

# Trajectory Planning for Two Manipulators to Deform Flexible Materials Using Compliant Motion

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**Abstract**—Coordinating two robot manipulators to handle flexible materials has a wide range of applications in the manufacturing industry. However, this problem has not been seriously addressed until recently. The two robot manipulators have to follow complicated trajectories to maintain a minimum interaction force with the flexible beam. These trajectories are very complicated and not suitable for real time systems. A compliant motion scheme is proposed to reduce the interaction forces and moments. The stability of the proposed system is investigated. Experimental results encourage the proposed scheme.

## I. INTRODUCTION

**T**RAJECTORY planning for two robot manipulators to deform a flexible material has been addressed without taking into account the interaction between the robots and its environment. Recently, handling of non-rigid bodies has been performed indirectly by using special tools such as vacuum pad [1]. This places an extra loading effort on the end-effector.

Zheng and Chen [3] proposed the use of minimization of forces and moments exerted on the end-effectors as a criterion of the trajectory planning. They studied the trajectory planning of two manipulators to deform a flexible beam. Optimal motion trajectories were derived to generate zero interaction moments. While the beam is being deformed, forces and moments should be applied to the beam. For example, in the manufacturing of Printed Wiring Boards (PWB) (Fig. 1), a book is formed from a large number of elastic sheets and laminated to form a PWB. Some of the sheets are covered with electronic circuits and should be carefully aligned. For the alignment purpose, each sheet is equipped with four holes that should be aligned with four alignment pins on the assembly bed. It is difficult to align the four holes simultaneously because of the small tolerance between the pins and the holes. Thus, the manipulators should bend the sheet to align the middle holes with the corresponding pins. Then, they unfold the sheet to align the rest of the holes.

The optimal trajectories are complicated and not suitable for implementation in real time systems. Actually, the optimal trajectories required the evaluation of two elliptic integrations at each point [2][3]. Al-Jarrah and Zheng [4] investigated the effects of the approximation of

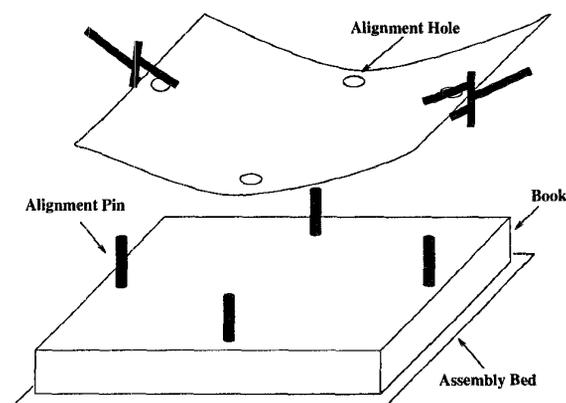


Fig. 1. Two end-effectors aligning a flexible sheet to form a PWB

the optimal trajectories on the end-effectors in terms of both forces and moments. They provided some efficient approximation methods of the optimal trajectories.

In this paper, we proposed a compliant control scheme to minimize the forces and moments exerted on the end-effectors. In the compliant motion, instead of rejecting or resisting the external forces, the manipulator should accommodate with them [5]. The compliant control scheme is attractive in this scope for more than one reasons. First, the nature of the bending process motivates this algorithm; when two persons try to bend a beam cooperatively, they will sense the forces and moments exerted on their hands, then modify the position and orientation by backing up if the force is large, or moving on if its fair. The second reason is the fact that the robots are equipped with an internal position control system that in most of times can not be accessed or modified to accept a force control method. Thus an external control loop should be designed to make use of the information about the interaction forces and provides a force control.

