ECE7850: Spring 2017
Hybrid Systems: Theory and Applications

Lecture 1: Course Info and Hybrid System Examples

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Course Info

- Instructor: Wei Zhang
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- Time: Tu/Th 11:10am – 12:30pm
- Location: Macquigg Lab 155
- Office Hour: Thursday 1-2pm
- Website: http://www2.ece.ohio-state.edu/~zhang/HybridSystemCourse/HybridSystemsCourse_Sp17.html
- Prerequisite:
  - ECE 5750 – Linear System Theory
  - Solid math background is essential
- **Grading Policy**
  - **Homework (30%)**
    - Assigned biweekly (roughly)
    - May involve open-ended questions
    - Must be typeset using Latex
    - Can be quite challenging!
  - **Midterm (30%)**: Date & Time: TBD (may be an evening exam)
  - **Final Project (40%)**:
    - Project proposal due shortly after midterm
    - Project report due in the final exam week;
    - 15-minute presentation at the end of the semester
    - Some ideas of project topics
      - Nontrivial extension of the results introduced in class
      - Nontrivial application of HS in your research area
      - Comprehensive literature review on a topic in HS not covered in the class
Course Materials:

• No required textbook!

• Lecture notes are developed based on
  - Important papers in the field of hybrid systems
  - “Switching in systems and control”, D. Liberzon, 2003
  - “Predictive Control for linear and hybrid systems”, F. Borrelli, A. Bemporad and M. Morari, 2013
Tentative Topics

- Introduction to Hybrid Systems
  - Examples, Modeling frameworks, Solution and execution, Filippov solution, zeno phenomena

- Stability Analysis and Stabilization
  - Stability under arbitrary switching, stability under constrained switching, Multiple-Lyapunov function, LMI based synthesis using multiple-Lyapunov function; control-Lyapunov function approach

- Discrete Time Optimal Control of Hybrid Systems
  - Switched LQR problem, MPC of switched Piecewise Affine Systems, Infinite-horizon optimal control and its connection to stability/stabilization

- Reachability analysis and computation:
  - Forward/backward reachable sets, HJI based reachability, zonotope based method, applications and automated vehicles

- Continuous Time Optimal Control of Hybrid Systems
  - Theory of numerical optimization in infinite-dimensional space, applications to optimal control of switched nonlinear systems
Special Notes

- Advanced but not seminar type of course (many assignments)

- Goal: prepare and train the students to develop new theories

- Growing field with important emerging applications
  - Networked control systems, Cyber-Physical Systems, Robotics, Intelligent transportation

- Caveat:
  - No standard textbooks
  - Few existing HS courses have a good balance among different topics
  - We will try to cover a wide range of major topics in depth
  - Each topic requires good understanding of some background materials that will be introduced at very fast pace
  - Mathematical maturity is essential!
What is Hybrid Systems

- Roughly: dynamical systems with combined continuous and discrete dynamics
  - Continuous state $x(t)$ continuous input $u(t)$
  - Discrete state/mode $q(t)$ discrete input $\sigma(t)$

- Coupled continuous-discrete dynamics
  - Discrete mode evolution:
    - $q^+ = g(x, q, \sigma)$
  - Mode-dependent continuous dynamics:
    - $\dot{x} = f(x, q, u)$

- Interactions:
  - Continuous state evolution $x$ triggers discrete mode transition
  - “Guard”: subset of state space; mode transition occurs when state hitting guard
  - Reset map: continuous state may jump during mode transition

- Mode transition modifies continuous dynamics characteristics
Hybrid System Example 1: Bouncing Ball

- Bouncing ball:
  - State of system: \[
  \begin{cases} 
  \dot{x}_1 = p \\
  \dot{x}_2 = \dot{x}_1 
  \end{cases}
  \]

  - Mode 1: Free fall:

  - Mode 2: Collision:
Hybrid System Example 2: Water Tank

- **Goal:** keep water level above references
- **Two modes:** left/right
- **Dynamics:**
  - **Guard:**
Hybrid System Example 3: Converter

- Two modes:

- Objectives: minimize output voltage error under uncertain $v_s, r_o$
Hybrid System Example 4: Air Traffic Control

- Unicycle aircraft model: 
  \[
  \begin{bmatrix}
  \dot{x}_1^a \\
  \dot{x}_2^a \\
  \end{bmatrix} = \begin{bmatrix} v \cos \theta_a \\
  v \sin \theta_a \end{bmatrix}, \quad 
  \begin{bmatrix}
  \dot{x}_1^b \\
  \dot{x}_2^b \\
  \end{bmatrix} = \begin{bmatrix} v \cos \theta_b \\
  v \sin \theta_b \end{bmatrix}
  \]

- Simple collision avoidance protocol:
  - Left if \(\|x^a - x^b\| < \alpha\) (\(\dot{\tau} = 1\), measure time)
  - Straight until \(\|x^a - x^b\| > \alpha\)
  - Right (\(\dot{\tau} = -1\))
  - Cruise

- This HS has 4 modes

- Continuous state: 
  \[
  \begin{bmatrix}
  x^a \\
  x^b \\
  \theta \\
  \end{bmatrix}, \quad \theta = \begin{bmatrix} \theta^a \\
  \theta^b \end{bmatrix}, \quad \tau
  \]
Hybrid System Example 4: Air Traffic Control

- Continue:
Hybrid System Example 5: Variable Structure Control

- Standard nonlinear dynamics: $\dot{x} = f(x, u)$
- Piecewise continuous control laws:
Hybrid System Example 5: Variable Structure Control

- Application in UAV control:
Hybrid System Example 6-1: Networked Control Systems

Simple NCS:

- $t_k$
- $\dot{x}(t) = f(x(t), u(t))$
- $u(t) = Kx(t_{k-\tau})$
- $e(t) = x(t) - x(t_{k-\tau}), z(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}$
Hybrid System Example 6-2: Event-Triggered Control

Event triggered control:

- Transmit: $z(t) \in E$
- $\dot{x}(t) = \tilde{f}(x(t), e(t))$
- $\dot{e}(t) = \tilde{f}(x(t), e(t))$
- $e(t_k^+) = 0$

- How to determine $E$ to ensure closed-loop stability?
Hybrid Systems Example 7: Embedded Systems

- Dynamic buffer management
  - Continuous state \(x\)
  - Discrete mode:

DBM Problem: Find best \(Q\) and switching strategy to minimize the total energy subject to constraints
• **Summary:**
  - Most general and natural modeling framework
  - Numerous applications
  - Further reading: reference papers in the “Application” category of the course website
  - Active area of research with many open challenges
  - This class is only an introduction to some important topics

• **Next time:**
  - Formal discussion on hybrid system models and solution concepts