

# Modeling and Optimization of Terminal Airspace and Aircraft Arrival Subject To Weather Uncertainties

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We develop an accurate model for the arrival traffic dynamics in terminal airspace that takes into account the weather forecast and runway configuration changes. The planning of runway configuration switching subject to weather constraints is formulated as a hybrid optimal control problem and a hierarchical approach is proposed to solve the problem. At the upper level of the hierarchy, a runway configuration sequence is determined based on weather forecast data and air traffic demand. At the lower level, the configuration switching times and the corresponding aircraft speeds are determined by solving a Mixed Integer Linear Program. We illustrate the utility of the approach by a case study inspired by operations in John F. Kennedy airport.

## Nomenclature

$G = (V, E)$	airspace graph: where $V$ is set of nodes, and $E$ is set of edges
$V_e$	set of entry nodes representing arrival meter fixes
$V_s$	set of sink nodes representing runways
$c \subset V_s$	mode of the graph
$z = (q, x, y)$	hybrid aircraft state: $q$ discrete, $x$ and $y$ continuous
$s$	aircraft speed (continuous control variable)
$\eta \in E \cup \{\text{hold}\}$	edge or holding pattern (discrete control variable)

## I. Introduction

Air Traffic Management (ATM) is responsible for sustainable, efficient and safe operation in civil aviation. Due to the continuous growth of air traffic demand, skyrocketing fuel prices and growing concerns over environmental impact of air transportation, a paradigm shift in the current ATM is being pursued. This paradigm shift is being addressed in Europe, within the framework of Single European Sky ATM Research (SESAR),<sup>1</sup> and in the United States, within the so-called Next Generation (NextGen) of air traffic.<sup>2</sup>

Air transportation in the United States is regulated in a divide-and-conquer manner, in which the airspace is divided into subregions, such as Air Route Traffic Centers (ARTCC), Sectors, and terminal areas. The different subregions are controlled by different groups of air traffic controllers so that the overall responsibility is decoupled. A terminal area refers to a region of airspace that is within approximately a 50 nautical mile (nmi) radius of an airport, and its control is divided between the Terminal Radar Approach Control (TRACON) and the Tower Control. It is the most crowded portion of the airspace and is often the throughput bottleneck of the airspace system. Due to its importance in safety and capacity of the airspace, a concept of operations for the NextGen terminal airspace, referred to as Super-Density Operations, has been proposed. The Super-Density Operations envision the use of advanced ground and flight deck automation, efficient Area Navigation (RNAV) and Required Navigation Performance (RNP) routes, optimized vertical profiles,

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and delegated interval management to maintain efficient utilization of terminal airspace even in adverse weather conditions. The main challenge in achieving highly efficient operations lies in weather and traffic uncertainties as well as the configuration changes in runways or airways necessitated by adverse weather conditions.

In airports with multiple intersecting runways, such as the John F Kennedy International Airport (JFK), a set of active runways, referred to as a runway configuration, for arrival or departure are chosen based on factors including the crosswind and tailwind magnitudes, visibility, traffic flow and noise abatement laws. The choice of runway configuration in JFK affects the arrival routes of incoming traffic to JFK, as well as to other nearby airports in the same TRACON airspace such as LaGuardia and Newark. Currently, unanticipated runway switching not only increases the workload of air traffic controllers and pilots, but also results in many approaching aircraft being put on holding pattern. Consequently, the capacity lost during the transitional period of a runway configuration switch is referred to as a perishable capacity and the delays are propagated into the airspace beyond the terminal area.

The work presented in this paper is motivated by the vision of the Super-Density Operations and the particular need for a better planning of runway configuration switching. More specifically, our goal is to develop (i) an accurate model for the arrival traffic dynamics in terminal airspace that takes into account weather uncertainties and runway configuration changes, and (ii) a traffic control algorithm based on the model that can reduce delays or other desired cost factors.

### **I.A. Previous Work**

The previous work on terminal airspace air traffic management falls into three categories depending on whether air traffic control, airspace management or runway configuration management is addressed. In the first category, optimization of aircraft landing times given fixed arrival routes and runways is addressed.<sup>3-6</sup> Bayen et al.<sup>3</sup> formulate a Mixed Integer Linear Program (MILP) to minimize aircraft delay given fixed arrival routes and approximate the MILP with a polynomial-time algorithm. Balakrishnan et al.<sup>6</sup> determine aircraft arrival sequence using Constrained Position Shifting in order to reduce delays while respecting the arrival spacing requirements. These works do not consider the complexity arising due to switching of the runway configuration nor the weather effects on the arrival route availability.

Next, research has studied effects of hazardous weather on the availability of predefined routes in the terminal area. The Route Availability Planning Tool (RAPT) has been developed based on the Corridor Integrated Weather System (CIWS) product<sup>7</sup> to help air traffic controllers assess the availability of departure routes in adverse weather.<sup>8</sup> Michalek et al.<sup>9</sup> use machine learning algorithms to determine routes that are robustly safe to fly through under weather uncertainties. Reconfiguration of airspace by designing routes that are safe with respect to hazardous weather is considered by Krozel et al. and Michalek et al.<sup>10,11</sup> These works do not consider control of the arrival traffic or runway configuration management in adverse weather.

Recently, research has begun to consider the problem of runway configuration planning. Roach<sup>12</sup> discusses configuration planning based on wind data and analyzes air traffic delays caused by non-prevailing wind conditions at Dallas/Fort Worth airport. In the work of Leihong et al.<sup>13</sup> wind forecast data is used in order to determine feasibility of runway configurations in a given future time horizon. The authors then formulate a Dynamic Programming algorithm to address runway configuration selection in order to maximize the throughput of the landing aircraft. Ramanujam et al.<sup>14</sup> determine a set of factors that are used in choosing a runway configuration and then apply machine learning in order to model the air traffic controllers' decisions in choosing runway configurations. These works do not consider the determination of the optimal switching times between the configurations and the control of the arrival traffic during the switching. In reality, in many instances the configuration sequence may be known to the air traffic controllers, while the switching times between configuration changes and the management of arrival traffic need to be determined optimally in order to minimize delays resulting from the transitional periods of the configuration switches.

### **I.B. Current work**

In this paper, we develop a novel stochastic hybrid system model to describe the dynamics of the arrival traffic. In this model, the discrete modes represent the runway configurations and the continuous states represent the locations of the aircraft in the terminal airspace. Weather uncertainties pose probabilistic constraints for discrete modes and in turn affect the evolution of the continuous dynamics. The runway switching problem is formulated as an optimal control problem of the stochastic hybrid system that requires

minimization of the total delay and fuel consumption subject to probabilistic configuration constraints as well as the separation constraints between the aircraft. An efficient algorithm is also proposed to solve this problem. The novelty of our formulation lies in its consideration of the weather uncertainties and the couplings between the runway switching and the incoming traffic patterns in the model.