The compliant control was addressed by several researchers. The core of the compliance motion is the concept of mechanical impedance [6] in which both the dynamic behavior and the position of the manipulator are controlled. Kazerooni et al. [5] addressed the compliant motion in the frequency domain as a stabilizing dynamic compensator. In their approach, the relation between the interaction forces and end-effector position is constant over a certain frequency range. Force control

Acknowledgment: This work was supported by NSF under grant IRI-9405276 and by ONR under grant N00014-90-J-1516. †Visiting scholar from Hanyang University, Seoul, Korea.

can be effectively achieved using adaptive compliant control schemes [7]. Compliant motion can be used in the coordination of two robots to perform complicated tasks. Tao et al. [8] proposed the use of a compliant control to coordinate two robots operated in a master/slave mode. There are several application of the compliant motion in the teleoperations systems [9][10][11].

Stability analysis of several compliant control schemes have been addressed by several researchers. The stability of the system depends on the desired impedance parameters ranges. The impedance of the environment greatly affects the stability analysis of the system [12] [13]. Kazerooni [14] provided a nonlinear approach to analyze the stability of the compliant motion. His analysis is based on the small gain theorem and Nyquist criterion.

This paper is organized as follows. The next section reviews the optimal trajectories of the two end-effectors to bend a flexible beam and the piece-wise linear approximation of these trajectories. The compliant control scheme is presented in the third section. The stability of the system is addressed in the fourth section. Experimental investigation is presented in fifth section. Finally, the paper is concluded in the sixth section.

## II. OPTIMAL TRAJECTORIES

To deform a beam, we need either to change the relative position or to change both position and orientation of the two end-effectors [3]. The optimal trajectory will be when the two end points of the beam coincide with its zero moment (ZM) points (inflection point) and the deflection angle of the beam and the orientation of the two end-effectors are the same (see Fig. 2). Under these conditions, the end-effectors are exposed to minimum forces and moments. The minimum moments are zero since the end-effectors are holding the ZM points. The force is given by:

$$F = \frac{(2R(\alpha))^2 EI_z}{L^2}, \quad (1)$$

where  $E$  is the stiffness of the beam,  $I_z$  is the moment of inertia of the beam cross-section,  $L$  is the length of the beam,  $\alpha$  is the deflection angle of the beam (Fig. 2), and  $R(\alpha)$  is calculated from the following elliptic integral:

$$R(\alpha, \phi_1) = \int_0^{\phi_1} \frac{d\phi}{\sqrt{1 - m \sin^2 \phi}}, \quad (2)$$

when  $m = \sin^2(\frac{\alpha}{2})$  and  $\phi_1 = \frac{\pi}{2}$ .

The optimal trajectories are functions of the orientation of the end-effectors. In order to maintain the minimum forces and moments, the relation between the end-effector orientation and the position should satisfy:

$$x = L \left( \frac{P(\alpha)}{R(\alpha)} - \frac{1}{2} \right), \quad (3)$$

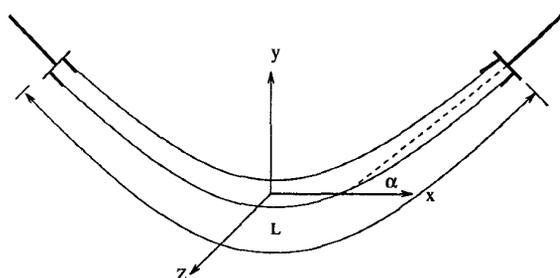


Fig. 2. Two Robots bending a flexible beam

and

$$y = \frac{L}{R(\alpha)} \sin\left(\frac{\alpha}{2}\right), \quad (4)$$

where  $P(\alpha)$  is calculated from another elliptic integral given by the following equation:

$$P(\alpha, \phi_1) = \int_0^{\phi_1} \sqrt{1 - m \sin^2 \phi} d\phi, \quad (5)$$

when  $m = \sin^2(\frac{\alpha}{2})$  and  $\phi_1 = \frac{\pi}{2}$ .

It is clear that we should calculate two elliptic integrals at each step of motion, which makes the process complicated and not suitable for real time applications. Moreover, for manipulators with point to point control, the behavior of the trajectories between points greatly affects the system performance.

