Ultra-wideband microwave imaging of breast cancer tumors via Bayesian inverse scattering

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We develop a new algorithm for ultra-wideband (UWB) microwave imaging of breast cancer tumors using Bayesian inverse scattering. A key feature of the proposed algorithm is that constitutive properties of breast tissues are reconstructed from scattered UWB microwave signals together with the confidence level of the reconstruction. Having such confidence level enables minimization of both false alarms and missed detections. Results from the application of the proposed algorithm demonstrate the accuracy in estimating both location and permittivity of breast tumors without the need for a priori knowledge of pointwise properties of the background breast tissue. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4865327]

I. INTRODUCTION

Microwave imaging of breast tissue with the aim of detecting and locating malignant tumors has been an active topic of research.1–8 The difference in electrical properties of tumors from those of healthy breast tissues enables tumor detection through electromagnetic inversion techniques. Microwave breast imaging provides a cheaper and safer alternative to existing imaging techniques such as magnetic resonance imaging (MRI) and X-ray mammography.1,2 A main challenge for accurate breast tumor localization using microwaves lies in the lack of precise knowledge of point-wise constitutive properties of the breast tissue, as well as of the skin layer and chest wall morphology. This leads to inaccurate extraction of the target (tumor) response and inaccurate computation of (background) Green’s function necessary for imaging. When the goal is solely to detect the location of the tumor rather than to produce a quantitative image of the breast constitutive properties, scattering contributions other than from the tumor are identified as clutter. Several signal-processing techniques were introduced to suppress or mitigate the effects of such clutter. In Refs. 6 and 7, the tumor-free response was assumed known and direct subtraction was used to obtain the tumor response. Time-reversal processing9–11 was then used to localize the tumor in the (assumed known) background medium. In Refs. 3–5, a detection algorithm based on time-reversal adaptive interference canceling (TRAIC) was developed. This algorithm also requires acquisition and storage of a baseline measurement of the healthy breast, which is unfeasible in many cases. A matched filter approach for target response estimation was introduced in Ref. 12. This requires only an estimation of the clutter template as an input. Another target estimation technique, which does not require any prior training, was devised in Ref. 13. It uses a data-adaptive filter and an envelope-detection algorithm. More recently, a beam subspace time-reversal technique was introduced in Ref. 14. It was shown to help reducing the interference due to dense fibroglandular tissues and focus at the correct target location.

Full quantitative inversion of the spatial distribution of breast dielectric properties has also been the focus of many studies, e.g., Refs. 15–17. In Ref. 15, the integral equation of electromagnetic scattering from breast tissue was linearized using the Born approximation and the conjugate gradient (CG) least-squares was used to solve the resulting ill-conditioned system coupled with regularization techniques such as Tikhonov regularization. The solution was then iterated using the distorted Born iterative method (DBIM). A similar approach was adopted in Ref. 18 based on a sparsity regularization imposed on the spatial distribution of breast tissue treated by contrast agents that influence only the tumor. Models with different levels of complexity and fidelity have been used for the breast tissue and morphology. In Ref. 19, a 3-D semi-ellipsoidal model was developed. Two-dimensional (2-D) MRI-derived breast phantoms was recently introduced in Refs. 3–6, 12. More realistic 3-D models were developed in Refs. 7 and 20. An experimental model that contains fat, skin and tumor phantoms was recently introduced in Ref. 21.

In this paper, we propose a Bayesian ultra-wideband (UWB) inverse scattering algorithm to reconstruct the electrical properties of the breast tissue, from which the presence of breast tumors can be easily inferred. Our approach requires a priori estimate of only the mean permittivity and conductivity of breast tissue, and only approximate knowledge of skin and chest wall properties. As noted in Ref. 15, the location and thickness of the skin layer can be estimated,22 or directly measured in a clinical setup23 using several techniques; hence, it can be assumed to be known a priori. Generally speaking, Bayesian inversion techniques combine (any) a priori information on the domain to be imaged with sensor acquisitions to produce an a posteriori probability density functions (PDFs) of the unknowns.24–31 Bayesian inversion provides a route for computing the

