Electromagnetic Time-Reversal Imaging and Tracking Techniques for Inverse Scattering and Wireless Communications

Dissertation

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By

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Abstract

Time-reversal (TR) was originated in acoustics as a technique for re-focusing waves around their source location. Under certain conditions, the wave equation is invariant under TR, therefore, waves emanated from a source or scattered from a passive target, and recorded by a transceivers array, will retrace their forward path and automatically focus at the source/target location if backpropagated in a time-reversed (last-in first-out) fashion from that array. Focusing resolution of time-reversed backpropagation in rich scattering environments beats that in free space, yielding what is known as ‘superresolution’. Moreover, under ultrawideband (UWB) operation, TR exhibits the distinctive property of ‘statistical stability’, which makes it an attractive technique for imaging in disordered media whose characteristics are not known deterministically (random media). Over the past few years, TR has been exploited in a variety of electromagnetic sensing and imaging applications such as ground penetrating radar, breast cancer detection, nondestructive testing, and through-wall imaging. In addition, TR has been extensively applied in UWB wireless communication providing myriad of advantages including reduced receiver complexity, power saving, increased system capacity, and enhanced information secrecy.
In this work, we introduce new TR-based signal processing techniques for imaging, tracking, and communicating with targets/users embedded in rich scattering environments. We start by demonstrating, both numerically and experimentally, the statistical stability of UWB TR imaging in inhomogeneous random media, under different combinations of random medium parameters and interrogating signal properties. We examine conditions under which frequency decorrelation in random media provides a more effective ‘self-averaging’ and therefore better statistical stability. Then, we devise a technique for detecting and tracking multiple moving targets in cluttered environments based on differential TR. This technique provides real-time tracking and exhibits superior clutter rejection at minimal processing costs. It also exploits the distinctive features of time-reversal such as statistical stability and superresolution.

Next, we develop an UWB inverse scattering technique for extended targets, with continuous permittivity and/or conductivity fluctuations, based on Bayesian compressive sensing. Bayesian inversion provides means for estimating the confidence level of the inversion, and for adaptively optimizing subsequent measurements. This technique is applied to a wide range of problems of practical interest such as underground crosshole sensing, medical imaging, and rough surface reconstruction. Finally, we develop new TR-based wireless communication techniques for UWB multiple-input single-output (MISO) and multiple-input multiple-output (MIMO) systems. We contrast relative strengths and limitations of those techniques for different scenarios of operation.
To my mother, my wife, and my family.
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Chapter 1: Introduction

1.1 Background and Motivation

The goal of inverse scattering is to estimate unknown parameters of target(s) of interest from noisy (cluttered) measurements. Target parameters may include location, size, orientation, and material properties [1, 2, 3, 4, 5, 6]. Electromagnetic inverse scattering finds many applications in medical imaging [7, 8, 9, 10, 11, 12], through-wall imaging [13, 14, 15], non-destructive testing [16, 17, 18], ground penetrating radar for buried targets detection [19, 20, 21, 22] and subsurface explorations [23, 24, 3, 4]. In all these applications, microwave signals are excellent candidates as they possess better penetration capabilities compared with optical or infrared signals for scenarios involving through-wall imaging, heavy dust and/or smoke, buildings debris, thick vegetation, or biological tissues [13, 14, 7]. Nevertheless, imaging obscured targets in clutter is still a challenging problem. Scattering from the target(s) of interest can be of the same order or even weaker than scattering from ambient clutter. Several techniques have been developed to mitigate the effect of clutter [25, 26, 27, 28]. Probably the most precise way to account for clutter is to iteratively reconstruct the background, then incorporate reconstructed background in imaging or tracking the desired target [13]. However, this can be too costly in many applications in particular
those involving real-time detection/tracking/classification. In the lack of deterministic information, the background can instead be treated as a random medium and the clutter as a consequential random process. The actual (measured) clutter can then be identified as a realization of that random process.