A piece-wise linear approximation of these trajectories is a simple approach that simplifies these computation. However the resultant force will be much larger than that of the optimal ones and the moment is no longer zero [4]. By using piece-wise linear approximation, the target bending angle  $\alpha$  is divided into a number of smaller angles. The optimal trajectories are described in terms of the orientation of the end effectors. These trajectories are approximated by linear functions during the motion between any set of two points. To describe such trajectories, the desired bending angle  $\alpha$  is divided to  $N$  discrete points such that, for  $k = 1 \dots N$ ,

$$\alpha(k) = \frac{k}{N}, \quad (6)$$

and  $x(\alpha(k))$  and  $y(\alpha(k))$  are then evaluated using (3) and (4). The motion for  $x$ ,  $y$ , and  $\alpha$  in between the points is linear.

## III. COMPLIANT CONTROL

In a position controlled system, the interaction forces between the manipulator and the environment are rejected and treated as disturbances to the system. However, when the robot manipulator makes a contact with the environment, the forces exerted on the end-effector should be accommodated rather than resisted [5]. To accomplish this,

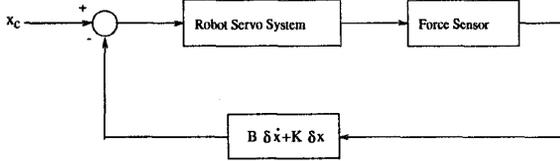


Fig. 3. Compliant control scheme

the dynamic behavior of the manipulator is modulated either by modifying the robot joints' servo gains or by using an external control loop that produces the same results [9]. The later has the advantage of high performance position tracking while keeping the force control in effect. Moreover, the transition from free motion to constraint one is done naturally without any additional cost.

In this paper, an external control loop is used such that the interaction forces drive a mechanical admittance to alter the commanded trajectories using a negative feedback scheme as shown in Fig. 3. If there is no contact between the manipulator and the environment, the system will precisely track the commanded position. If a contact is made, then the external feedback loop will be excited, and the robot will comply with the interaction forces by a deviation of  $\delta x$  in its position.

The relation between the deviation  $\delta x$  from the commanded position is given by:

$$B\delta\dot{x} + K\delta x = F, \quad (7)$$

where  $B$  is the friction of the damper, and  $K$  is the stiffness of the spring [9]. By applying Laplace transform on (7), we get :

$$(Bs + K)\delta x(s) = F(s). \quad (8)$$

The quantity  $(Bs + K)$  represents the mechanical impedance.

In the case of bending a flexible object, the forces and moments exerted on the end-effectors comprise the bending forces and moments and the gravitational effects. The manipulators should comply with the forces and moments due to the bending process and reject the effects of the gravity. If the manipulators comply with the gravity, the object will always moves down which is unacceptable. Thus, in our system  $\delta x$  is a vector that represents the deviation in the motion along the x-axis on and the rotation about the z-axis.

Consider the case where  $B$  and  $K$  are given by:

$$B = \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} \quad (9)$$

and

$$K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}. \quad (10)$$

Then, the compliant controller can be written as two separate filters; one for the motion along x-axis and one for the

rotation about z-axis. Under this assumption, we have:

$$\delta x = \frac{F}{b_1 s + k_1}, \quad (11)$$

and

$$\delta \alpha = \frac{M}{b_2 s + k_2}. \quad (12)$$

Let  $b$  represents both  $b_1$  and  $b_2$  and  $k$  represents both  $k_1$  and  $k_2$ . The impulse response of the filter for both force and moment is:

$$h(t) = \frac{1}{b} \exp\left(-\frac{k}{b}t\right). \quad (13)$$

A discrete filter of the same impulse response has the following transfer function:

$$H(z) = \frac{\frac{1}{b}z}{z - \exp\left(-\frac{k}{b}T\right)}, \quad (14)$$

where  $T$  is the sampling period. Thus, the discrete filter can be written as:

$$\delta x(nT) = \exp\left(-\frac{k}{b}T\right)\delta x((n-1)T) + \frac{1}{b}F(nT). \quad (15)$$

