ECE7850: Spring 2017
Hybrid Systems: Theory and Applications

Lecture 1: Course Info and Hybrid System Examples

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Course Info

- Instructor: Wei Zhang

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- Time: Tu/Th 11:10am – 12:30pm

- Location: Macquigg Lab 155

- Office Hour: Thursday 1-2pm

- Website: http://www2.ece.ohio-state.edu/~zhang/HybridSystemCourse/HybridSystemsCourse_Sp17.html

- Prerequisite:
  - ECE 5750 – Linear System Theory
  - Solid math background is essential
- **Grading Policy**
  - **Homework (30%)**
    - Assigned biweekly (roughly)
    - May involve open-ended questions
    - Must be typeset using Latex
    - *Can be quite challenging!*
  - **Midterm (30%)**: Date & Time: TBD (may be an evening exam)
  - **Final Project (40%)**: 
    - Project proposal due shortly after midterm
    - Project report due in the final exam week;
    - 15-minute presentation at the end of the semester
    - Some ideas of project topics
    - Nontrivial extension of the results introduced in class
    - Nontrivial application of HS in your research area
    - Comprehensive literature review on a topic in HS not covered in the class
Course Materials:

• No required textbook!

• Lecture notes are developed based on

  - Important papers in the field of hybrid systems


  - “Switching in systems and control”, D. Liberzon, 2003

  - “Predictive Control for linear and hybrid systems”, F. Borrelli, A. Bemporad and M. Morari, 2013
Tentative Topics

- Introduction to Hybrid Systems
  - Examples, Modeling frameworks, Solution and execution, Filippov solution, zeno phenomena

- Stability Analysis and Stabilization
  - Stability under arbitrary switching, stability under constrained switching, Multiple-Lyapunov function, LMI based synthesis using multiple-Lyapunov function; control-Lyapunov function approach

- Discrete Time Optimal Control of Hybrid Systems
  - Switched LQR problem, MPC of switched Piecewise Affine Systems, Infinite-horizon optimal control and its connection to stability/stabilization

- Reachability analysis and computation:
  - Forward/backward reachable sets, HJI based reachability, zonotope based method, applications and automated vehicles

- Continuous Time Optimal Control of Hybrid Systems
  - Theory of numerical optimization in infinite-dimensional space, applications to optimal control of switched nonlinear systems
Special Notes

- Advanced but not seminar type of course (many assignments)
- Goal: prepare and train the students to develop new theories
- Growing field with important emerging applications
  - Networked control systems, Cyber-Physical Systems, Robotics, Intelligent transportation
- Caveat:
  - No standard textbooks
  - Few existing HS courses have a good balance among different topics
  - We will try to cover a wide range of major topics in depth
  - Each topic requires good understanding of some background materials that will be introduced at very fast pace
  - Mathematical maturity is essential!
What is Hybrid Systems

- Roughly: dynamical systems with combined continuous and discrete dynamics
  - Continuous state $x(t)$  continuous input $u(t)$
  - Discrete state/mode $q(t)$  discrete input $\sigma(t)$

- Coupled continuous-discrete dynamics
  - Discrete mode evolution:
    - $q^+ = g(x, q, \sigma)$
  - Mode-dependent continuous dynamics:
    - $\dot{x} = f(x, q, u)$

- Interactions:
  - Continuous state evolution $x$ triggers discrete mode transition
    - "Guard": subset of state space; mode transition occurs when state hitting guard
    - Reset map: continuous state may jump during mode transition
  - Mode transition modifies continuous dynamics characteristics
Hybrid System Example 1: Bouncing Ball

- **Bouncing ball:**
  - State of system: \[
  \begin{align*}
  \dot{x}_1 &= p \quad \text{(position)} \\
  \dot{x}_2 &= \dot{x}_1 \quad \text{(velocity)}
  \end{align*}
  \]

- **Mode 1: Free fall:**
  \[
  \begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2
  \end{bmatrix} =
  \begin{bmatrix}
  x_2 \\
  -mg
  \end{bmatrix} \iff \text{true until hitting ground}
  \]

- **Mode 2: Collision:**
  \[
  t_1: \text{collision time}
  \]
  \[
  \begin{cases}
  x(t_1^+) = x_1(t_1) \\
  x_2(t_1^+) = -c x_2(t_1)
  \end{cases}
  \]
  \[
  x_1 = x_2 \\
  \dot{x}_2 = -mg
  \]
  \[
  x_2^+ := -c x_2 \quad \text{reset map}
  \]
  \[
  x_i = 0 \& x_2 < 0 \quad \text{(guard condition)}
  \]
  \[\text{HIS Model: have one mode}\]

Hybrid System Example 2: Water Tank

- **Goal:** keep water level above references
  
  \[ q=1 \quad q=2 \]

- **Two modes:** left/right

- **Dynamics:**
  
  \[ \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} w - v_1 \\ -v_2 \end{bmatrix} \equiv f_1(x) \]

  \[ q=1 \text{ (left):} \]

  \[ \dot{x} = \begin{bmatrix} -v_1 \\ w - v_2 \end{bmatrix} \equiv f_2(x) \]

  \[ q=2 \text{ (right):} \]

- **Guard:**
  
  From \( q=1 \) to \( q=2 \):
  \[ \{ (x_1, x_2) \in \mathbb{R}^2 \mid x_2 \leq r_2 \} \]

  From \( q=2 \) to \( q=1 \):
  \[ \{ (x_1, x_2) \in \mathbb{R}^2 \mid x_1 \leq r_1 \} \]