The paper is organized as follows: In Section II we define the problem of runway configuration and aircraft scheduling and develop a model for the arrival traffic in terminal airspace. In Section III we define our solution approach for addressing the problem. In Section IV we show a case study on applying the model and solution approach. Finally, we summarize our results and directions for future work in Section V.

## II. Problem Model

The set of runways that are selected for landing at any airport is referred to as the arrival runway configuration, and will simply be referred to as the runway configuration in the rest of this paper. The configuration may change several times in a day because it is selected by considering various factors such as wind direction and magnitude, noise level, visibility and air traffic patterns. The choice of configuration affects the air traffic routes in the terminal airspace. The airspace model developed here captures the air traffic routes and the runway configurations, while the hybrid dynamic model of the aircraft captures the motion of aircraft on these arrival routes.

### II.A. Hybrid Model of Arrival Traffic

#### II.A.1. Airspace model

Aircraft are often required to enter and leave the terminal airspace through some fixed locations called meter fixes. For each meter fix, there are usually several predefined paths leading to different runways in the airport. This pathway structure within the terminal area can be easily described by a directed graph  $G = (V, E)$ , where each node  $v \in V$  in this graph represents a waypoint (including runways) in the terminal airspace, and each edge  $e = (v_1, v_2) \in E \subset V \times V$  represents a directed aircraft route from node  $v_1$  to node  $v_2$ . Each edge is associated with a length  $l_e$  as well as a set of neighbors  $\mathcal{N}(e)$  consisting of the edges that share a common node with  $e$ . A node is called an entry node if it connects the terminal airspace to the en-route airspace and is called a sink node if it corresponds to a runway. The set of entry and sink nodes are denoted by  $V_e$  and  $V_s$ , respectively. Every directed path is a connected set of edges that starts at an entry node  $v_e \in V_e$  and ends at a sink node  $v_s \in V_s$ . The set of edges  $e = (v_1, v_2)$  with  $v_2 \in V_s$  is denoted by  $E_s$ .

A mode of the graph is characterized by a set of sink nodes  $c \subset V_s$ . There is a one-to-one correspondence between the graph modes and the runway configurations. We will use the terms mode and runway configuration interchangeably in the rest of this paper. If the graph is in mode  $c$  then the configuration includes runways which are represented by nodes in  $c$ . In this case, the edge  $e = (v_1, v_2)$  with  $v_2 \in V_s$  is available as an aircraft route if and only if  $v_2 \in c$  and this edge is referred to as a final edge.

The control input for the graph is the choice of graph mode over a time horizon. This choice over an interval of time  $[t_I, t_F]$  is represented as:

$$\sigma = [(t_I, c_0), (t_1, c_1), \dots, (t_N, c_N)], \quad (1)$$

where  $0 < N < \infty$ ,  $t_I \leq t_1 \leq \dots \leq t_N \leq t_F$  and  $c_k \subset V_s$  for  $k = 0, 1, \dots, N$ . In this sequence, the pair  $(t_I, c_0)$  is the initial condition and the pair  $(t_k, c_k)$ ,  $k \geq 1$ , indicates that at time  $t_k$  the graph mode changes from  $c_{k-1}$  to  $c_k$ . As a consequence, in the time interval  $[t_k, t_{k+1})$  the graph mode is given by  $c_k$ .

#### II.A.2. Aircraft dynamics

Let  $[t_I, t_F]$  be a time interval of interest for optimizing arrival traffic. Suppose there are  $N_a$  scheduled arrivals during this interval with the  $i^{th}$  aircraft crossing one of the entry nodes at time  $t_0^i$ . Once aircraft enter the terminal airspace, they should travel along the pathways defined by graph  $G$ . For aircraft  $i$ , let  $q^i(t) \in E$  be the edge it is on at time  $t$ ,  $x^i(t) \in \mathbb{R}_+$  be its current distance from the first node of edge  $q^i(t)$ ,  $y^i(t) \in \mathbb{R}_+$  be the total distance it has traveled since time  $t_0^i$  and  $z^i(t) = (q^i(t), x^i(t), y^i(t))$  be its hybrid state.

The evolution of the hybrid state  $z^i(t)$  is controlled by air traffic controllers through speed adjustment, path selection and holding pattern assignment. We assume speed changes and holding patterns occur only when the aircraft is at one of the nodes in the graph. We denote a generic air traffic control command as

$u = (s, \eta)$  where  $s \in \mathbb{R}_+$  is the speed magnitude assignment and  $\eta \in E \cup \{\text{hold}\}$  is the discrete control command specifying whether the aircraft needs to enter a holding pattern (when  $\eta = \text{hold}$ ) with speed  $s$  at the current node or travel along the new edge specified by  $\eta$  (when  $\eta \in E$ ) with speed  $s$ .

Suppose that aircraft  $i$  is at node  $v \in V$  at some time  $\hat{t} \geq t_0^i$  and receives a control  $u = (s, \eta)$ . If  $\eta = \text{hold}$ , then evolution of the hybrid state is given by:

$$\begin{bmatrix} q^i(t) \\ \dot{x}^i(t) \\ \dot{y}^i(t) \end{bmatrix} = \begin{bmatrix} q^i(\hat{t}) \\ 0 \\ s \end{bmatrix}.$$

On the other hand, if  $\eta \in \mathcal{N}(q^i(\hat{t}))$ , then the hybrid state first undergoes an instantaneous reset to  $z^i(\hat{t}_+) = (\eta, 0, y^i(\hat{t}))$ , where  $\hat{t}_+$  denotes the time immediately after  $\hat{t}$ . The dynamics after time  $\hat{t}_+$  is governed by:

$$\begin{bmatrix} q^i(t) \\ \dot{x}^i(t) \\ \dot{y}^i(t) \end{bmatrix} = \begin{bmatrix} \eta \\ s \\ s \end{bmatrix}.$$

The above two evolutions continue until the aircraft finishes the number of holding patterns assigned or reaches the next node, at which time it will receive a new control command and the process repeats. The evolution stops once the aircraft reaches one of the sink nodes, which can be determined through the hybrid state by checking whether the edge  $q^i(t)$  is a final edge and  $x^i(t) = l_{q^i(t)}$ . The time aircraft  $i$  reaches a sink node is denoted by  $t_f^i$ .

Let  $n^i$  denote the number of edges in the aircraft path from the source to the sink node. The set of all controls for aircraft  $i$  is given by  $u^i = (s^i, \eta^i)$ , where the continuous control is  $s^i = (s_1^i, \dots, s_{n^i}^i)$  and the discrete control is  $\eta^i = (\eta_1^i, \dots, \eta_{n^i}^i)$ .

## II.B. Constraints

There are constraints on the airspace due to weather conditions and on the aircraft due to separation requirements between the aircraft.