Consider an approximation of the optimal trajectories. The compliant controller alter this approximation according to (7) to reduce the forces and moments exerted on the end-effectors due to the bending process. The deviation due to the bending moment is added to the bending angle  $\alpha$ . Thus, if the moment is not zero, the deflection angle of the beam will increased and this reduces moment and forces exerted on the end-effectors [4]. The motion of the end-effectors along the x-axes are deviated from the approximated trajectories by adding the filter output to the x-component of the trajectories at each point. The later will result in reducing the approximation error and thus reducing the bending forces and moments.

#### IV. STABILITY ANALYSIS

The stability of the compliant motion scheme depends on many factors. The model of the robot positioning system is very important in the stability analysis. Most of the researchers used second order linear control systems to model the position of the end-effector as a function of the command [12][13][14].

We can model the end-effector positioning system along each axis as a second order system [13]. Let  $x$  be one of these components, then the end effector position  $x$  as a function of the desired position  $x_d$  (see Fig. 4) can be expressed as:

$$\frac{x(s)}{x_d(s)} = G(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}, \quad (16)$$

where  $w_n$  is the natural frequency and  $\xi$  is the damping ratio.

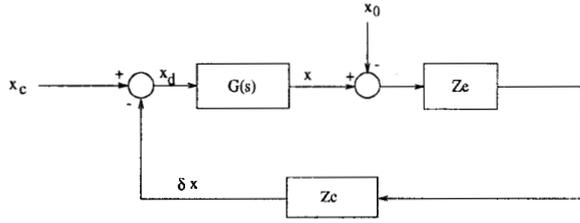


Fig. 4. Closed loop system block diagram

From Fig. 4, the relation between the desired position and the commanded one is:

$$x_d = x_c - \delta x. \quad (17)$$

Moreover, the interaction force is transformed into deflection using the compliant controller impedance  $Z_c$ . However, this force is related to the end-effector position by the environment impedance  $Z_e$ . Substituting in (17) we get:

$$x_d = x_c + \frac{Z_e(x(s) - x_0(s))}{Z_c}, \quad (18)$$

where  $x_0$  is the impedance center of the environment. substituting (18) in (16) we get:

$$\frac{x(s)}{x_c(s)} = \frac{G(s)}{1 + G \frac{Z_e}{Z_c}}. \quad (19)$$

To study the stability, we will assume that the environment impedance is dominated by the stiffness [7]. Thus, in our case we have :

$$Z_e = K_e, \quad (20)$$

and

$$Z_c = (bs + k). \quad (21)$$

Substituting the above in (19) we get:

$$\frac{x(s)}{x_c(s)} = \frac{w_n^2(bs + k)}{bs^3 + (2\xi w_n b + k)s^2 + (bw_n^2 + 2\xi w_n k)s + w_n^2(k_e + k)}. \quad (22)$$

This equation can be used to study the stability of the compliant motion scheme. Using Routh's stability criterion and assuming that the impedance parameters in (22) are not negative, we get:

$$2b^2\xi w_n^3 + 4\xi^2 w_n^2 bk + 2\xi w_n k^2 - w_n^2 bk_e > 0 \quad (23)$$

It is clear that the stability of the system for a given set of the impedance parameters ( $b, k$ ) depends on the stiffness of the environment. For two robots handling a flexible material, this stiffness is determined by the stiffness of the beam. If the stiffness of the beam is very large, then a large value of the controller stiffness is required. Also,