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confidence interval of the inversion and hence for adaptively optimizing subsequent measurement(s), if needed. In particular, Bayesian inference applied to compressive sensing was considered in Refs. 33 and 34, where sparsity priors were imposed on a compressible (sparse) set of unknowns. That problem was solved efficiently using a relevance vector machine (RVM) technique. Bayesian compressive sensing was also applied to microwave imaging of sparse discrete scatterers using single-frequency data, with either a contrast source formulation or a first-order Born approximation. The projection matrix can now be written in matrix form as

\[ \mathbf{G}_{t,k} = \begin{bmatrix} \mathbf{A}_{t,k} & \mathbf{B}_{t,k} \\ \mathbf{C}_{t,k} & \mathbf{D}_{t,k} \end{bmatrix}, \]

where

\[ \mathbf{A}_{n,p} = \text{Re}\{\mathbf{g}_{p,n}(\mathbf{r}_n, \omega_k)\}, \]

\[ \mathbf{B}_{n,p} = -\text{Im}\{\mathbf{g}_{p,n}(\mathbf{r}_n, \omega_k)\} \times \left( \frac{-1}{\omega_k \varepsilon_0} \right), \]

\[ \mathbf{C}_{n,p} = \text{Im}\{\mathbf{g}_{p,n}(\mathbf{r}_n, \omega_k)\}, \]

\[ \mathbf{D}_{n,p} = \text{Re}\{\mathbf{g}_{p,n}(\mathbf{r}_n, \omega_k)\} \times \left( \frac{-1}{\omega_k \varepsilon_0} \right) \]

and \( \mathbf{g}_{p,n}(\mathbf{r}_n, \omega_k) = \int_{\mathcal{D}} \mathcal{E}_{0}(\mathbf{r}', \omega_k) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega_k) d\mathbf{r}' \).

The remainder of this paper is organized as follows: In Sec. II, we present the mathematical formulation of UWB Bayesian inverse scattering. In Sec. III, we present examples of imaging normal and tumorous breasts based on 2-D numerical simulations. Conclusions and possible future extensions are discussed in Sec. IV.

II. UWB BAYESIAN INVERSE SCATTERING

The scattered field at location \( \mathbf{r} \) and frequency \( \omega_k \) resulting from a total field \( \mathcal{E}_{t}^{\text{tot}} \) generated by transmitter \( t \), is given by

\[ \mathcal{E}_{t}^{\text{tot}}(\mathbf{r}, \omega_k) = \int_{\mathcal{D}} \tau(\mathbf{r}', \omega_k) \mathcal{E}_{t}^{\text{tot}}(\mathbf{r}', \omega_k) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega_k) d\mathbf{r}', \]  

(1)

where \( \mathbf{G} \) is the (scalar) Green’s function of the background medium, \( \mathcal{D} \) is the support of the scattering object, and we assume a 2-D problem under TM illumination for simplicity. In the above, \( \tau \) is the complex-valued contrast function given by

\[ \tau(\mathbf{r}, \omega_k) = [\varepsilon_{t}(\mathbf{r}, \omega_k) - 1] - j \frac{\sigma(\mathbf{r}) - \langle \sigma \rangle}{\omega_k \varepsilon_0}, \]

\[ = \Delta \varepsilon(\mathbf{r}) - j \frac{1}{\omega_k \varepsilon_0} \Delta \sigma(\mathbf{r}), \]

(2)

where \( \varepsilon_t \) and \( \sigma \) are the relative permittivity and the conductivity, respectively, and \( \langle \varepsilon_t \rangle \) and \( \langle \sigma \rangle \) represent the corresponding mean values in the background medium. The real and imaginary parts of the scattered field, recorded at \( N_s \) sensors positioned at \( \mathbf{r}_n \), with \( n = 1, \ldots, N_s \), can be stacked in a column vector as \( \mathbf{e}_{t,k} = [\text{Re}\{\mathcal{E}_{t}^{\text{tot}}(\mathbf{r}_1, \omega_k)\}, \ldots, \text{Re}\{\mathcal{E}_{t}^{\text{tot}}(\mathbf{r}_{N_s}, \omega_k)\}, \text{Im}\{\mathcal{E}_{t}^{\text{tot}}(\mathbf{r}_1, \omega_k)\}, \ldots, \text{Im}\{\mathcal{E}_{t}^{\text{tot}}(\mathbf{r}_{N_s}, \omega_k)\}]^T \). By discretizing the domain of investigation (DOI) into \( N_p \) pixels, each of size \( D_p \), with \( p = 1, \ldots, N_p \), and assuming a pulse basis function expansion for the contrast, the projection matrix can be written as

\[ \mathbf{F}^{-1} = \begin{bmatrix} \mathbf{A}_{t,k} & \mathbf{B}_{t,k} \\ \mathbf{C}_{t,k} & \mathbf{D}_{t,k} \end{bmatrix}, \]