Ever since the capabilities of time-reversal (TR) techniques were first illustrated by Fink and collaborators [29] for acoustic (scalar) waves, considerable theoretical and experimental effort has been put into their extension to electromagnetic and elastic waves and their use to augment detection and imaging algorithms in rich-scattering environments [30]. TR techniques are based on the invariance of the wave equation under time reversal (in lossless, reciprocal and time-invariant media). When waves emanated from a source (or reflected by a passive scatterer) are recorded by a sensor array, time-reversed, and retransmitted (in a first-in last-out fashion) in the same medium, they will automatically retrace their ‘paths’ and focus at the (original) source/scatterer location, where the field adds coherently throughout the frequency band of operation. Indeed, TR harnesses multipathing to enhance focusing resolution beyond the classical diffraction limit. This feature is known as superresolution [31, 32, 33, 34, 35]. Another feature of TR is self-averaging under ultra-wideband (UWB) operation, meaning that, under certain conditions, imaging in random media is independent of the specific realization of the random medium, but depends only on its statistical properties. This is known as statistical stability [36, 37, 38, 39, 40]. True TR invariance is broken in lossy media, which is invariably the case for electromagnetic waves in Earth media [41, 42, 43]. In this case, compensation techniques [44] or the techniques to be presented in Chapter 5 can be used, under some limits.
Backpropagation of time-reversed waves can take place either in the original physical medium (physical TR) or in a synthetic imaging medium (synthetic TR). The former approach finds applications in areas such as lithotripsy [45], and wireless communications [46]. Synthetic TR, on the other hand, is typically used for detection and imaging of targets [19, 20, 15, 47, 48, 49, 50, 51, 27], with applications in ground penetrating radar [19, 20], through-wall imaging [15], and medical imaging [50, 51, 52, 53], for example.

In wireless communications, TR provides a simple and cost-efficient solution to the problem of delay spread in UWB systems [54, 55, 56, 57, 46, 58, 59, 60, 61]. With the continuous increase in demand for higher data rates in wireless communications, UWB systems are becoming more popular [62]. Despite their advantages, UWB systems are very sensitive to delay spreads caused by multipath in rich scattering environments and require complex and expensive receivers to compensate for delays of different replicas of the desired received signal. TR moves the burden of equalization from the receiver side to the transmitter and channel sides, thus allowing for simpler and cheaper receivers at the expense of some added complexity to the transmitter [58]. TR provides both temporal and spatial focusing of the signal at the receiver [63, 64]. Not only does it mitigate negative effects of multipath, but also it is capable of actually harnessing multipath to achieve better focusing resolution (superresolution) both in time and space [64, 54, 55, 65, 33]. The temporal focusing property amounts to a type of pre-equalization procedure that reduces (or cancels) intersymbol interference (ISI) at the receiver [66]. Spatial focusing combats channel fading [63], maximizes delivered power to the intended receiver, and therefore enables
power saving at the transmitter side and/or increasing channel capacity and communication range. Spatial focusing also reduces power leakage to other locations. This is very important to reduce interuser interference (IUI) in multiuser configurations, which in turn allows for a more effective use of space-division multiple access (SDMA) to boost the system capacity [57, 59, 60, 67]. Spatial focusing also adds a degree of physical layer security (covertness) to the systems, making it hard for eavesdroppers away from the intended receiver’s location to decode the signal using traditional decoding techniques [66, 68]. In general, the communication link can be viewed as an inverse source problem, where the receiver tries to infer the maximum amount of information about the state of the transmitter [69]. The increase in effective degrees of freedom in multipath environments is used efficiently by TR to increase the amount of information communicated per channel use.

In previous works by our group, several studies on UWB TR were conducted, and novel TR-based signal processing techniques for detection and imaging of obscured targets were developed. Those include: (i) Investigating the superresolution of polarimetric TR in continuous random background media, and studying the effect of depolarization on the re-focusing resolution [33]. (ii) Frequency dispersion and loss compensation for UWB TR techniques utilizing the short-time Fourier transform (STFT) [44]. (iii) Developing a time-domain fully polarimetric selective focusing method based on time-reversal operator decomposition (DORT) method [39]. (iv) Studying the sensitivity of TR methods to external perturbation such as noise, clutter and losses [48]. (v) Developing an UWB imaging technique based on simultaneous utilization of spatial and UWB frequency data [47]. In addition, an UWB time-domain radar for TR applications was built [70].
1.2 Contributions

The contributions of this work can be summarized as follows:

- **Statistical stability of UWB TR imaging:** A numerical study on the statistical stability of UWB time-reversal imaging in continuous random media has been carried out and validated using UWB time-domain measurements. Additionally, a new frequency-synthesized technique for UWB time-reversal-based imaging has been developed [71, 72].

