Lecture 1: Course Info and Hybrid System Examples

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Course Info

- Instructor: Wei Zhang
- Teaching support: Hua Chen
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- Time: Tu/Th 11:10am – 12:30pm
- Room: Caldwell Lab 0133
- Office Hour: Thursday 4-5pm
- Webpage: http://www2.ece.ohio-state.edu/~zhang/ECE7850HybridSystems.html
- Prerequisite:
  - ECE 5750 – Linear System Theory
  - Solid math background is essential
Grading Policy

- Group homework (30%)
  - Assigned biweekly (roughly)
  - Group with 2 or 3 students
  - Must be typeset using Latex
  - Can be quite challenging!

- Midterm (30%)
  - Open lecture notes and homework solutions
  - Date & Time: TBD (most likely evening exam around mid March)
Final Project (40\%):

- Team with 1 or 2 students
- Project proposal due by 04/04/2014;
- Project report due in the final exam week;
- 15-minute presentation at the end of the semester

Some ideas of project topics
- Nontrivial extension of the results introduced in class
- Nontrivial application of HS to a practical problem in your research area
- Comprehensive literature review on a topic in HS not covered in the class

The result and report are expected to be of conference/journal paper quality
References

- “Switching in systems and control”, D. Liberzon, 2003
- “Predictive Control for linear and hybrid systems”, F. Borrelli, A. Bemporad and M. Morari, 2013
- Representative papers in hybrid systems

Major conference Hybrid Systems

- International Conference on Hybrid Systems: Computation and Control (HSCC)
- Submission due around mid-October each year
Tentative Topics

- **Introduction to Hybrid Systems**
  - Examples, Modeling frameworks, Solution and execution, Filippov solution, zeno phenomena

- **Stability Analysis and Stabilization**
  - Stability under arbitrary switching, stability under constrained switching, Multiple-Lyapunov function, LMI based synthesis using multiple-Lyapunov function; control-Lyapunov function approach

- **Discrete Time Optimal Control**
  - Switched LQR problem, MPC of switched Piecewise Affine Systems, Infinite-horizon optimal control and its connection to stability/stabilization

- **Continuous Time Optimal Control**
  - Theory of numerical optimization in infinite-dimensional space, Applications to optimal control of switched nonlinear systems

- **Reachability analysis and computation:**
  - HJI based reachability analysis

- **Estimation of Stochastic Hybrid Systems**
  - Joint state and mode estimation using Interacting Multiple Model (IMM) estimator
Special Notes

- Advanced but not seminar type of course
- Goal: prepare and train the students to develop new theories
- Growing field with important emerging applications
  - Networked control systems, Cyber-Physical Systems, Robotics, Intelligent transportation
- No standard textbooks
  - Existing courses tend to focus on a single topic
  - We will try to cover a wide range of major topics in depth
  - mostly from “control/dynamical systems” perspective
- Caveat:
  - First time offering……
What is Hybrid Systems

- Roughly: dynamical systems with combined continuous and discrete dynamics

- Key features:
  - Discrete mode evolution:
    - $q^+ = g(x, q, \sigma)$
  - Mode-dependent continuous dynamics:
    - $\dot{x} = f(x, q, u)$
  - Interactions:
    - Continuous state evolution $x$ triggers discrete mode transition
    - Mode transition modifies continuous dynamics characteristics
Hybrid System Example 1: Bouncing Ball

- Bouncing ball:
  - Free fall:
    \[
    \begin{align*}
    \dot{x}_1 &= p \quad \text{(position)} \\
    \dot{x}_2 &= \dot{x}_1 \quad \text{(velocity)}
    \end{align*}
    \]
  - Collision:
    \[\begin{align*}
    x_1(t_c) &= \alpha x_1(t_c) \\
    x_2(t_c) &= -c x_2(t_c)
    \end{align*}\]
    \[c \in [0,1]\]

  \[\dot{x}_1 = \frac{\dot{x}_2}{g} \quad \text{the until hit ground}\]

  \[\begin{align*}
  x_1 &= 0 \quad \text{if } x_2 < 0 \quad \text{(guard condition)}
  \end{align*}\]

  \[\begin{align*}
  x_1 &= x_2 \\
  \dot{x}_2 &= g \quad \text{state reset}
  \end{align*}\]
Hybrid System Example 2: Water Tank

- **Goal:** keep water level above references

  \[ q_0 = 1 \quad q = 2 \]

- **Two modes:** left/right

- **Dynamics:**

  state: \[ x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \]

  \[ q = 1 \ (\text{left}): \quad \dot{x} = \begin{bmatrix} w - v_1 \\ -v_2 \end{bmatrix} \leq f_1(x) \]

  \[ q = 2 \quad \dot{x} = \begin{bmatrix} -v_1 \\ w - v_2 \end{bmatrix} \leq f_2(x) \]