where

\[ \mathbf{A}_{n,p} = \text{Re}\{\mathbf{g}_{p,n}(\mathbf{r}_n, \omega_k)\}, \]

\[ \mathbf{B}_{n,p} = -\text{Im}\{\mathbf{g}_{p,n}(\mathbf{r}_n, \omega_k)\} \times \left( \frac{-1}{\omega_k \varepsilon_0} \right), \]

\[ \mathbf{C}_{n,p} = \text{Im}\{\mathbf{g}_{p,n}(\mathbf{r}_n, \omega_k)\}, \]

\[ \mathbf{D}_{n,p} = \text{Re}\{\mathbf{g}_{p,n}(\mathbf{r}_n, \omega_k)\} \times \left( \frac{-1}{\omega_k \varepsilon_0} \right) \]

and \( \mathbf{g}_{p,n}(\mathbf{r}_n, \omega_k) = \int_{\mathcal{D}} \mathcal{E}_{0}(\mathbf{r}', \omega_k) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega_k) d\mathbf{r}' \).

We seek maximum a posteriori (MAP) estimates for the weights \( \mathbf{w} = \text{argmax}_{\mathbf{w}} p(\mathbf{w}|\mathbf{e}') \). We then use this posterior PDF of \( \mathbf{w} \) conditioned on the measurement system noise \( \mathbf{e}' \). From Bayes’ rule, the posterior PDF is given by

\[ p(\mathbf{w}|\mathbf{e}') = \frac{p(\mathbf{e}'|\mathbf{w}) p(\mathbf{w})}{p(\mathbf{e}')} \]

Direct Computation of the likelihood \( p(\mathbf{e}'|\mathbf{w}) \) from (8) requires computationally expensive Monte Carlo and Gibbs sampling, as done, e.g., in Refs. 29–31 and 43. Fortunately, two assumptions can be made here to expedite the computation: (i) Eq. (8) can be linearized under weak scattering conditions (first-order Born approximation), i.e., \( E^{\text{tot}} \approx E^{\text{inc}} \), with \( E^{\text{inc}} \) being the incident field from transmitter \( t \). In high contrast tissues where weak scattering would not hold, an iterative procedure can be used to refine this assumption (higher-order Born approximations). (ii) Imposing an hierarchical sparsity prior \( p(\mathbf{w}) \) based on the assumption that the number of non-zero spatial harmonics \( N_{nz} \) is much less than the size of \( \mathbf{w} \). This is justified since the constitutive properties of a breast tissue exhibit a degree of spatial correlation that makes the contrast function more sparse in the spatial-harmonics domain than in the spatial domain. In particular, the prior \( p(\mathbf{w}) \) can be defined through a vector of hyperparameters \( \mathbf{z} \) as follows:

\[ p(\mathbf{w}) = \int p(\mathbf{w} | \mathbf{z}) p(\mathbf{z}) d\mathbf{z}, \]

(9)

where the conditional PDF is defined as.
in which the hyperparameters are the reciprocals of the variances of the zero-mean normal distributions. Assuming independent zero-mean Gaussian noise with variance $\sigma_n^2$, the likelihood can be written as

$$p(\mathbf{e}^i|\mathbf{w}, \sigma_n^2) = (2\pi \sigma_n^2)^{-N/2} \exp\left(-\frac{1}{2\sigma_n^2} \| \mathbf{e}^i - \mathbf{G} \mathbf{F}^{-1} \mathbf{w} \|^2 \right),$$  \hspace{1cm} (12)

and $N$ is the total number of measurements stacked in $\mathbf{e}^i$. The sought posterior PDF, conditioned on the hyperparameters and the noise variance, can be written as

$$p(\mathbf{w}|\mathbf{e}^i, \mathbf{z}, \sigma_n^2) = \frac{p(\mathbf{e}^i|\mathbf{w}, \sigma_n^2)p(\mathbf{w}|\mathbf{z})}{p(\mathbf{e}^i|\mathbf{w}, \sigma_n^2)}. \hspace{1cm} (13)$$