### II.B.1. Airspace constraints

The weather can affect the dynamics of the graph by affecting availability of the edges. If a significant portion of an edge is blocked by a storm or hazardous weather, then no aircraft can be assigned to that edge. In addition, if a runway is prohibited from landing due to strong wind or other environmental conditions, then no aircraft can be assigned to any of the final edges leading to the sink node corresponding to the runway.

First, we discuss runway feasibility. We focus on the effects of wind on runway selection and do not consider other factors such as noise abatement which are dependent on the particular airport and procedures. The wind direction and magnitude is a major factor in determining whether a runway is safe for landing because aircraft cannot safely land if the component of the wind perpendicular to the landing direction, referred to as the crosswind, and parallel to the landing direction, referred to as the tailwind, are above certain thresholds. From the wind forecast data we assume we have the probabilities that the crosswind and tailwind to runway  $v$  are below the required thresholds and denote these probabilities by  $p_{v,cw}$  and  $p_{v,tw}$  respectively. We define a configuration or graph mode feasible if the probabilities of crosswind and tailwind threshold satisfaction are above a desired level  $\alpha_w \in (0, 1)$  for each runway in the configuration. The constraint for the graph mode sequence of equation (1) over the time horizon  $t \in [t_{k-1}, t_k]$  is given as:

$$c_k \in \{c \subset V_s \mid \forall v \in c, p_{v,cw}(t) \geq \alpha_w \wedge p_{v,tw}(t) \geq \alpha_w\}. \quad (2)$$

Next, we discuss the edge feasibility. We assume that we have the probability  $p_e(t)$  of edge  $e = (v_1, v_2)$  being open at time  $t$ . In order to assign aircraft to edge  $e$ , we require that the edge is open with high enough probability, that is,  $p_e(t) \geq \alpha_e$ , where  $\alpha_e \in (0, 1)$  is a parameter determined by the safety requirements. Consequently, the discrete aircraft control  $\eta$  at time  $t$  and at node  $v_1$  has to satisfy:

$$\eta \in \{e \in E \mid p_e(t) \geq \alpha_e\} \cup \{0, 1, \dots, H\}. \quad (3)$$

In the above, the maximum number of allowable holding patterns at a node is denoted by  $H$ . Also, a final edge  $e = (v_1, v_2) \in E_s$  must satisfy  $v_2 \in c$  where  $c$  is a feasible runway according to (2).

### II.B.2. State constraints

For safety requirements the aircraft on the same edge or neighboring edges must be separated by a given distance  $d$ . In addition, for safety due to wake vortex of aircraft, there are runway separation distance requirements based on leading and trailing aircraft types. Let the type of aircraft  $i$  be denoted by  $a^i$ . The runway separation distance between aircraft  $i$  and  $j$  is denoted by  $D(a^i, a^j)$ . Let matrix  $D \in \mathbb{R}_+^{n_t \times n_t}$  represent the runway separation requirement between all pairs of aircraft, in which  $n_t$  denotes the number of different aircraft types. In general, if the aircraft land on different runways, the separation requirement and hence the matrix  $D$  would also depend on the aircrafts' respective landing runways. For simplicity in notation, we drop this dependence here. The separation constraints along the edges and runways are encoded with the constraint  $h(z^i, z^j, t) \leq 0$ , where the function  $h$  is defined as:

$$h(z^i, z^j, t) = \begin{cases} x^j(t) - x^i(t) + d, & \text{if } q^j(t) = q^i(t), \\ x^j(t) - x^i(t) + d, & \text{if } q^j(t) \in \mathcal{N}(q^i(t)) \wedge x^i(t) = l_{q^i(t)}, \\ x^j(t) + D(a^i, a^j) - l_{q^j(t)}, & \text{if } q^i(t), q^j(t) \in E_s \wedge x^i(t) = l_{q^i(t)}. \end{cases} \quad (4)$$

The first constraint denotes the separation requirement for two aircraft on the same edge, the second denotes the separation requirement for aircraft on neighboring edges and the third denotes the separation requirement for landing aircraft. In all cases, it is assumed that aircraft  $j$  precedes aircraft  $i$ .

The constraint on the final state is the requirement that aircraft land at a runway by some time  $t \in [t_I, t_F]$  in the planning horizon:

$$z^i(t) \in \{(q, x, y) \mid q \in E_s \wedge x(t) = l_q\}. \quad (5)$$

### II.C. Optimization

For each aircraft, we penalize a function of the aircraft state by defining a running cost function  $L(z^i)$ . This function could for example denote the total distance or travel time of the aircraft and hence in general is a function of the discrete state representing edges in aircraft path and the continuous state representing the distance travelled. We associate a cost due to switching from graph mode  $c_i$  to mode  $c_j$ ,  $S(c_i, c_j)$ , due to overhead associated with switching runway configuration. Let  $u = (u^1, u^2, \dots, u^{N_a})$  denote the sequence of inputs to all the aircraft. The objective function to be minimized is formulated as:

$$J(u, \sigma) = \sum_{i=1}^{N_a} \int_{t_0^i}^{t_f^i} L(z^i(t)) dt + \sum_{k=1}^{N-1} S(c_k, c_{k+1}). \quad (6)$$

The constraints of the optimization are those on the graph mode sequence (2), the aircraft edges (3), the aircraft state (4), (5) and the range of allowable aircraft speed. The optimization problem formulated above is a constrained hybrid optimal control problem. There are discrete control inputs consisting of the runway sequence selection, aircraft path and holding pattern assignments, and continuous inputs including the switching times between the runway configurations and the speed assignment along the edge for each aircraft.

## III. Hierarchical Solution Approach

We develop a hierarchical approach to solve the problem formulated above. The approach consists of two stages: In the first stage, the optimal runway sequence and the aircraft paths are determined. In the second stage, the optimal switching times and the speed and holding pattern control inputs along the paths for each aircraft are determined. The approach is hierarchical because the choice of the runway configuration is based on weather and traffic demand factors and is not affected by individual aircraft behavior, while this choice affects the arrival paths and hence the control of the individual aircraft.

### III.A. Stage I

Here, the optimization variables are the runway mode sequence and the sequence of edges that describe the path of each aircraft. These variables are determined based on weather forecast and established procedures.

### III.A.1. Mode sequence determination

Given that it is not feasible to switch configurations frequently, we can always choose the planning horizon small enough such that there is one runway configuration switch. Consequently, we assume there are only two graph modes during the planning horizon  $[t_I, t_F]$ . The initial condition for the mode is  $c_0$ . Due to wind or traffic demand, the initial mode becomes infeasible and hence a switch to another mode  $c_1$  is required. The new mode is chosen such that it is feasible with respect to wind, that is,  $c_1$  satisfies (2) for all  $t \in [t_s, t_F]$ , where  $t_s$  is the switching time to be determined. If there are multiple modes that are feasible with respect to wind, the configuration that accommodates the traffic demand is selected. While in this stage the graph mode sequence is determined, the switching time  $t_s$  will be determined in Stage II.