- **Differential TR:** Novel UWB techniques for imaging and tracking moving targets in clutter, based on differential TR, were developed [73]. The proposed techniques were applied to through-wall detection and tracking.

- **Targets characterization:** Scattering matrices, employed in TR imaging algorithms, were investigated in an attempt to provide additional classification information for scatterers with different shapes, sizes, and material properties [73].

- **Bayesian inverse scattering:** Information-theoretic UWB inverse scattering techniques, for continuous media, based on Bayesian compressive sensing, were developed. These techniques were applied to problems of practical interest such as underground crosshole and borehole sensing, breast imaging, and rough surface reconstruction.

- **TR for wireless communications:** New time-reversal-based techniques, for increasing the capacity and improving the performance of ultrawideband wireless communications, were developed. The proposed techniques were applied to
multiple-input single-output (MISO) and multiple-input multiple-output (MIMO) wireless communication systems operating in rich scattering environments [74].

1.3 Organization of this Dissertation

The rest of this dissertation is organized as follows: In Chapter 2, we present a study on the statistical stability of UWB TR DORT and MUSIC imaging in random media under different combinations of random medium parameters and interrogating signal properties. Then, in Chapter 3, we experimentally verify the statistical stability of UWB TR-based imaging of targets in discrete random media. Chapter 4 introduces two differential TR algorithms for identifying, imaging, and tracking moving targets in clutter. In Chapter 5, we develop UWB inverse scattering techniques in continuous random media based on Bayesian compressive sensing. Chapter 6 considers the application of different time-reversal (TR) signal processing and beamforming techniques to multiple-input single-output (MISO) and multiple-input multiple-output (MIMO) wireless communication systems. Finally, conclusions and future work are provided in Chapter 7.
Chapter 2: Statistical Stability of Ultrawideband
Time-Reversal Imaging in Random Media

In TR imaging, acoustic or electromagnetic signals acquired by the sensing apparatus are time-reversed and synthetically backpropagated in a synthetic (i.e., mathematical) imaging domain [29, 64]. Most often, the synthetic domain does not correspond to the physical domain where actual propagation has occurred because of the lack of deterministic information about the latter, as mentioned above. Under suitable conditions, the spectral components (phase and amplitude) at two frequencies $f_1$ and $f_2$ of an UWB pulse propagating in random media become progressively more decorrelated with an increase in the distance $|f_1 - f_2|$, as these components interact differently with the intervening medium. As a result, combining a sufficiently wideband of frequencies to construct the image yields “self-averaging”, which surrogates ensemble averaging. This means that, a stable image can be obtained from a single realization, given a sufficiently wide bandwidth, without having to average over many measurements from different random medium realizations, which may not be available in practice. This striking property has been referred to statistical stability of UWB TR [37, 40, 48]. Statistical stability of physical (but not imaging) TR, where time-reversed signals are physically “backpropagated” in the same medium as the forward propagation occurs, was considered in [36, 37, 38, 40]. There, it was verified
that TR from extended sources is stable under sufficiently wideband operation. In addition, statistical stability in time-varying media was studied in [75].

A TR-based signal processing technique, that is often used when multiple targets are present, is the time-reversal operator decomposition (DORT, under its French acronym) [76, 77, 78, 79, 39]. DORT is based upon an eigenspace analysis of the time-reversal operator (TRO). If the scatterers are well-resolved by the TR array, eigenvectors in the signal subspace of the TRO can be used to yield selective focusing (beamforming) on each scatterer individually. Another effective subspace technique for TR applications is the Multiple SIgnal Classification (MUSIC) algorithm, which can be applied on the null subspace to obtain a collective image of all targets.