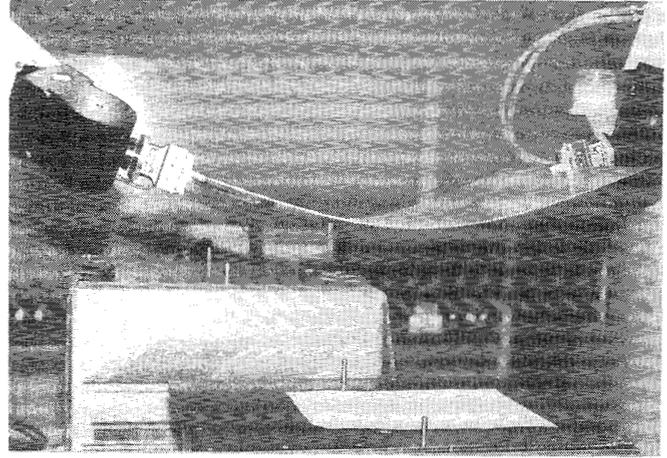


Fig. 5. Two PUMA robots align an aluminum sheet

generally, from (23), small values of the controller parameters, damping and stiffness, can result in instability. In our analysis of the stability, we neglected any time delay in the control loop. Analysis of the effect of time delay in the control loop can be found in [13].

In the experimental part of this paper, two PUMA 560 robots will be used. If we consider some values of the parameters in (16) we can get some limits on the choices of the parameters in the controller. The natural frequency of the PUMA 560 robot is not lower than 2Hz, and  $2\xi w_n \geq 25$  rad/sec [8]. Substituting these lower limits in (23) with the stiffness of the environment is estimated to be  $k_s = 11500$  N/mm we get:

$$k > 550, \quad (24)$$

and

$$b > 6.92. \quad (25)$$

As we will see in the next section, these limits are very close to that found experimentally.

## V. EXPERIMENTAL STUDY

The system consists of two PUMA 560 robots and a supervisor computer. One of the two robots is equipped with a multi-axis force/torque sensor (Intelligent Multi-axis Force/Torque Sensor System). The sensor is connected to a local controller that calculates the force/torque components from the raw data and provides a temperature compensation and a filter. The supervisor computer communicates with the sensor controller via a serial port.

The experiments (Fig. 5) resemble the production of PWB example as mentioned in the introduction of this paper. In both of the experiments the robots had to move to a pre-specified position and pick up the aluminum sheet. After that, the robots move to another specified position where the bending process occurred. The sheet is bent to make the two middle holes aligned with pins first. Finally, the robots extend the sheet to align the rest of the holes.

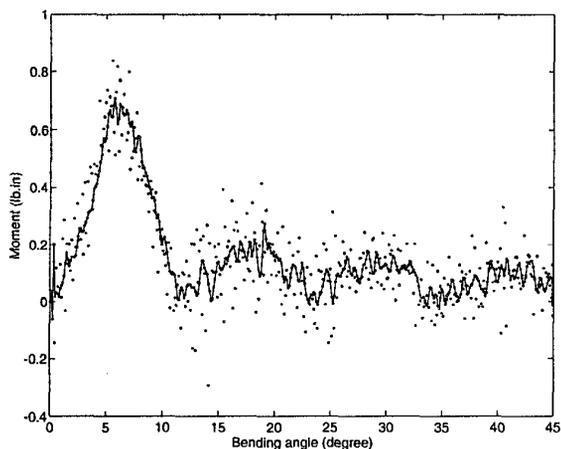


Fig. 6. Moment exerted on the end effector in a piece-wise linear approximation of the optimal trajectories

Since the moment will be zero in the optimal case, we will consider it as a criterion for the system performance. Because of the behavior of the bending force is similar to that of the bending moment [4], we will only consider the bending moment for presentation. We first run the system without compliant control. The used trajectories were obtained in [3]. However, to control the robot motion, we used a piece-wise linear approximation of the optimal trajectories. Thus, only at the end-points of every linear piece, the bending moment was zero. Between two end-points, the bending moment could be very high. Fig. 6 shows the results when the linear piece is a 0.2 radian interval. One can see that the bending moment is peaked between two end-points. The data from the force sensor is very noisy, this is due to the vibration of the end-effector during the robot motion. The MA robot is positionally controlled with each joint has its own controller. The joint processor receive a command every 28ms. Thus we can send a position command to joint servos every 28ms (ALTER mode in VAL II language). The gravitational effects, friction, and higher order dynamics start to have more effects as the frequency of the commanded position increases and explain the observed vibrations.