### III.A.2. Aircraft path determination

In most airports, the path the aircraft travels prior to landing is chosen based on predefined arrival routes, such as those prescribed in established Standard Arrival Routes (STARs). An example of a STAR is shown in Appendix V.A. We use the established procedures to determine the edges that need to be selected for the aircraft path. If with high probability an edge is infeasible due to weather as described in Equation (3) the aircraft will be assigned to a new edge which is not blocked due to weather. These new edges could be determined or designed from the forecast data.<sup>9,10</sup> The remaining control inputs for the aircraft are the speed and holding patterns along each edge which are determined in Stage II.

## III.B. Stage II

The wind magnitude and direction based on the forecast data are uncertain and have low resolution in time, for example, hourly forecast data. As such we do not have an exact time at which infeasibility of a runway configuration occurs and there is some flexibility in choosing the switching time between the configurations in order to minimize the overhead in the configuration switch. Let  $\alpha$  denote the first time the infeasibility due to wind based on the forecast is encountered. If the forecast interval is  $\delta_w$  minutes, we define  $\alpha_1 = \alpha - \delta_w$  and  $\alpha_2 = \alpha + \delta_w$ . We assume that the configuration switch must occur at a time:  $t_s \in [\alpha_1, \alpha_2] \subset [t_I, t_F]$ . Our objective is the determination of  $t_s$  such that the cost of interest (6), is optimized. In order to impose the state constraints (4) and (5) we formulate an equivalent characterization of these constraints based on conversion of a separation constraint in terms of distance to a separation constraint in terms of time.

### III.B.1. Separation constraints along edges

Consider the first separation constraint in Equation (4). Suppose aircraft  $i$  and  $j$  fly on an edge  $e = (v_1, v_2)$ , with aircraft  $i$  preceding aircraft  $j$ . The aircraft fly with constant speeds of  $s^i$  and  $s^j$  respectively along the edge. Let  $x = x^i - x^j$  and  $t_1^i, t_1^j$  be the time at which aircraft  $i, j$  depart from node  $v_1$  respectively. Suppose  $x(t_1^j) \geq d$ , that is, the distance between the two aircraft at the time aircraft  $j$  crosses node  $v_1$  is greater than the minimum required distance. Then, in order to ensure separation constraint along the edge, due to constant aircraft speed along the edge it is sufficient to impose  $x(t_2^i) \geq d$ . This constraint can be converted to a constraint on the time of crossing node  $v_2$  as  $t_2^j \geq t_2^i + \frac{d}{s^j}$ . For aircraft on neighboring edges, the second constraint in (4) must hold. The same argument shows that separation distance requirement in this case can be converted to separation requirement for time of crossing the common node of the neighboring edges.

Next, we find bounds on feasible times of arriving at the nodes along the path of each aircraft. Consider aircraft  $i$  flying through edge  $e = (v_1, v_2)$  with constant speed  $s^i \in [s_l^i, s_u^i]$ . Let  $t_1^i$  and  $t_2^i$  denote the time of arrival of aircraft at nodes  $v_1$  and  $v_2$ , respectively. Then,  $t_2^i \in I_0^i = [\tau_l^i, \tau_u^i]$ , where  $\tau_l^i = t_1^i + \frac{L_e}{s_u^i}$  and  $\tau_u^i = t_1^i + \frac{L_e}{s_l^i}$ . In addition, if the aircraft is to perform a number  $n_H \geq 0$  of holding patterns at node  $v_1$ , each for a duration of  $t_{hp}$ , then  $t_2^i \in I = \cup_{k=0}^{n_H} I_k^i$ , where  $I_k^i = kt_{hp} + [\tau_l^i, \tau_u^i]$ . Based on this analysis, we find upper and lower bounds on the arrival time of aircraft at each node along the path of the aircraft. In addition, given an arrival time  $t_2^i \in I$  we can uniquely determine the minimum number of holding patterns required at node  $v_1$  and the speed of aircraft along the edge  $e$ . Hence, the assignment of speed along edge  $e = (v_1, v_2)$  and holding pattern at node  $v_1$  can be converted to the assignment of time of arrival of aircraft at  $v_2$ .

Let the arrival time at node  $v \notin V_s$  for aircraft  $i$  be written as  $t_v^i \in I_v^i$ , where  $I_v^i$  may be a union of disjoint intervals due to presence of holding patterns as discussed above. Let  $d_t = \frac{L_e}{s_l^i}$ , where  $s_l^i$  is the minimum aircraft speed through node  $v$ . For aircraft  $i$  and  $j$  flying through the same edge or neighboring

edges, the separation constraint can be written as  $t_v^j \geq t_v^i + d_t$ , where  $t_v$  is the time of crossing the common node  $v$  of the aircrafts' corresponding edges. Let  $N_v$  denote the number of aircraft that fly through node  $v$  in the planning horizon. Suppose aircraft are ordered according to the nominal time of arrival at node  $v$ . To account for the first two separation constraints in (4) we minimize deviation of arrival time of each aircraft from its nominal arrival time  $\bar{t}_v^i$  at this node through formulating the constrained optimization problem:

$$\begin{aligned} \min \quad & \sum_{i=1}^{N_v} t_v^i - \bar{t}_v^i \\ \text{s.t.} \quad & t_v^i \in I_v^i, \quad i = 1, \dots, N_v, \\ & t_v^i - t_v^{i+1} + d_t \leq 0, \quad i = 1, \dots, N_v - 1. \end{aligned} \quad (7)$$

### III.B.2. Optimal switching time and separation constraints at runways

Consider the last constraint in Equation (4) which is the spacing requirement for aircraft  $i$  and  $j$  landing on runway  $v \in V_s$  through edge  $q$  at times  $t_f^i$  and  $t_f^j$  respectively with  $t_f^j > t_f^i$ . Let  $s_0^j$  be the arrival speed of aircraft  $j$  and  $D_t(a^i, a^j) = \frac{D(a^i, a^j)}{s_0^j}$  denote the runway separation requirement in time. The runway separation requirement can be written as a constraint on the landing times of the two aircraft:  $t_f^j \geq t_f^i + D_t(a^i, a^j)$ . Based on the lower and upper bounds of the speed of the aircraft and the maximum number of holding patterns, we find possible intervals for landing time of the aircraft. In addition, since the switching time between the configurations,  $t_s$ , affects the landing runway for the aircraft and consequently the path of the aircraft, the feasible landing time interval is also dependent on the switching time. Hence, aircraft  $i$  landing time must be inside the set of feasible intervals:  $t_f^i \in I^i(t_s) = \cup_{k=0}^H k t_{hp} + [\tau_l^i(t_s), \tau_u^i(t_s)]$ .