This chapter is divided into two main parts. First, we present a new frequency-synthesized algorithm for constructing UWB TR images. Conventional TR images are typically obtained by backpropagating time-reversed wavefields in a synthetic imaging domain and capturing the image at an appropriate time reference “t=0”, associated with the assumed instant when the target has initiated signal transmission (active target/passive radar mode) or the assumed two-way travel time (passive target/active radar mode) [39]. However, the “t=0” reference may not always be known precisely and/or may not even be the optimal focusing instant. For example, if the (assumed) imaging domain properties do not correspond to those of the physical medium (e.g., different mean permittivities and hence different travel-times), the optimal focusing instant will be unknown. The proposed algorithm avoids this problem by employing a spatiotemporal peak search to automatically determine both the focusing location and optimal time instant (of focusing). This algorithm is very efficient in terms of processing time, and information about the imaging domain can be stored and reused
to construct images of different targets (in a fixed domain). We start by presenting
the algorithm in connection to DORT and MUSIC imaging of a point-like scatterer
embedded in a homogeneous background using a linear TR array. We compare the
resulting DORT and MUSIC images and consider the effect of interrogating signal pa-
rameters (center frequency and bandwidth) on the image quality. Next, we apply the
same approach to full-aspect arrays, where we introduce “volumetric” beamforming
and “volumetric” MUSIC techniques.

In the second part of the chapter, we study the statistical stability of time-reversal
imaging in continuous random media. In this case, frequency components that con-
stitute the image and which are sufficiently apart can become “incoherent”, in the
sense of having decorrelated amplitude and phase relationships among them. Under
UWB operation, this “frequency decorrelation” leads to self-averaging in the time
domain and therefore statistically stable images. We carry out a parametric study
on statistical stability of time-reversal imaging with respect to (i) interrogating sig-
nal properties (center frequency and bandwidth), (ii) random medium parameters
(permittivity variance and correlation length), and (iii) sensor array size and spatial
deployment (full- or partial-aspect).

2.1 Frequency-Synthesized Images

2.1.1 Frequency-Synthesized DORT

To obtain selective imaging of a scene comprising multiple targets/scatterers,
eigenstructure (or subspace) methods such as DORT and/or MUSIC can be applied.
In this case, a TR transceiver array of \( N \) elements is used to produce an \( N \times N \)
multistatic data matrix (MDM) \( \mathbf{K}(t) \), where the matrix element \( [\mathbf{K}(t)]_{ij} \) corresponds
to the scattered field (time-domain waveform) recorded by element \( i \) from the individual excitation of element \( j \) [80]. A Fourier transformation can be applied to obtain a set of MDMs \( \mathbf{K}(\omega) \) at a discrete set of frequencies within the bandwidth of operation. The time-reversal operator (TRO) at each frequency \( \mathbf{T}(\omega) \) is defined as the self-adjoint matrix \( \mathbf{T}(\omega) = \mathbf{K}^\dagger(\omega)\mathbf{K}(\omega) \), where \( \dagger \) denotes a conjugate transpose (note that time-reversal is equivalent to complex conjugation in the frequency domain).

An eigenvalue decomposition of the TRO provides \( N \) orthonormal eigenvectors and associated eigenvalues. In the scalar case and for well-resolved point-like targets, each target is associated with one eigenvalue/vector pair: the eigenvalue bears information on the scattering coefficient and the eigenvector bears information on the target’s location [48]. Extended targets will, in general, be associated with more than one significant eigenvalue/vector pair depending on their electrical sizes and orientations w.r.t the array. Here, we assume point-like targets; however, under certain conditions, these techniques can be applied to extended targets as well. A detailed analysis on the application of DORT to extended targets classification and imaging was presented in [73].

If the number of array elements \( N \) is larger than the number of point-like scatterers \( M \), only \( M \) eigenvalues will be significant, with the associated eigenvectors spanning the signal subspace. The remaining \( N - M \) eigenvectors constitute the null subspace. The DORT method utilizes the signal subspace eigenvectors to construct separate images of each target (selective focusing) [76, 39]. The complex conjugate of the \( m^{th} \) eigenvector (corresponding to the \( m^{th} \) target) is given by

\[
\mathbf{q}_m(\omega) = \frac{\mathbf{g}_m(\omega)}{\|\mathbf{g}_m(\omega)\|} e^{j\omega \tau} \quad (2.1)
\]
where $g_m(\omega)$ is the steering vector of location $m$, given by

$$g_m(\omega) = [G_{(m,1)}(\omega), ..., G_{(m,N)}(\omega)]^T$$

(2.2)

where $G_{(m,n)}(\omega)$ is the Green’s function between the location of target $m$ and the $n^{th}$ element of the array, and $\|g_m\| = \sqrt{\langle g_m, g_m \rangle}$, where $\langle a, b \rangle = b^\dagger a$ denotes inner product, and $\tau$ represents a possible time shift that may result from the lack of precise knowledge about the time reference $t = 0$, as discussed before.

