1. Download the MPT3 toolbox and install it. Implement the MPC controller for the Cessna citation airport example discussed in Lecture Note 9. No need to go through the intermediate steps. Directly implement the MPC control with all the constraints. Please try several different initial states and attach your code and simulation results. Note: to obtain discrete-time model from continuous time model, please refer to Lecture Note 28 of [http://ee263.stanford.edu/lectures.html](http://ee263.stanford.edu/lectures.html)

2. Consider the double integrator example (Example 2 of Lecture Note 11) with the same parameters (terminal constraint $X_f = X$ and no terminal cost).

   (a) Use the MPT3 toolbox to compute the feasible sets: $X_0$, $X_1$, and $X_2$. Plot these sets in figures.

   (b) Read the MPC textbook [BBM17] Chapter 11.2.
      i. Use the toolbox to compute the maximal control invariant set
      ii. Solve the corresponding unconstrained LQR problem and use the toolbox to compute the maximal positive invariant set for the closed-loop system $x_{k+1} = (A + BK^*)x_k$ where $K^*$ is the optimal infinite-horizon LQR control gain.
      iii. Discuss the relations of the two sets.

   (c) Use the above positive invariant set computed through LQR as terminal constraint and the infinite horizon LQR value function $V^*(z)$ as the terminal cost. Implement the MPC and show the simulation results.

3. Consider the general nonlinear $N$-horizon optimal control problem:

   $\mathcal{P}_N(x(t)) : \quad V_N(x(t)) = \begin{cases} 
   \min_{U_0} & J_N(x(t), U_0) \triangleq J_f(x_N) + \sum_{k=0}^{N-1} l(x_k, u_k) \\
   \text{subj. to:} & x_{k+1} = f(x_k, u_k), k = 0, \ldots, N - 1 \\
   & x_k \in \mathcal{X}, u_k \in \mathcal{U}, k = 0, \ldots, N - 1 \\
   & x_N \in \mathcal{X}_f, \quad x_0 = x(t) 
   \end{cases}$

   Let $\pi^*_N = \{\mu^*_N, \mu^*_{N-1}, \ldots, \mu^*_1\}$ be the optimal $N$-horizon policy. Assume that Assumptions 1 and 2 in Lecture Note 11 hold.

   Consider a new receding-horizon controller, for which the above optimal control problem is solved online in every two sampling instants (i.e. at time $t = 0, 2, 4, \ldots$) and the first two steps of obtained control sequences are applied to the system.
(a) Write down the closed-loop system dynamics analytically using the notations introduced above.

(b) Show that if $X_f$ is chosen as a control-invariant set, then the problem is persistently feasible (you need to generalize the persistent feasibility concept to this case).

(c) Show that with $X_f = X$ and $J_f \equiv 0$, the closed-loop system is exponentially stable for sufficiently large prediction horizon $N$.

References