Using (12) and (10) in (13),

$$p(\mathbf{w}|\mathbf{e}^i, \mathbf{z}, \sigma_n^2) = \left(\frac{2\pi}{2Np+1}\right)^{Np+1/2} \exp\left(-\frac{1}{2} (\mathbf{w} - \mathbf{w}\text{MAP})^T \Sigma^{-1} (\mathbf{w} - \mathbf{w}\text{MAP}) \right),$$ \hspace{1cm} (14)

where covariance matrix of the weights $\Sigma = (\sigma_n^{-2} (\mathbf{G} \mathbf{F}^{-1})^T \mathbf{G} \mathbf{F}^{-1} + \text{diag}(\mathbf{z}))^{-1}$, and the MAP estimate of the weights $\mathbf{w}\text{MAP} = \sigma_n^{-2} \Sigma (\mathbf{G} \mathbf{F}^{-1})^T \mathbf{e}^i$. The hyperparameters and the noise variance can be efficiently computed using a fast RVM which searches for the most relevant weights that best-fit the measurements and, at the same time, seeks to avoid over-fitting noisy measurements by setting irrelevant weights to zero. In this way, the RVM avoids direct inversion of the projection matrix $\mathbf{G}$, which can be ill-conditioned in practice. It is of note also that the RVM only requires an initial guess of the noise level $\sigma_n^2$ that is used to adjust the strength of the sparsity regularization. Further details on the RVM can be found in Refs. 27, 28, 33, 35, 36, and 40.

III. NUMERICAL EXAMPLES

A. Breast modeling

To construct a 2-D model of a typical breast permittivity distribution, we follow the same basic procedure as outlined in Refs. 3–6: We first use a magnetic resonance image of the breast as shown in Fig. 1(a), compute its logarithm, then

FIG. 1. Breast tissue modeling. (a) Raw MRI image. (b) Associated permittivity distribution. The outer skin and the chest wall have relative permittivities equal to 30 and 50, respectively; however, the color bar is truncated to enhance the visibility. (c) Associated conductivity distribution. Note the conformal array of fifteen sensors indicated close to the outer skin. (d) Pixelated permittivity distribution of the breast tissue. The generic directions of the underlying nonuniform polar grid used are indicated in the figure. (e) Pixelated permittivity distribution as a function of pixels index number. (f) Spatial-frequency spectrum of the permittivity corresponding to the distribution in (e). Note the prevalence of lower frequencies due to the spatial correlation of the tissue properties.
adjust its mean to have $\varepsilon_r = 9$, and scale it to have variability of 16%. Finally, the outer skin and the chest wall are added with permittivities of 30 and 50, respectively, as shown in Fig. 1(b). We follow the same procedure for the conductivity, except that the mean conductivity is set to 0.4 S/m, and the skin and the chest wall have the same conductivity of 0.4 S/m, as shown in Fig. 1(c). The tumor is inserted as a circular region with radius of 5 mm, $\varepsilon_r = 50$, and $\sigma = 1$ S/m. Other accurate breast and tumor models can be found in Refs. 15 and 20. Fifteen sensors are deployed in a matching fluid ($\varepsilon_r = 9$) conformal to the skin, as shown in Fig. 1(b). The interrogating signal has frequency range of 1–7.5 GHz with 53 steps. The scattered field from the breast tissue is computed by subtracting a synthetic scattered field computed numerically from a model that takes into account the skin and the chest wall but assumes uniform tissue properties.
(\(\varepsilon_r = 9\) and \(\sigma = 0.4\) S/m), from the (noisy) measurements. A systematic approach to quantify the information content of scattering data, relative to measurement setup parameters, such as frequency, transmitter/receiver selection and noise level, was presented in Ref. 44. This approach can be used as a prior step to our inversion algorithm to optimize the measurement setup. For the inverse problem, the DOI, which is the breast tissue confined within the outer skin and the chest wall, is discretized using a non-uniform polar grid as shown in Fig. 1(d). The grid has 30 pixels along the “\(\rho\)"-direction and 30 pixels along the “\(\phi\)"-direction. The discretized permittivity profile versus the pixel index along \(\rho\)- and \(\phi\)-directions is shown in Fig. 1(e). Note that this image is a distorted version of the actual breast profile; yet, it still exhibits sufficient degree of smoothness that makes the (spatial harmonics) spectrum sparse, as clearly visible in Fig. 1(f).