To construct the frequency-synthesized image of target $m$, $q_m(\omega)$ is projected onto normalized steering vectors of a synthetic imaging domain. For an arbitrary point $p$ within the imaging domain, this projection is given by

$$P^m_p(\omega) = \left\langle \frac{g_p(\omega)}{\|g_p(\omega)\|}, q_m(\omega) \right\rangle$$

(2.3)

In the time domain, the projection is computed by taking the inverse Fourier transform of $P^m_p(\omega)$, that is

$$P^m_p(t) = \mathcal{F}^{-1}(P^m_p(\omega))$$

(2.4)

The focusing time $t^m_f$ and the focusing location $p^m_f$ are the instant and location at which the peak of $P^m_p(t)$ occurs, that is

$$\{p^m_f, t^m_f\} = \arg\max_{\{p,t\}} (P^m_p(t))$$

(2.5)

Finally, the normalized image functional at point $p$ is given as

$$I^m_p = \left| \frac{P^m_p(t^m_f)}{\max_{\{p,t\}}(P^m_p(t))} \right|$$

(2.6)

As an example, consider the imaging of a point-like target located in free-space using a linear TR array of z-polarized point sources in a two-dimensional scenario,
Figure 2.1: Normalized DORT image (in dB scale) of a point-like target imaged by a linear array. Array elements are indicated by white dots. The target location is indicated by the small white circle. The center frequency is 500 MHz and the bandwidth is 400 MHz.

Figure 2.2: Projection (in the frequency domain) of the eigenvector of a target on the imaging domain at the focusing location. The target is point-like and embedded in a uniform medium. (a) Magnitude. (b) Phase.
Figure 2.3: Profiles of normalized DORT images produced using four different sets of interrogating signal parameters. (a) Co-range. (b) Cross-range.

as illustrated in Fig. 2.1. The magnitude and phase of the projection at the focusing location in the frequency domain are plotted in Fig. 2.2. The magnitude is flat over all frequencies and equals to one (maximum value by definition), while the phase behavior is almost linear. This shows that perfect correlation exists between the target eigenvector and the steering vector at the focusing location. Also, in this case, there is a deterministic relationship among frequency components.

The resulting normalized DORT image is shown in Fig. 2.1. The impact of the interrogating signal bandwidth $B$ (we assume a rectangular spectrum throughout, where $B = f_{\text{max}} - f_{\text{min}}$), and center frequency $f_c$ on the image is considered in Fig. 2.3, where the co- and cross-range profiles of the imaging functional are plotted. Fig. 2.3(a) shows that by reducing the fractional bandwidth (whether by reducing the bandwidth or increasing the center frequency), an increase in the sidelobe level along the co-range results. Increasing the center frequency, however, reduces the main lobe width. Note that, for sufficiently far targets imaged by linear arrays, the co-range
profile is simply the spatial mapping of the projection $P_p(t)$ at the focusing location (as in conventional beamforming). Fig. 2.3(b) shows that the cross-range profile is hardly affected by the bandwidth. Conversely, it is seen that increasing the center frequency enhances cross-range resolution (from an overall increase on the electrical size of the array), as expected.

