1. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}$, where $a_j, j = 1, \ldots, 4$ are the columns of $A$. Suppose you know (i) $a_1$ and $a_3$ are independent; (ii) $a_1 = 2a_3 + 2a_4$, and $a_2 = 2a_3 + a_4$. What is the rank($A$)? Briefly justify your answer.

2. Given the matrix $A = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$.
   (a) Find the eigenvalues and the corresponding eigenvectors of $A$.
   (b) Find a similarity transformation matrix $T$ and a diagonal matrix $D$ such that $A = TDT^{-1}$.

3. Let $L : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is a linear transformation defined as follows:
   $$L(x) = \begin{bmatrix} x_1 + 2x_2 + 3x_3 \\ 2x_1 + 3x_2 + 4x_3 \end{bmatrix}, \quad \forall x \in \mathbb{R}^3$$
   - Find a matrix representation of $L(\cdot)$, i.e., find a matrix $A$ such that $L(x) = Ax$.
   - Suppose we use a new basis $\{a\}$ for the source space $\mathbb{R}^3$ and a new basis $\{b\}$ for the target space $\mathbb{R}^2$, where
     $$\{a\} = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} \right\}, \text{ and } \{b\} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$$
   Find the matrix representation for $L$ under the new bases.

4. Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 17.5 N at a distance of 0.600 m from the hinges, and the second child pushes at a distance of 0.450 m. What force must the second child exert to keep the door from moving? Assume friction is negligible.

5. The latitude of Los Angeles is around 35° North of the equator. The radius of earth is approximately 6.37 \times 10^6 meters. The earth rotates around its axis (points towards north pole) at roughly 7.27 \times 10^{-5} deg/sec. What is the speed of a person in Los Angeles due to the earth’s rotation about its axis.

6. For any vector $a \in \mathbb{R}^3$, let $[a]$ denote its skew-symmetric representation as defined in lecture note 1. Show the following property:
   $$[a \times b] = [a][b] - [b][a]$$