In order to determine the optimal switching time and aircraft landing times, we formulate the cost function as the total differences between the nominal landing time  $\bar{t}_f^i$  and the actual landing time  $t_f^i$  for all aircraft, that is,  $J(t_s, u_f) = \sum_{i=1}^{N_a} t_f^i - \bar{t}_f^i$ . Here,  $u_f$  denotes the vector of speed and holding pattern assignments for all aircraft along the final edge of their paths. In addition, we assume aircraft are ordered according to their nominal landing times. Hence, we formulate the following optimization problem:

$$\begin{aligned} \min \quad & \sum_{i=1}^{N_a} t_f^i - \bar{t}_f^i \\ \text{s.t.} \quad & t_f^i \in I^i(t_s), \quad i = 1, \dots, N_a, \\ & t_f^i - t_f^{i+1} + D_t(a^i, a^{i+1}) \leq 0, \quad i = 1, \dots, N_a - 1. \end{aligned} \quad (8)$$

Note that it is easy to formulate a cost function which would penalize early and late landing time using  $J(t_s, u_f) = \sum_{i=1}^{N_a} |t_f^i - \bar{t}_f^i|$ . Let  $u_f(t_s)$  denote the optimal input for a given switching time and define  $J(t_s) = J(t_s, u_f(t_s))$ . Then, the optimal switching time is determined as  $t_s^* = \min_{t_s \in [\alpha_1, \alpha_2]} J(t_s)$  and the optimal speed and holding pattern control on the last edge of the aircraft is given as  $u_f(t_s^*)$ .

### III.B.3. Numerical solution of the optimization problems

The hybrid optimal control problem in (6) has been reduced to a set of optimization problems; Problem (7) at each node that is not a runway and Problem (8) at the runway nodes. In order to solve these optimization problems, we can formulate a Mixed Integer Linear Program (MILP) as follows:<sup>3</sup> Let  $\delta_k \in \{0, 1\}$  for  $k = 0, 1, \dots, H$  denote binary variables associated to each discrete interval  $I_k$ . Then,  $t \in \cup_{k=0}^H I_k$  can be equivalently written as  $t \in \sum_{k=0}^H \delta_k I_k$  with  $\sum_{k=0}^H \delta_k = 1$ . If the upper and lower bounds on arrival time satisfy  $t_{hp} \leq \tau_u - \tau_l$ , then, the intervals  $I_k$  overlap, hence the arrival time is inside one connected interval and this constraint can simply be cast as a linear constraint without the need for binary variables. In summary, a MILP, or at best a Linear Program (LP), for each node needs to be solved to find arrival times that satisfy separation constraints. As for landing times, Problem (8) is a MILP for a given switching time  $t_s$ . Since the allowable range of  $t_s$ ,  $[\alpha_1, \alpha_2]$ , is usually small (less than one or two hours) and precisions to order of minutes for determination the switching time is sufficient, we discretize  $[\alpha_1, \alpha_2]$  and solve the MILP at the discrete values to determine optimal switching time, aircraft landing times and aircraft control inputs along their final edges. The number of binary variables would be at most  $N_a \times H \times V_a$  where  $N_a$  is the number of aircraft in the planning horizon and  $V_a$  is the maximum number of nodes along each aircraft path.

## IV. Case Study

We focus on the John F. Kennedy (JFK) airport and consider an instance of optimally planning its runway configurations and aircraft arrival schedules. The airspace graph is derived based on abstraction of the airspace structure of the JFK airport. The aircraft arrival rates and aircraft types are generated according to the actual counts and probabilities observed in practice as will be described.

### IV.A. Airspace Graph

The JFK TRACON consists of a region of approximately 60 nmi radius centered on the JFK airport and includes many airports in this area, with the major ones being LaGuardia (LGA) and Newark (EWR). A representation of the arrival and departure paths and the runway directions for the major airports in this TRACON is shown in Figure 1a. In our case study, we focus only on the JFK airport in the TRACON.

There is a number of Standard Arrival Routes (STARs) leading to the JFK airport. An example of such route from the West direction is shown in the Appendix Figure 5. These diagrams describe the routes aircraft take for arrival and locations for assigning holding patterns. Once the aircraft reach the last meter fix in the arrival path, they may follow verbal commands from the air traffic controllers or follow an Instrument Approach Plate (IAP) to make their final descent to the runway. JFK airport has two sets of intersecting runways as represented in Figure 1a and shown in the airport diagram provided in Appendix Figure 6.

There are multiple possibilities for arrival runway configurations. For this case study, we consider the configuration  $c_0 = \{22L, 22R\}$ , which is a common arrival configuration in low traffic, and  $c_1 = \{31L, 31R\}$ , which is used if landing in  $c_0$  is not possible due to wind. Based on studying JFK runway configurations, STAR files and our discussions with the air traffic controllers at the JFK airport<sup>a</sup>, we identify three main arrival directions to JFK and create a graph which models the arrival airspace structure. This graph is shown in Figure 1b. In this figure we superimposed the left and right runways in each set of parallel runways for simplicity, that is, the runway set  $\{22L, 22R\}$  is shown as one runway and the runway set  $\{31L, 31R\}$  is shown as one runway. The entry nodes are  $\{v_1, v_2, v_3\}$  corresponding to three major entry meter fixes. Two of the sink nodes  $v_7$  and  $v_8$ , corresponding to runways  $\{22L, 22R\}$  and  $\{31L, 31R\}$  respectively, are shown. The paths from each entry meter fix to each of the two runway configurations are depicted.

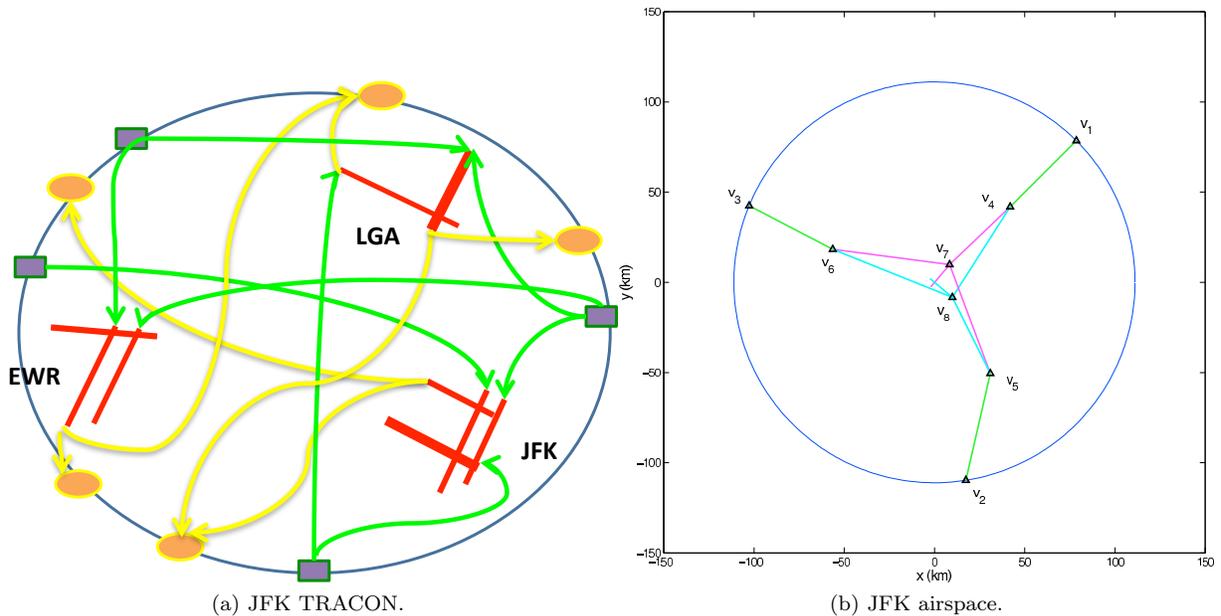


Figure 1: JFK TRACON model (a) and JFK arrival airspace graph (b). The figures are abstractions based on the actual runway directions and the arrival routes.