### 2.1.2 Frequency-Synthesized MUSIC

Instead of using the signal subspace as in DORT, MUSIC employs the null subspace to construct a simultaneous image of all targets. The null subspace projection vector at point $p$ ($n_p(\omega)$) can be obtained using signal subspace eigenvectors as follows

$$n_p(\omega) = \frac{g_p(\omega)}{\|g_p(\omega)\|} - \sum_{m=1}^M P_p^m(\omega)q_m(\omega)$$

(2.7)
The above equation is illustrated schematically in Fig. 2.4 (for \( N = 3 \) and one significant eigenvector, that is \( M = 1 \)). The time domain version of (7) provides \( N \) signals that, once transmitted by the array in a time-reversed fashion, produce a beam focusing on \( p \) and null fields at each of the \( M \) targets locations. For imaging
purposes, the norm of \( n_p(\omega) \) is first computed and then all frequencies are combined by taking an inverse Fourier transform, yielding a null subspace projection in the time domain \( N_p(t) \) as follows

\[
N_p(t) = \mathcal{F}^{-1}(\|n_p(\omega)\|)
\]  

(2.8)

All frequency components involved in the above inverse Fourier transform are real; hence, it is irrelevant here to define a focusing time instant. The focusing location is the minimizer of \( N_p(t) \) which holds for any \( t \), so for instance

\[
p_f = \arg \min_p N_p(t = 0)
\]  

(2.9)

The MUSIC imaging functional is defined as the reciprocal of the null subspace projection, that is

\[
I_{p}^{MU} = \frac{\min_p N_p(t = 0)}{N_p(t = 0)}
\]  

(2.10)

The MUSIC image of the same point-like target considered before is plotted in Fig. 2.5. The co-range and cross-range profiles for different interrogating signal parameters are shown in Fig. 2.6. From these plots we observe that, in contrast to the prior DORT images, the (cross-range) resolution of the MUSIC images is mostly independent of the center frequencies and bandwidths considered. However, this is only valid for point-like targets in uniform media, where UWB operation does not add extra information over that offered by narrowband operation. This will not hold for imaging in random media as will be discussed later on.

2.1.3 Volumetric beamforming: full-aspect DORT and MUSIC

We next apply the above DORT and MUSIC frequency-synthesized algorithms to a full-aspect (circular) array configuration. The DORT image is shown in Fig. 2.7.
Figure 2.7: Same as Fig. 2.1 using a full-aspect circular array (volumetric DORT).

Figure 2.8: Normalized volumetric DORT image profiles along $x$-direction produced by using four different sets of interrogating signal parameters.

The image profiles along the horizontal direction are shown in Fig. 2.8. Similarly to the linear array cross-range profile, the full-aspect image resolution is effectively independent of the bandwidth, but is enhanced by increasing the center frequency of operation. Similar to the linear array co-range profile, the full-aspect image has
sidelobe levels that increase with a decrease on the fractional bandwidth. However, the sidelobe levels are considerably lower than in the previous linear array case. The full-aspect MUSIC image is shown in Fig. 2.9. It retains the excellent cross-range resolution of conventional MUSIC along all directions. Such full-aspect imaging can be viewed as a form of “volumetric beamforming”.

2.2 Statistical Stability Analysis

2.2.1 Theory and approach

So far, we have considered imaging of targets in uniform (homogeneous) background media. Often times in practice, the exact background distribution is unknown and/or difficult (that is, computationally very costly) to reconstruct. In such cases, the background can be treated as a random medium where there is a mismatch between the actual (physical) background and the synthetic (imaging) domain, each
corresponding to a possible (different) realization. In this section, we extend our previous discussion to targets embedded in continuous random media. In this case, the image itself becomes a random variable, whose variance can be used as a measure of statistical stability.

Consider a point-like source (active target) embedded in a random medium realization as shown in Fig. 2.10. The signal transmitted by the target is recorded by the TR array and used to reconstruct the target location within a synthetic imaging domain with the dimensions shown. The synthetic imaging domain is chosen as a uniform medium with permittivity equal to the mean permittivity of the random medium.
Figure 2.11: Frequency domain projections of the recorded signal vector, from different realizations of the forward problem, at the source location in the imaging domain. (a) Magnitude. (b) Phase. Dotted curves represent the equivalent projections in a known (uniform) background.