  There is state jump (no reset)
Hybrid System Example 3: Converter

- Two modes:

  \[ d(k) \]

  \[ S_1 = 1 \]
  \[ S_2 = 0 \]

  \[ kT_s < t < (k+1)T_s \]

- Objectives: minimize output voltage error under uncertain \( v_s, r_o \)

  Two modes:
  \[ q=1 \]
  \[ \dot{x} = f(x) = A_1 x + g_1 v_s \] \[ kT_s < t < (k + d(k))T_s \]

  States (e.g., \( v_c, i_c \))

  \[ q=2 \]
  \[ \dot{x} = A_2 x \] \[ t \in (kT_s, (k+1)T_s) \]

  If \( d(k) \) is constant:

  mode transition triggered by time only

  \[ t > (k+d(k))T \]

  using feedback

  \[ d(k) = M(x(k)) \] \[ q=1 \] \[ q=2 \]
Hybrid System Example 4: Air Traffic Control

- Unicycle aircraft model: 
  \[
  \begin{bmatrix}
  \dot{x}_1^a \\
  \dot{x}_2^a \\
  \dot{x}_1^b \\
  \dot{x}_2^b \\
  \end{bmatrix} =
  \begin{bmatrix}
  v \cos \theta_a \\
  v \sin \theta_a \\
  v \cos \theta_b \\
  v \sin \theta_b \\
  \end{bmatrix}
  \]

- Simple collision avoidance protocol:
  - Left if \(|x^a - x^b| < \alpha\) (\(\dot{\tau} = 1\), measure time)
  - Straight until \(|x^a - x^b| > \alpha\)
  - Right (\(\dot{\tau} = -1\))
  - Cruise

- This HS has 4 modes

- Continuous state: 
  \[
  \begin{bmatrix}
  x_a^a \\
  x_b^a \\
  \theta^a \\
  \theta_b \\
  \end{bmatrix}, \quad \tau 
  \]
  time counter

  heading angle
Hybrid System Example 4: Air Traffic Control

- Continue:

\[
\begin{align*}
\dot{\mathbf{x}} &= f(\mathbf{x}, \theta) \\
\dot{\theta} &= 0 \\
\dot{t} &= 0
\end{align*}
\]

**Cruise**

\[
\begin{align*}
\mathbf{x}^c &= \mathbf{x}^a \\
\theta^c &= 0 \\
t^c &= 0
\end{align*}
\]

\[\|\mathbf{x}^a - \mathbf{x}^b\| \leq \alpha\]

\[\theta_{ab}^+ = \theta_{ab} + \frac{\alpha}{4}\]

**Right**

\[
\begin{align*}
\mathbf{x}^r &= f(\mathbf{x}, \theta) \\
\theta^r &= 0 \\
t^r &= -1
\end{align*}
\]

\[\theta_{ab}^+ = \theta_{ab} + \frac{\alpha}{4}\]

**Straight**

\[
\begin{align*}
\dot{\mathbf{x}} &= f(\mathbf{x}, \theta) \\
\dot{\theta} &= 0 \\
\dot{t} &= 0
\end{align*}
\]

\[\theta_{ab}^- = \theta_{ab} - \frac{\alpha}{4}\]

**Stop**

\[
\begin{align*}
\mathbf{x}^s &= f(\mathbf{x}, \theta) \\
\theta^s &= 0 \\
t^s &= 1
\end{align*}
\]

\[\theta_{ab}^- = \theta_{ab} - \frac{\alpha}{4}\]

\[t \geq 10 \text{ (min)}\]

**Question:** whether the protocol is safe?
Hybrid System Example 5: Variable Structure Control

- Standard nonlinear dynamics: \( \dot{x} = f(x, u) \)

- Piecewise continuous control laws:

  \[
  \begin{align*}
  \text{partition state space into regions} \\
  \text{define control laws: } u(x) = \begin{cases} 
  u_1(x) & \text{if } x \in \Omega_1 \\
  u_2(x) & \text{if } x \in \Omega_2
  \end{cases}
  \end{align*}
  \]

  In this case, two modes

  Guard \( 1 \rightarrow 2 \) : \( \Omega_2 \)

  \( 2 \rightarrow 1 \) : \( \Omega_1 \)

  no reset
Hybrid System Example 5: Variable Structure Control

- Application in UAV control:
Hybrid System Example 6-1: Networked Control Systems

Simple NCS:

- $t_k$: $k^{\text{th}}$ transmission time
- $\dot{x}(t) = f(x(t), u(t))$
- $u(t) = Kx(t_{k-1})$
- $e(t) = x(t) - x(t_{k-\tau}), z(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}$

$$\begin{align*}
\dot{e}(t) &= f\left( x(t), \tau, x(t_{k-\tau}) \right) \\
&= f\left( z(t), \tau \right) \\
\Rightarrow \\
\hat{z} &= f
\end{align*}$$
Hybrid System Example 6-2: Event-Triggered Control

Event triggered control:

- Transmit: $z(t) \in E$
- $\dot{x}(t) = \tilde{f}(x(t), e(t))$
- $\dot{e}(t) = \tilde{f}(x(t), e(t))$
- $e(t_{k+}^+) = 0$

How to determine $E$ to ensure closed-loop stability?
Hybrid Systems Example 7: Embedded Systems

- Dynamic buffer management
  - Continuous state $x$
  - Discrete mode:

DBM Problem: Find best $Q$ and switching strategy to minimize the total energy subject to constraints
• Summary:
  • Most general and natural modeling framework
  • Numerous applications
  • Further reading: reference papers in the “Application” category of the course website
  • Active area of research with many open challenges
  • This class is only an introduction to some important topics

• Next time:
  • Formal discussion on hybrid system models and solution concepts