<sup>a</sup>JFK Tower Control visit, November 2010

## IV.B. Data for Scenario Set-up

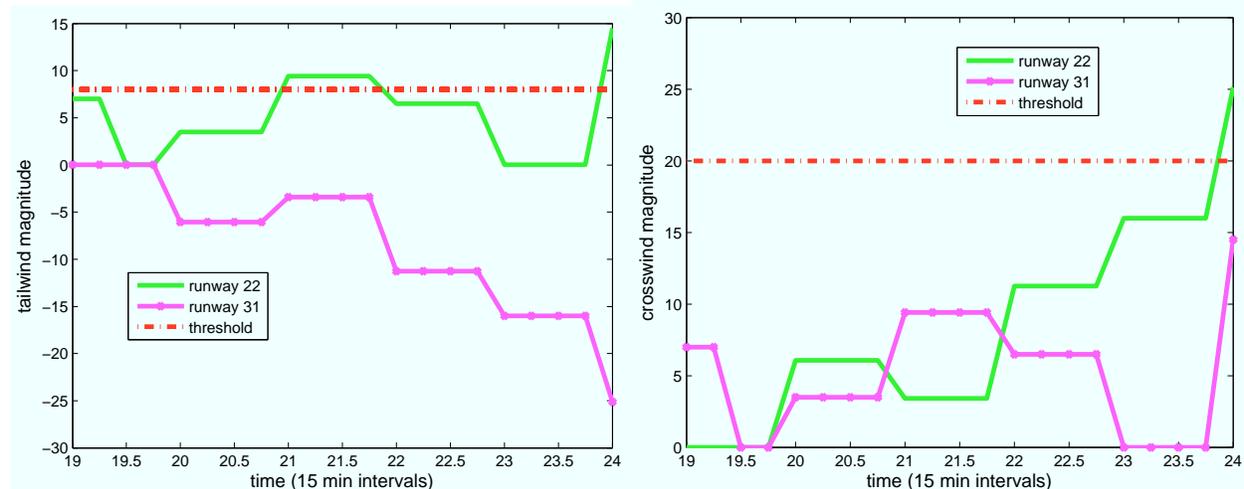
We use the ASPM<sup>b</sup> database in order to instantiate wind forecast data and aircraft arrival times. The day under consideration is 07/01/2009 and the time horizon is the interval [19.00, 24.00] hours during which high wind magnitudes were recorded. The data in ASPM is recorded at 15-minute intervals. Although the wind data is the recordings of actual wind magnitude and direction, we treat this as an uncertain wind forecast for our optimization problem. The aircraft arrival times at the entry meter fixes are generated randomly but with the number of arrivals in each 15-minute interval set according to the ASPM arrival counts. The probability of arrival of aircraft types {Heavy, B757, Large, Small} and the landing speed of these aircraft types were set to {0.390, 0.066, 0.179, 0.365} and {150, 130, 130, 90} knots respectively.<sup>13</sup> The aircraft entry nodes were assigned randomly, with equal probability for each entry node. The required runway separation distance in minutes is shown in Table 1 and is derived based on the data on required separation distance in nautical miles and the average landing speed of aircraft.<sup>13</sup>

Table 1: Runway separation time in minutes. Leading/trailing aircraft as rows/columns respectively.

	Heavy	B757	Large	Small
Heavy	1.60	2.31	2.31	4.00
B757	1.60	1.85	1.85	3.33
Large	1.00	1.15	1.15	2.67
Small	1.00	1.15	1.15	1.67

### IV.B.1. Stage I optimization

We computed the crosswind and tailwind for the two modes,  $c_0 = \{22L, 22R\}$  and  $c_1 = \{31L, 31R\}$  as shown in Figure 2. The crosswind and tailwind thresholds were set to 20 and 8 knots respectively. The initial mode is  $c_0$  and we find that this mode becomes infeasible due to high tailwind at approximately hour 21.00, while runway configuration  $c_1$  remains feasible. Although the ASPM data indicated that a configuration switch occurred at hour 23.00, due to infeasibility of  $c_0$  at approximately hour 21.00 and due to consideration of uncertainty in wind data, we choose the range of allowable switching time as  $[\alpha_1, \alpha_2] = [20.30, 21.30]$ . We aim to choose the switching time in this interval so that aircraft delay is minimized. We consider all aircraft in the JFK airspace in a two-hour planning horizon [20.00, 22.00].



(a) Tailwind at configurations  $c_0$  and  $c_1$ .

(b) Crosswind at configurations  $c_0$  and  $c_1$ .

Figure 2: Wind impact on the runway configurations  $c_0 = \{22L, 22R\}$  and  $c_1 = \{31L, 31R\}$ .

<sup>b</sup>Aviation Systems Performance Metric: <http://www.aspm.faa.gov>

Aircraft paths were set based on the airspace graph and the entry nodes of the aircraft. The paths to the two runway configurations under study from each entry node are shown in Figure 1b. For example, an aircraft arriving at entry node  $v_1$  would take  $\{(v_1, v_4), (v_4, v_7)\}$  to land on either 22L or 22R, and would take  $\{(v_1, v_4), (v_4, v_8)\}$  to land on either 31L or 31R. In order to determine the speed and holding pattern on the edges along the path for each aircraft and the optimal switching time we used Stage II solution approach.