The projection at point $p$ in the imaging domain is given by

$$P_p(\omega) = \left\langle \frac{g_p(\omega)}{\|g_p(\omega)\|}, \frac{\tilde{g}_s(\omega)}{\|\tilde{g}_s(\omega)\|} \right\rangle$$

(2.11)

where $\tilde{g}_s(\omega)$ is the steering vector of the source in the random medium, and $g_p(\omega)$ is the steering vector of location $p$ in the (uniform) imaging domain. The mismatch between $\tilde{g}_s(\omega)$ and $g_p(\omega)$ makes $P_p(\omega)$ a complex random variable. Its amplitude and phase depend on the random medium realization as shown in Fig. 2.11. This figure also shows that for a given realization, the projection is “incoherent”, i.e., $P_p(\omega)$ has nondeterministic variation with frequency (compare with the dotted curves of the uniform background case).

By decomposing the interrogating signal into $N_f$ discrete frequency components, the projection can be assembled into the following complex random vector

$$p = [P_p(\omega_1), ..., P_p(\omega_{N_f})]^T$$

(2.12)
In the time domain, the projection at \( t = 0 \) (normalized by the number of frequency components to ensure a fair comparison among images produced using different bandwidths) is given by

\[
P_p(t = 0) = \frac{1}{N_f} \sum_{i=1}^{N_f} [p]_i
\]  

(2.13)

The mean square of \( P_p(t = 0) \) is given by

\[
E[P_p^2(t = 0)] = \frac{1}{N_f^2} \sum_{i=1}^{N_f} \sum_{j=1}^{N_f} [R(p, p)]_{ij}
\]

(2.14)

where \( R(p, p) = E[pp^\dagger] \) is the autocorrelation matrix of \( p \) and \( E[.\] denotes expected value. The matrix \( R(p, p) \) can be expanded as

\[
R(p, p) = E[p]E[p^\dagger] + C(p, p)
\]

(2.15)

where \( C(p, p) \) is the autocovariance matrix of \( p \) [81]. Substituting (2.15) in (2.14) yields

\[
E[P_p^2(t = 0)] = \frac{1}{N_f^2} \sum_{i=1}^{N_f} \sum_{j=1}^{N_f} E[p]_iE[p^\dagger]_j + \frac{1}{N_f^2} \sum_{i=1}^{N_f} \sum_{j=1}^{N_f} [C(p, p)]_{ij}
\]

(2.16)

The first term in the right hand side is \( E[P_p(t = 0)]^2 \), so the variance of \( P_p(0) \) is given by

\[
\text{var}(P_p(t = 0)) = \frac{1}{N_f^2} \sum_{i=1}^{N_f} \sum_{j=1}^{N_f} [C(p, p)]_{ij}
\]

(2.17)

A typical covariance matrix is plotted in Fig. 2.12 (\( l_c, \sigma, \epsilon_{rm} \) are medium parameters defined in the next section). The fact that the magnitude of off-diagonal elements is much smaller than that of the diagonal elements is an indication that sufficiently spaced frequency components are decorrelated. In such case, the sum of \( N_f \) frequency components mimics an ensemble average over \( N_f \) (uncorrelated) realizations. As a result, the image variance should decay asymptotically with the bandwidth as \( \simeq 1/N_f \) under the assumption that the \( N_f \) components have equal variance.
2.2.2 Simulation results

In this section, we use the average (over $p$) in (2.17) as a measure of stability. We will call it simply the image variance. We carry out a parametric study on the effect of different signal and medium parameters on the image stability. We consider both the image variance itself and its fractional decrease with bandwidth. The latter is used as a measure of frequency decorrelation.

The background medium in the forward problem is a realization of a continuous random permittivity with a clipped Gaussian distribution and a Gaussian spatial correlation function [82]. This distribution is characterized by three parameters (see Fig. 2.10): the mean permittivity $\epsilon_{rm}$, the standard deviation of the permittivity $\sigma$, and the spatial correlation length $l_c$ [82, 83]. For this analysis, we choose $\epsilon_{rm} = 5$, and compute the image variance using Monte Carlo technique by ensemble averaging over one hundred realizations. Note that in practice, one has access to only one realization.
Figure 2.13: Beamforming results for $f_c = 500$ MHz and different bandwidths. (a) Image average for $B = 100$ MHz. Actual source location is indicated by ‘o’. Estimated source locations from different realizations are indicated by ‘x’s. The r.m.s. error in the source localization is 35 cm. (b) Image variance for $B = 100$ MHz. (c) Image average for $B = 400$ MHz. The r.m.s. error in source localization is 11 cm. (d) Image variance for $B = 400$ MHz.

of the random medium and hence only one image; nevertheless, this image variance sets an upper bound on the deviation of the image from the (stable) ensemble average according to the Chebyshev inequality [37]. All simulations are carried out using the finite-difference time-domain method [84].
Figure 2.14: Image variance versus bandwidth and center frequency, for $l_c = 0.44$ m.

