ECE5463: Introduction to Robotics

Lecture Note 10: Generalized Force and Statics of Open Chains

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Spring 2018
Outline

- Wrench
- Statics of Open Chains
Wrench

• Consider a rigid body with body frame and consider a force \( f \) acting on a point \( r \) on the rigid body

• Define an arbitrary stationary frame \( \{a\} \) and let \( r_a \) and \( f_a \) be the \( \{a\} \)-frame representations of \( r \) and \( f \) vectors. This force create a torque or moment \( m_a \in \mathbb{R}^3 \) in frame \( \{a\} \)

\[
m_a = r_a \times f_a
\]

• Similar to twist, we can merge the moment and force into a single 6D vector. This vector is called the spatial force or wrench.

\[
\mathcal{F}_a = \begin{bmatrix} m_a \\ f_a \end{bmatrix}
\]
Wrench-Twist Pair and Power

• Recall that for a point mass with linear velocity $v$ and linear force $f$. Then we know that the power (instantaneous work done by $f$) is given by $f \cdot v = f^Tv$

• This relation can be generalized to spatial force (i.e. wrench) and spatial velocity (i.e. twist)

• Suppose a rigid body has a twist $\gamma_a = (\omega_a, v_a)$ expressed in $\{a\}$, and a force $f$ is applied at a point $r$ on the rigid body with wrench $F_a$. Then the power is simply

$$\gamma_a \cdot F_a = \gamma_a^T F_a = \omega_a^T m_a + v_a^T f_a$$
Rotational Power

• Consider a point mass with a pure rotational velocity $\omega_a = \dot{\theta} \hat{\omega}_a$, and a moment $m_a$, relative to frame $\{a\}$

• Our previous discussion indicates that its power is

\[
\omega_a^T m_a = \dot{\theta} \cdot (\hat{\omega}_a^T m_a) \triangleq \dot{\theta} \cdot \tau
\]

• $\tau = \hat{\omega}_a^T m_a = m_a^T \hat{\omega}_a$ is the projection of the moment onto the rotation axis, i.e. the effective part of the moment.

• Often times, $\tau$ is also referred to as "torque" with the understanding that it is a scalar quantifying the effectiveness of a moment (i.e. vector torque) relative to some rotation axis.
Wrench Representations in Different Frames

- The wrench $\mathcal{F}_a$ can be expressed in another frame $\{c\}$, provided $T_{ac}$ is known.
- This is not simply rewriting the coordinates of the vectors $m$ and $f$ in $\{c\}$.
- We have to change the vector representation of the point $r$ from $r_a$ (vector from the origin of $\{a\}$ to $r$, expressed in $\{a\}$) to $r_c$ (vector from the origin of $\{c\}$ to $r$, expressed in $\{c\}$).
Wrench Representations in Different Frames

- The power generated by an \((\mathcal{F}, \mathcal{V})\) pair must be the same regardless of the frame in which it is represented.

- Consider two frames \(\{a\}\) and \(\{c\}\). We must have

\[
\mathcal{V}_c^{T} \mathcal{F}_c = \mathcal{V}_a^{T} \mathcal{F}_a = ([\text{Ad}_{T_{ac}}] \mathcal{V}_c)^{T} \mathcal{F}_a = \mathcal{V}_c^{T} ([\text{Ad}_{T_{ac}}])^{T} \mathcal{F}_a
\]

- Since the above relation should hold for all possible twist \(\mathcal{V}_c\), we must have

\[
\mathcal{F}_c = [\text{Ad}_{T_{ac}}]^{T} \mathcal{F}_a
\]

- We are often interested in fixed space frame \(\{s\}\) and body frame \(\{b\}\), we can define a **spatial wrench** \(\mathcal{F}_s\) and **body wrench** \(\mathcal{F}_b\). They are related by

\[
\mathcal{F}_b = [\text{Ad}_{T_{sb}}]^{T} \mathcal{F}_s
\]
Example of Wrench

The robot hand is holding an apple with a mass of 0.1kg in a gravitational field \( g = 10 \text{ m/s}^2 \) (rounded to keep the numbers simple) acting downward on the page. The mass of the hand is 0.5kg. What is the force and torque measured by the six-axis force–torque sensor between the hand and the robot arm?

\[
\begin{align*}
\text{Example 3.28.} & \quad \text{The robot hand in Figure 3.22 is holding an apple with a mass of } 0.1 \text{ kg in a gravitational field } g = 10 \text{ m/s}^2 \text{ (rounded to keep the numbers simple) acting downward on the page. The mass of the hand is 0.5kg. What is the force and torque measured by the six-axis force–torque sensor between the hand and the robot arm?}
\end{align*}
\]
Statics of Open Chains

• Now consider an open-chain robot with \( n \) joints. Let \( \tau \in \mathbb{R}^n \) be the joint torques vector.

• Applying torques to joints will result in motion of the robot and forces of the end effector. By conservation of power:

\[
\text{Power at the joints} = (\text{Power to move the robot}) + (\text{Power at the end-effector})
\]

• At static equilibrium (i.e. no power is used to move the robot), we have

\[
\tau^T \dot{\theta} = \mathcal{F}_b^T \mathcal{V}_b = \mathcal{F}_b^T J_b(\theta) \dot{\theta}
\]

• We can pick \( \dot{\theta} \) infinitesimally small, but in arbitrary direction in \( \mathbb{R}^n \).

\[
\Rightarrow \quad \tau = J_b^T(\theta) \mathcal{F}_b
\]

• If we use the fixed space frame, we will have \( \tau = J_s^T(\theta) \mathcal{F}_s \)
End-Effector Force Analysis

- If an external wrench $\mathcal{F}$ is applied to the end-effector, the joint torques that can generate opposing wrench $-\mathcal{F}$ is given by

$$\tau = J^T(\theta)(-\mathcal{F})$$

- What is the end-effector wrench generated by a given joint torque vector $\tau$?
  - the answer is $(J^T(\theta))^{-1} \tau$ provided $J^T(\theta)$ is invertible
  - If $J^T(\theta)$ is not invertible, the problem is not well defined.
  - An interesting case is when $J^T(\theta)$ has a nontrivial null space:
    $$Null(J^T(\theta)) = \{\mathcal{F} \in \mathbb{R}^6 : J^T(\theta)\mathcal{F} = 0\}$$
    - The wrench that lies in the null space causes no torques, i.e., the balance equation is satisfied with $\tau = 0$; the resisting forces are supplied completely by the robot’s mechanical structure.
Example of Statics of Open Chains

What are the wrenches that can be resisted by the manipulator with $\tau = 0$?
More Discussions

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More Discussions

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