To get rid of the noise a low pass filter (LPF) with cutoff frequency of 10 Hz is added. The filter was first implemented using infinite impulse response (IIR) filter. However, this filter produced a phase lag which affected the stability of the compliant controller. Thus a finite impulse response (FIR) filter was used to smooth the noise.

The compliant control in (8) was implement using a digital equivalent (impulse invariant method). We added compliance in two directions. The first direction was a long the bending force, while the other was along the bending moment. The force sensor is mounted at a distance  $L_x$  From the center of the gripper (see Fig. 7). Thus,

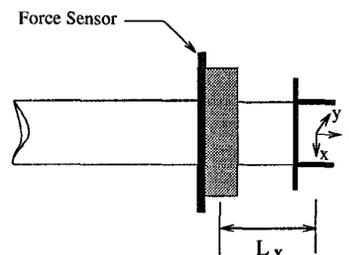


Fig. 7. Force sensor

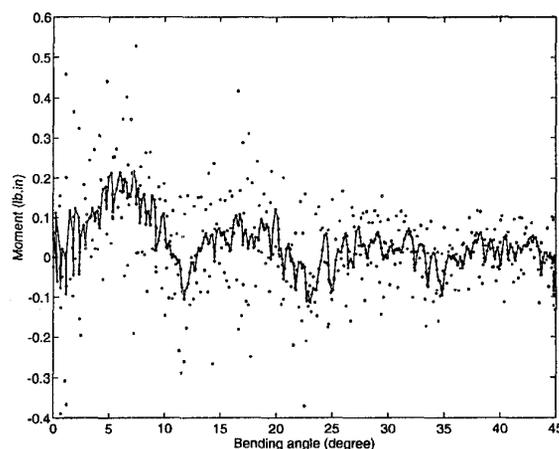


Fig. 8. Moment exerted on the end effector in a piece-wise linear approximation of the optimal trajectories with compliant control

the forces exerted on the end-effector will cause a moment on the force sensor. Since this moment is not exerted on the end-effector, we subtracted it from the force sensor torque measurements. With a sampling rate of 35.7 Hz, the parameters were selected to be:

$$B = \begin{bmatrix} 7 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad (26)$$

and

$$K = \begin{bmatrix} 575 & 0 \\ 0 & 10000 \end{bmatrix}. \quad (27)$$

The compliant controller clearly reduced the moment as shown in Fig. 8. Ideally, this moment can be further reduced to a very small value (for all practical purposes zero). However, If the matrix  $B$  is reduced further, the system starts to oscillate as shown in Fig. 9. This is due to the fact that the friction, gravity, and higher order dynamic start to dominate the system characteristics. Moreover, due to the oscillation in the response the Robot will crash and stop if decrease this matrix more since the robot inertia starts to dominate the dynamics of the system.

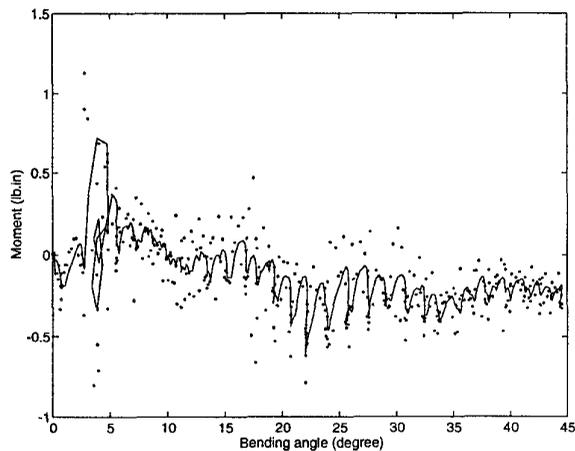


Fig. 9. Moment exerted on the end effector with compliant controller outside the stability boundaries

## VI. CONCLUSIONS

A compliant control is proposed in this paper to alter the approximation of the optimal trajectories of two robots bending a flexible beam. The compliant controller was able to reduce the moments and forces exerted on the end-effectors.