- **Guard:**

  \[ q = 1 \text{ to } q = 2 : \quad \{(x_1, x_2) \in \mathbb{R}^2 | x_2 \leq r_2 \} \]

  \[ q = 2 \text{ to } q = 1 : \quad \{(x_1, x_2) \in \mathbb{R}^2 | x_1 \leq r_1 \} \]

  **State:** There is no jump upon switch
Hybrid System Example 3: Converter

- Two modes:
  - Mode 1: $S_1 = 1$, $S_2 = 0$
  - Mode 2: $S_1 = 0$, $S_2 = 1$

- Objectives: minimize output voltage error under uncertain $v_s$, $r_o$

Example courtesy: Dr. Geyer
**Hybrid System Example 4: Air Traffic Control**

- **Unicycle aircraft model:**
  \[
  \begin{bmatrix}
  \dot{x}_1^a \\
  \dot{x}_2^a
  \end{bmatrix} = \begin{bmatrix}
  v \cos \theta_a \\
  v \sin \theta_a
  \end{bmatrix},
  \begin{bmatrix}
  \dot{x}_1^b \\
  \dot{x}_2^b
  \end{bmatrix} = \begin{bmatrix}
  v \cos \theta_b \\
  v \sin \theta_b
  \end{bmatrix}
  \]

- **Naïve collision avoidance protocol:**
  - Left if \( |x^a - x^b| < \alpha \) \((\dot{t} = 1)\) \( \leftarrow \) \( t \leq \beta \)
  - Straight until \( |x^a - x^b| > \alpha \) \( \leftarrow \)
  - Right \((\dot{t} = -1)\)
  - Cruise

**HS has 4 modes:**

Define \( \alpha = \begin{bmatrix} \alpha^a \\ \alpha^b \end{bmatrix} \), \( \theta = \begin{bmatrix} \theta^a \\ \theta^b \end{bmatrix} \), and \( t \).
Hybrid System Example 4: Air Traffic Control

- Continue:
Hybrid System Example 5: Variable Structure Control

- Standard nonlinear dynamics: \( \dot{x} = f(x, u) \)

- Piecewise continuous control laws:
  
  - Partition entire domain into several regions

  define: \( u(x) = \begin{cases} 
  u_1(x), & \text{if } x \in \Omega_1 \\
  u_2(x), & \text{if } x \in \Omega_2 
  \end{cases} \)

  In this case, system has two modes.

  \[ q=1, \quad \dot{x} = f(x, u_1(x)) \]

  \[ q=2, \quad \dot{x} = f(x, u_2(x)) \]

  \[ \text{Guard: } 1 \Rightarrow 2 : \Omega_2 \]

  \[ 2 \Rightarrow 1 : \Omega_1 \]

  \[ \text{Reset: } f(1 \Rightarrow 2, x) = x \]

- Prof. Utkin here at OSU is one of the originators in variable structure control
Hybrid System Example 5: Variable Structure Control

- Application in UAV control:

[Reference: DGHVZT11]
Simple NCS:

- $t_k$: $k$th transmission time
- $\dot{x}(t) = f(x(t), u(t))$
- $u(t) = Kx(t_{k-\tau})$
- $e(t) = x(t) - x(t_{k-\tau})$, $z(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}$

$\dot{z}(t) = f(z(t), \tau, x(t_{k-\tau}))$

$z(t) - e(t) = f(z(t), \tau)$

$\Rightarrow \dot{e}(t) = f(z(t), u(t)) \Rightarrow \text{until periodic reset}$
Hybrid System Example 6-2: Event-Triggered Control

Event triggered control:

- Transmit: $z(t) \in E$
- $\dot{x}(t) = \tilde{f}(x(t), e(t))$
- $\dot{e}(t) = \tilde{f}(x(t), e(t))$
- $e(t_{k}^{+}) = 0$

How to determine $E$ to ensure closed-loop stability? Stay tuned.
Hybrid Systems Example 7: Embedded Systems

- Dynamic buffer management
  - Continuous state $x$: amount of data in the buffer
  - Discrete mode:

DBM Problem: Find best $Q$ and switching strategy to minimize the total energy subject to constraints

[Reference: ZH08]
Summary:
- Most general and natural modeling framework
- Numerous applications
- Further reading: reference papers in the “Application” category
- Active area of research with many open challenges
- This class is only an introduction to some important topics

Next time:
- Formal discussion on hybrid system models and solution concepts