#### IV.B.2. Stage II optimization

Problems (7) was solved for each of the nodes  $v_4, v_5, v_6$  to determine arrival times at these nodes such that aircraft separation constraint is maintained along the first three edges  $(v_1, v_4), (v_2, v_5), (v_3, v_6)$ . The spacing requirement at these nodes was set to 2 minutes which results in a separation distance greater than 5 nmi in the 2-d plane. Then, Problem (8) was solved for each switching time in the interval  $[20.30, 21.30]$  in order to find the optimal switching time and to determine the optimal speed and holding pattern assignment for each aircraft along its final edge. The upper and lower bounds on speed of each aircraft were set to  $1.12\bar{s}$  and  $0.88\bar{s}$ , in which  $\bar{s}$  denotes the nominal speed of the aircraft along an edge. The nominal speed was determined based on aircraft type and distance from the runway. For example, at entry nodes  $v_1, v_2, v_3$ , the nominal speed of the aircraft was set to twice its landing speed and at nodes  $v_4, v_5, v_6$  it was set to 1.5 times its landing speed. Each holding pattern had a duration of 3 minutes<sup>3</sup> and the maximum number of holding patterns at each node for each aircraft was set to  $H = 2$ . Hence, for each aircraft at each node, there were 3 binary variables  $\delta_k^i$ ,  $k = 0, 1, 2$  associated with zero, one and two holding patterns. In order to minimize the number of holding patterns, we penalized each holding pattern by including a cost term  $w(\delta_1^i + \delta_2^i)$ , with  $w > 0$  a weight which was set to 10 in the following simulation.

#### IV.C. Simulation Results

The result of Stage II optimization for an instance of randomly generated arrival data is summarized as follows: In the 2-hour planning horizon there were 85 aircraft in the JFK airspace. We used CPLEX optimization software package to solve Problem (7) for the arrival times at nodes  $v_4, v_5, v_6$  and then used CPLEX to solve Problem (8) by discretizing the time interval  $[20.30, 21.30]$  into 60 minutes and solving a MILP for each of the 60 switching times. Aircraft that arrived at an entry meter fix before/after the switching time were assigned to mode  $c_0/c_1$  respectively. The optimized cost  $J(t_s)$ , which is the total deviations from the nominal landing time summed with the cost of holding patterns, as a function of the switching time  $t_s$  is plotted in Figure 3. From this computation, we find the optimal switching time to be  $t_s^* = 20.37$ . In this optimal solution 5 aircraft are put on hold at node  $v_4$ , each for one holding pattern. Notice that the cost varies greatly based on the switching time. This illustrates that choosing the switching time based on the arrival data can significantly decrease the number of holding patterns and delays. We note that although the recorded switching time from the ASPM database was 23.00, the actual configuration switch may have occurred earlier, as the ASPM configuration data is written manually and may have delays or errors.

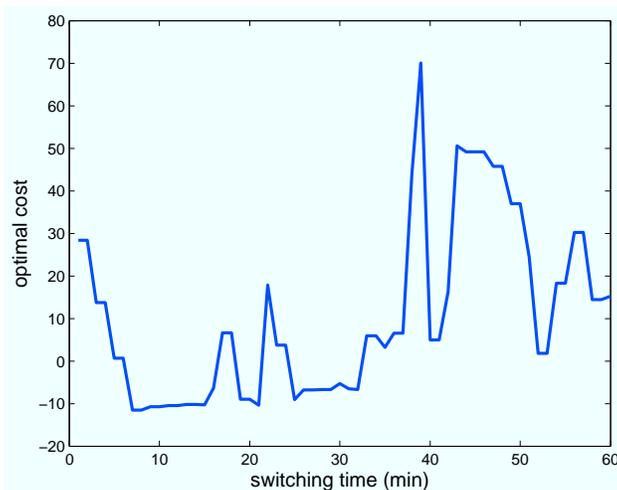
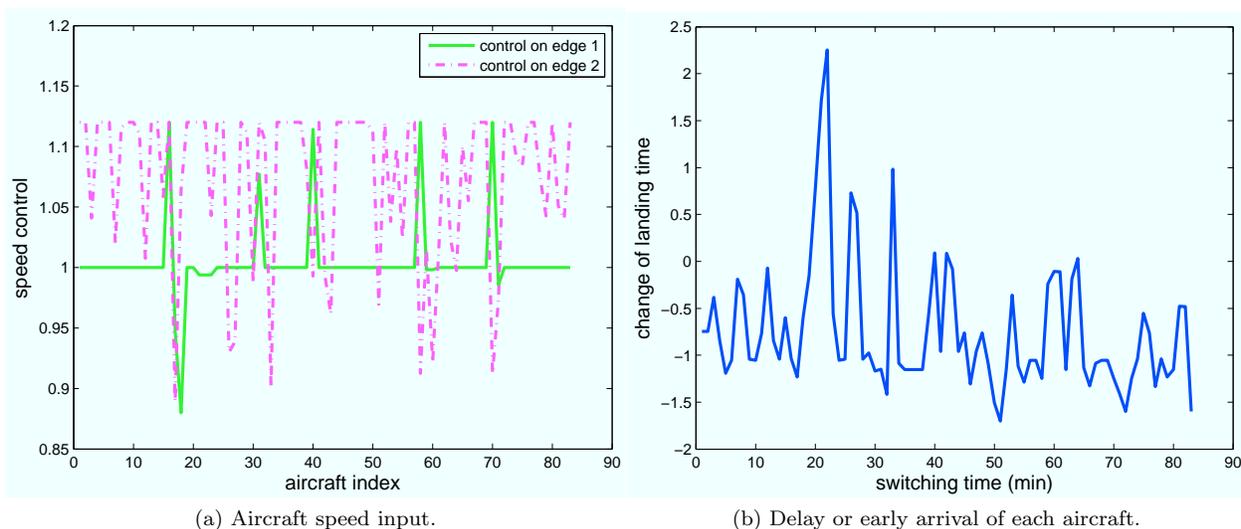


Figure 3: Total cost for each switching time in the interval  $[20.30, 21.30]$  hour.

For the optimal switching time, the fraction of decrease or increase of the speed of each aircraft from its nominal value along the first and second edges of aircraft path are shown in Figure 4a. In addition, the change of the landing times from the nominal landing times of the aircraft are shown in Figure 4b. At the optimal switching time, 9 out of 83 aircraft were delayed with a maximum delay of 2.25 minutes.



(a) Aircraft speed input. (b) Delay or early arrival of each aircraft.  
 Figure 4: Aircraft speed control and total delay/savings in minutes for the optimal switching time.

In terms of computation complexity, the number of binary variables in the CPLEX solver was equal to  $N_a \times H \times V_a$  where  $N_a = 85$  is the number of aircraft,  $H = 3$  and  $V_a = 2$  denote three possibilities for holding pattern at each of the 2 nodes along the path of the aircraft. The average running time for the CPLEX solver was 0.99 seconds on a processor with 2.66 GHz processing speed and 4 GB memory.

## V. Conclusions

In this article, we developed a hierarchical approach to determine the switching time of runway configurations as well as traffic control strategies during the transitional period of a configuration switch in order to minimize the overall traffic delays. The framework is based on deriving an accurate model of arrival airspace structure and is consistent with the TRACON and Tower Control procedures commonly used in practice. We have illustrated our results with a case study inspired by the JFK airspace geometry, arrival and wind data. The case study shows how the framework can be applied to real scenarios to plan the runway configuration and the benefits gained by optimizing aircraft arrival during transitional periods of configuration switching. In the future, we will apply our framework to more realistic air traffic, airspace and weather forecast data.