An experiment using two PUMA560 robots was carried out and verified the proposed algorithm. Furthermore, stability of the compliant controller were investigated experimentally.

## REFERENCES

- [1] Douglas E. Ruth and Prasanna Mulgaonkar, "Robotic lay-up of prepreg composite plies," Proc. 1990 IEEE Int. Conf. on Robotics and Automation, Cincinnati, Ohio, May 13-18, 1990, pp.1296-1300.
- [2] Yuan F. Zheng and Ming Z. Chen, "Trajectory planning for two manipulators to deform flexible beams," Proc. 1993 IEEE Int. Conf. on Robotics and Automation, Atlanta, Georgia, May 2-6, 1993, pp.1029-1024.
- [3] Yuan F. Zheng and Ming Z. Chen, "Trajectory planning for two manipulators to deform flexible beams," Robotics and Autonomous Systems, 12(1994), pp. 55-67.
- [4] Omar Al-Jarrah and Yuan F. Zheng, "Efficient trajectory planning for two manipulators do deform flexible materials" Proc. IROS'94, Munich, Germany, Sept. 1994.
- [5] H. Kazerooni, T. B. Sheridan, and P. K. Houpt, "Robust compliant motion for manipulators, Part I-II," IEEE Journal of Robotics and Automation, vol. RA-2, no. 2, pp. 83-105, 1986.
- [6] N. Hogan, "Impedance control: an approach to manipulation, Part I-III," ASME Journal of Dynamic Systems, Measurements and Control, vol. 107, pp. 1-24, 1985.
- [7] R. Colbaugh and A. Engelmann, "Adaptive compliant motion control of manipulators: theory and experiments," Proc. 1994 IEEE Int. Conf. on Robotics and Automation, San Diego, California, May 8 - 13, 1994, pp. 2719-2726.
- [8] Jain M. Tao, J. Y. S. Luh, and Yuan F. Zheng, "Compliant coordination control of two moving industrial robots," IEEE Trans. Robotics and Automation, vol. 6, no. 3, June 1990, pp. 322-330.
- [9] Paul S. Schenker, Antal K. Bejczy, Won S. Kim, and Sukhan Lee, "Advance man-machine interfaces and control architecture for dexterous teleoperations," IEEE Oceans'91, October 1-3, Honolulu, Hi, "Underwater Robotics" Session.
- [10] H. Das, H. Zak, W. S. Kim, A. K. Bejczy, and P. S. Schenker "Performance experiments with alternative advanced teleoperator control modes for a simulated solar maximum satellite repair," 5th Annual Space operations, Appl. Research Symp., July 9-11, 1991, Houston, Tx.
- [11] A. K. Bejczy and Z. F. Szakaly "A harmonic motion generator (HMG) for telerobotic applications," Proc. 1991 IEEE Int. Conf. on Robotics and Automation, Sacramento, CA, April 1991.
- [12] Yangsheng Xu and Richard P. Paul "On position compensation and force control stability of a robot with compliant wrist," Proc. 1988 IEEE Int. Conf. on Robotics and Automation, Philadelphia, Pennsylvania, April 14-19, 1988, pp. 1173-1178.
- [13] Dale A. Lawrence "Impedance control stability properties in common implementations," Proc. 1988 IEEE Int. Conf. on Robotics and Automation, Philadelphia, Pennsylvania, April 14-19, 1988, pp. 1185-1190.
- [14] H. Kazerooni and T. I. Tsay "Stability criteria for robot compliant maneuvers," Proc. 1988 IEEE Int. Conf. on Robotics and Automation, Philadelphia, Pennsylvania, April 14-19, 1988, pp. 1166-1172.