for each device, if the market price
is higher than the bidding price, its bid is not cleared and the device will consume no energy during the next market clearing cycle; if the market price is lower than the bidding, then it wins the bid and consumes $Q_i^b \cdot \delta$ amount of energy, where $T$ is the length of the market clearing cycle. According to our general framework, such a bidding strategy can be viewed as using a step function to approximate the energy curve $h_i$. This methodology is illustrated in Fig. 8, where $P_i^{bid}$ denotes the bidding price, and $c_1$ and $c_2$ are computed based on $x_i(t_k)$, $\theta_i^b$ and $\theta_i^m$. For notation convenience, define $c_1 = c_1(u_1, \theta_i^b, \theta_i^m)$ and $c_2 = c_2(u_2, \theta_i^b, \theta_i^m)$, where $u_1$ and $u_2$ are the temperature setpoint control corresponding to $c_1$ and $c_2$, respectively. Using the second-order ETP model (2) and control policy (3), $u_1$ and $u_2$ for the $i$th device can be obtained as:

$$\begin{align*}
    u_1 &= T_i^c(t_k) + \delta / 2 \\
    u_2 &= LA_i^{-1} e^{A_i T_i} (A_i z_i(t_k) + B_{on}^i) - LA_i^{-1} B_{on}^i + \delta / 2
\end{align*}$$

(13)

where $L = [1, 0]$, and the power state of the $i$th TCL is on at $t_k$.

The positions of $c_1$ and $c_2$ are critical to the effectiveness of the bidding. Here we propose to bid the middle point of $c_1$ and $c_2$, which gives:

$$P_i^{bid} = \frac{c_1 + c_2}{2}$$

(14)

In this case, the sufficient bidding consists of the bidding price and bidding power, i.e., $[\theta_i^b, [P_i^{bid}, Q_i^{bid}]]$. After the coordinator collects all the bids, he can then construct function $h_i$ to compute the energy price.

Remark 4: The proposed bidding strategy assumes the knowledge of ETP model parameters $\theta_i^m$. In practice it may be difficult to derive these parameters. Our future work includes estimating the ETP model parameters by joint state and parameter estimation using extended Kalman filter or particle filter. In addition, the ETP model used in the framework may be inaccurate in terms of characterizing the energy consumption of TCLs. In practice the proposed bidding strategy may be affected by model errors, but this problem is out of the scope of this paper.

V. CASE STUDIES

In this section, we present some realistic simulation results to demonstrate the effectiveness of the proposed approach.

A. Simulation Setup

The proposed framework is validated in a 5-minute double auction market, where a second-order ETP model is used for the air conditioners. The ETP model parameters are determined by various building parameters, such as floor area, ceiling height, glass type, glazing layers and material, area per floor, etc. Realistic default values are used in GridLAB-D. For detailed description of these parameters and their relations to the ETP model parameter, please refer to [14]. In the simulation, we generate 1000 sets of building parameters. A few important parameters are randomly generated using the same approach in [15], and the rest take their default values in GridLAB-D. Throughout the simulation, we assume that the air conditioner consumes 5kW power on average. The power of the unresponsive loads is assumed to be 12MW, and the feeder power constraint is 15MW.

We use the weather data and the Typical Meteorological Year (TMY) data for Columbus, OH, obtained from [16], [17], which include the air temperature and the solar gain.
The energy price data is derived from PJM’s wholesale energy market [18] and modified to a retail rate in $/kWh plus a retail modifier as defined by AEP’s tariff [19].

B. Simulation Results

Different outside air temperature traces are used to validate the proposed framework. First, the simulation is performed using the outside air temperature record on August 20, 2009 in Columbus, OH. It covers a horizon of 24 hours, and the power trajectory is shown in Fig. 9 (average power during each market clearing cycle). When a market clearing price is given, the associated power response can be always found on the demand curve in Fig. 3 or Fig. 4, and we call it the cleared power. This cleared power is the coordinator’s estimation on the aggregated power based on all the user’s bids. Simulation results show that the cleared power accurately captures the real power trajectories, which implied that the proposed bidding strategy enables the coordinator to effectively construct the demand curve and make optimal pricing decisions.

The market clearing prices for the entire day are presented in Fig. 10. The average energy price during the entire day is 0.0728$/kWh. It can be observed that the energy prices are higher than the base price during congestion, which effectively caps the aggregated power at key times.

The demand curve and the market clearing process are shown in Fig. 11 and Fig. 12, which correspond to 08:20 AM and 12:30 PM, respectively. When the total power demand is less than the feeder power constraint, the clearing price is equal to the base price (Fig. 11), otherwise the clearing price is determined by the intersection of the demand curve and the power constraint curve, as shown in Fig. 12. Then we apply the energy price and the temperature record on August 16, 2009 in Columbus, OH. The power trajectory for this case is presented in Fig. 13. It is clear that the cleared power can accurately capture the real aggregated power, and the power can be effectively capped under the feeder power constraint. As the outside air temperature on August 16 is higher than that of August 20, more power congestion can be observed for this case.

Furthermore, to demonstrate the optimality of the proposed pricing strategy, we compare it with a base scenario in terms of social welfare. In the base scenario, when there is no congestion, the market clearing price is equal to the wholesale energy price. When the power congestion occurs, the clearing price is the wholesale price multiplied by a fixed ratio $\gamma$, which is greater than 1 to cap the aggregated power. Among all the possible ratios, we choose the minimum one that can effectively cap the aggregated power below the feeder capacity. In this simulation we apply the weather and price data on August 17, 2009, in which case $\gamma = 2.6$. The social welfare of the two pricing strategies during congestion is shown in Fig. 14. The simulation results demonstrate that the optimal pricing strategy improves the social welfare of the random pricing by 22.1% on average, and the proposed optimal pricing strategy always outperforms the base scenario in terms of social welfare.

VI. Conclusion

This paper presents a market-based coordination framework for thermostatically controlled loads, where a coordinator uses price incentives to manage a group of users under a given feeder power constraint. The optimal pricing strategy is derived, and the bidding strategy is also designed for the coordinator to compute the optimal price based on online bidding information. Simulation results are presented to validate the proposed framework. Future work includes formulating the fully dynamic market-based coordination framework with multiple periods and extending the results to other responsive loads such as plug-in electric vehicles, washers, dryers, among others.

REFERENCES