## Acknowledgments

The authors would like to thank Keenan Roach, Rich DeLaura and John Robinson for insightful discussions on runway configuration switching. We are grateful to Hamsa Balakrishnan and Diana Michalek for organizing a site visit to Boston Logan and JFK Tower Control facilities.

This work was supported by NSF under grant CNS-0931843, by AFOSR under grant FA9550-06-1-0312 and by the Natural Sciences and Engineering Research Council of Canada (NSERC).

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## Appendix

A few of the diagrams associated with JFK Airport that were considered in abstracting the graphical model of this airspace are included.

### V.A. Standard Arrival Routes

A Standard Arrival Route (STAR) indicates a nominal path for the aircraft approach towards an airport. The STAR may include a number of meter fixes at which aircraft could enter into a holding pattern.

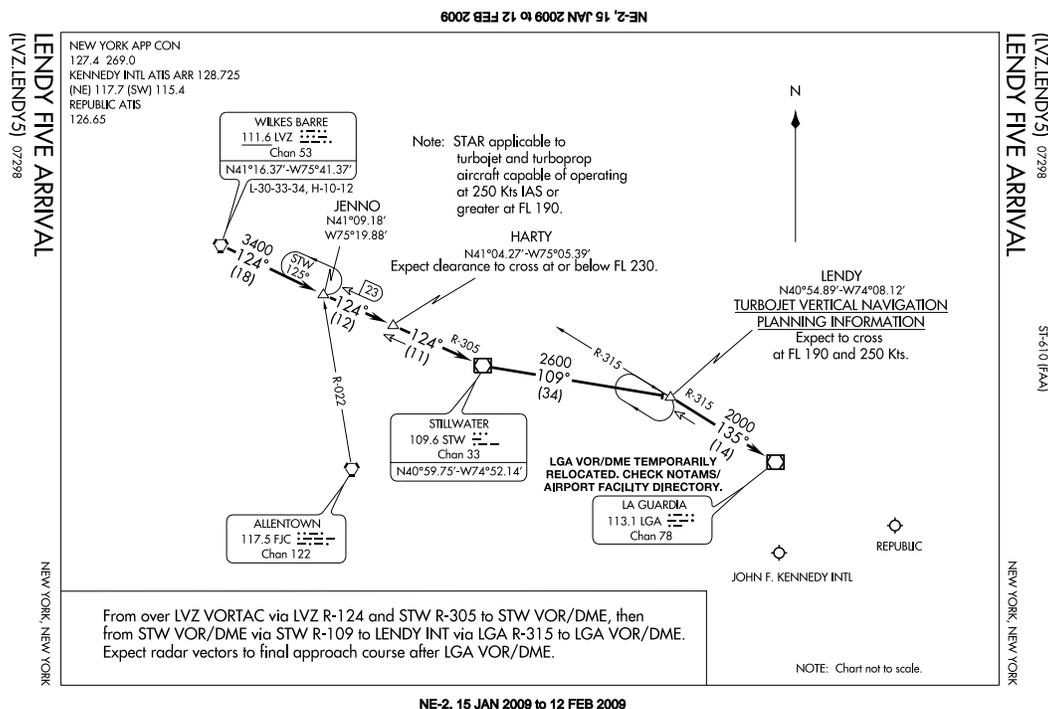


Figure 5: JFK Standard Arrival Route (STAR) corresponding to the meter fix LENDY. As shown in the diagram, aircraft may enter a holding pattern at JENNO or LENDY meter fixes.

## V.B. Runways

Runway numbers indicate the runway direction relative to the magnetic North direction. For example, an arrival on *22L* indicates that aircraft will be landing with a heading of approximately 220 degrees measured from the magnetic North along runway *22L*. A runway can normally be used in two directions and is numbered for each direction separately. JFK airport has two pairs of parallel runways labeled as {*22L/4R*, *22R/4L*} and {*13L/31R*, *13R/31L*} as shown in the figure below. This leads to arrivals from four possible directions and to several possible arrival runway configurations. For example, a common configuration in periods of high traffic is {*22L*, *22R*, *31L*}.

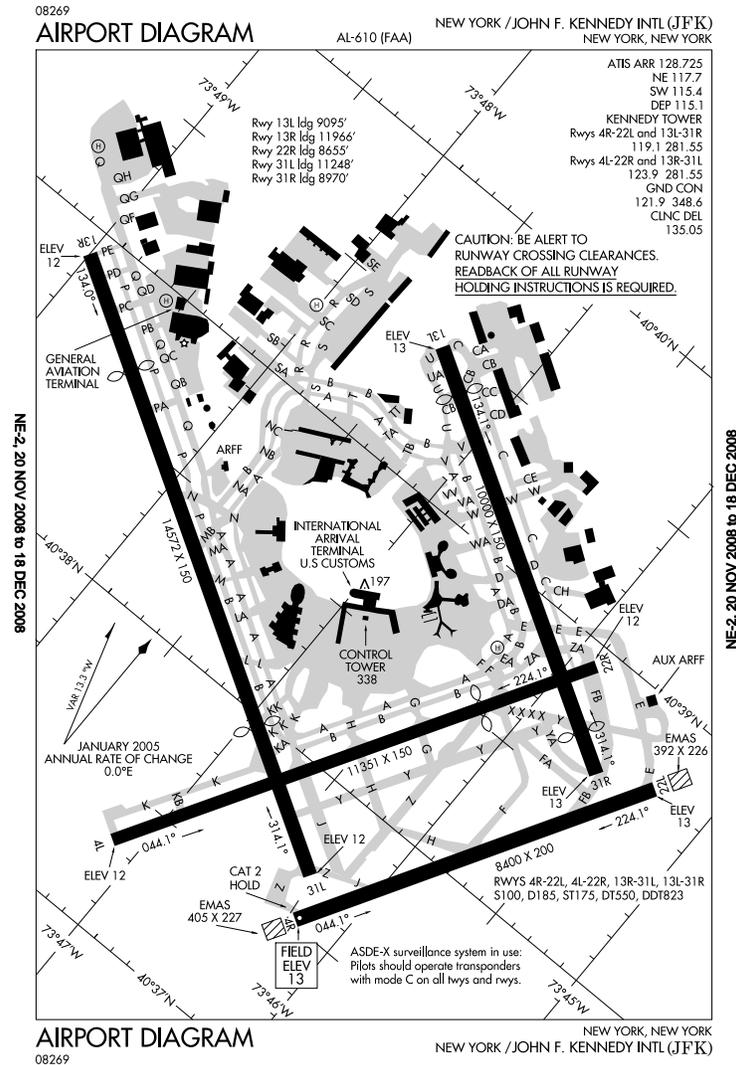


Figure 6: JFK airport diagram.