**B. Adjusting the strength of sparsity regularization**

A key input parameter to the RVM is an initial guess of the noise level of the measurements, \(\sigma_i^2\). This parameter adjusts the strength of the sparsity regularization. Higher \(\sigma_i^2\) instructs the RVM to set more weights to zero as the signal-to-noise ratio (SNR) is estimated to be low, and vice versa. To illustrate the effect of this parameter choice, the reconstructed permittivity profiles and the respective estimated standard deviations using different strengths of sparsity regularization for a breast with one tumor are shown in Fig. 2. In

![Fig. 3. Reconstructed permittivity profiles from three iterations of the proposed Bayesian inversion algorithm with \(\sigma_i^2 = 10^{-1} \times \langle |(e^*|^2\rangle\). (a)-(c) A healthy breast. (d)-(f) A breast with one tumor. (g)-(i) A breast with a multifocal tumor with two foci. The actual tumors locations are indicated by small circles. SNR = 20 dB.](image)
Figs. 2(a) and 2(b), it is seen that investigating the reconstructed permittivity profile in a stand-alone fashion, i.e., without consideration of the standard deviation, yields very poor results: not only there is a missed detection of the actual tumor but also a false alarm on the presence of tumors at the lateral bases of the breast. On the other hand, when a maximum standard deviation cutoff is used (for example, reconstructed pixels where the standard deviation exceeds a certain multiple of the minimum standard deviation are deemed inaccurate and hence suppressed), the truncated image successfully detects the tumor in the subsequent domain of confidence (DOC), as shown in Figs. 2(c) and 2(d). To extend the DOC to encompass the entire DOI, larger $r_i$ needs to be plugged into the RVM, as in Figs. 2(e) and 2(f). However, as $r_i$ is further increased, we start losing some of the image features, as illustrated in Figs. 2(g) and 2(h).

C. Inversion results

We apply the proposed Bayesian inversion algorithm to reconstruct for the permittivity distribution of the breast tissue. Note that the contribution of the conductivity to the complex-valued contrast function $\tau$ is much less than that of the permittivity for realistic conductivity values of breast tissues in the utilized frequency band. Hence, we solve for the permittivity contrast alone in what follows. The projection matrix is computed numerically using the finite-difference time-domain method. The model used to compute $G$ and $E^{\text{tot}}$ accounts for the skin and the chest wall, and, in the first iteration, assumes uniform (mean values) tissue properties. Fig. 3 shows the reconstructed permittivities of a healthy breast in plots (a)–(c), along with the case of an isolated tumor in plots (d)–(f), and a case of a multifocal tumor with two foci in plots (g)–(i). During the iterative procedure, the reconstructed profile of a given iteration is used to recompute $E^{\text{tot}}$ used for the inversion in the next iteration. These results demonstrate the ability of the proposed algorithm to locate tumors accurately along with their local permittivity contrasts, using only a few iterations and in the absence of pointwise or detailed knowledge of background tissue properties. For comparison, we also apply the conventional (non-Bayesian) DBIM with CG solver to the two-tumors case considered above. Reconstructed images for three iterations are shown in Fig. 4. Images produced by CG DBIM are of no better quality than those produced by the proposed Bayesian approach and we should again stress that the Bayesian approach is more advantageous for the three following main reasons: (i) In addition to reconstruction images, it provides the confidence level of the inversion, from which false alarms and misdetections can be minimized. (ii) It provides faster convergence for capturing the correct permittivity contrast of the tumors. (iii) It is faster; one RVM solver takes only 30 s to run on a machine with average central processing unit (CPU) speed of 2.7 G-cycle/s, compared to 230 s for one CG solver. It should also be pointed out any of these times is much smaller than the time required to run the numerical solver for higher-order Born iterations.

IV. CONCLUSION

We have applied a new Bayesian inverse scattering methodology to the problem of ultra-wideband microwave imaging of breast cancer tumors. Besides providing accurate estimates of the constitutive properties of the breast tissues under imaging, a salient feature of the proposed approach is that it also provides estimates on the confidence level of the inversion, thus enabling the potential minimization of false alarms and missed detections.

By exploiting the spatial correlation exhibited by constitutive properties of breast tissues, the present approach allows for the effective use of sparsity priors in the spatial harmonics domain. Consequently, the posterior PDF can be computed very efficiently using a fast relevant vector machine. Future extensions to further increase the efficacy and accuracy of the proposed methodology include incorporating frequency-dispersion models of the breast tissue, the use of high-fidelity 3-D anatomical models for the breast and chest wall geometries, and possibly combining the proposed algorithm with time-reversal-based imaging techniques to produce spatially localized strategies for enhanced inversion.

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52 Note that the term “compressive sensing” is used here (and in the cited references above) in a broad sense.