**Interrogating signal parameters**

Fig. 2.13 shows the image average and variance for narrowband and wideband operations around the same center frequency $f_c$. It is clear from this figure that the narrowband image is unstable: the image variance is large and spreads out throughout the imaging domain. On the contrary, the wideband images are stable: their variance is small and confined to a small region around the true target location.

For a given frequency band, the image variance can be computed for all possible $f_c, B$ combinations within that band, as illustrated in Fig. 2.14. This figure shows that for a given $B$, the variance increases with larger $f_c$. This is because higher frequency components are better able to resolve the random medium fluctuations for the present choice of $l_c$. 

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Random medium parameters

The dependence of the image variance on the correlation length $l_c$ for different $B$ and $f_c$ is illustrated in Fig. 2.15(a) and (b), respectively. We can distinguish two main regimes. The first regime consists of $l_c \ll \lambda_c$, where the random medium effectively behaves as a uniform medium with (effective) permittivity $\epsilon_{rm}$. As $l_c$ increases, the signal is gradually more affected by the medium fluctuations and consequently the variance increases. The second regime consists of $l_c \geq \lambda_c$. In this case, the variance is large but increases only slightly with $l_c$. For $l_c \gg \lambda_c$, the medium behaves again as a uniform medium (at least locally in the imaging domain), but this time with random permittivity that varies from one realization to the other. Note also that, as seen from Fig. 2.15(b), the transition between these two regimes occurs at shorter correlation lengths when $f_c$ increases, as expected.

The variance we have computed so far corresponds to images captured at $t = 0$. However, when there is any mismatch between the average permittivity of the actual background and that of the imaging domain, $t = 0$ is no longer the optimal focusing instant, as we discussed before. This is typical in the limit of long $l_c$. Perhaps a more accurate measure of stability is therefore to consider the variance of the null subspace projection $N_p(t = 0)$, that is, the reciprocal of the MUSIC imaging functional. MUSIC would provide target’s image at the same location as DORT image had the latter been captured at the optimal focusing instant. The counterpart of Fig. 2.15 using the null subspace variance measure is shown in Fig. 2.16. The main difference is that, in the long $l_c$ regime, the variance decreases monotonically with $l_c$. This shows that MUSIC images are more robust to permittivity mismatch.

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Figure 2.15: Image variance versus correlation length $l_c$ for (a) $f_c = 500$ MHz and varying bandwidth $B$, and (b) for $B = 400$ MHz and varying center frequency $f_c$.

Figure 2.16: Same as Fig. 2.15 using MUSIC.

Array geometry

We next illustrate the effect of the TR array geometry on the image variance. For full-aspect circular arrays, reducing the array radius $R$ decreases the two-way travel path of the electromagnetic signal; therefore, the image variance decreases as shown
Figure 2.17: Effect of array geometry. (a) Image variance versus correlation length for $f_c = 500$ MHz, $B = 400$ MHz, and three array geometries. (b) Fractional decrease in image variance with bandwidth for $l_c = 2.94$ m and $f_c = 500$ MHz.

in Fig. 2.17(a). The use of a linear array with same number of elements and (nearest) distance $L = R$ from the source, increases the image variance significantly. In addition, the image variance becomes less sensitive to bandwidth increase, as evident by the fractional decrease in the image variance seen in Fig. 2.17(b). The covariance matrices of full-aspect array and linear array images are plotted in Fig. 2.18, for comparison. It is clear that the frequency correlation in the linear array case is much stronger than in the full-aspect case. This is due to the fact that a larger region of the domain is “spanned” by the interrogating field using a full-aspect geometry as opposed to a linear-aspect geometry.

2.3 Conclusion

We have presented a new frequency-synthesized technique for constructing ultrawideband images using time-reversal DORT and MUSIC imaging. The proposed
Figure 2.18: Covariance matrices for $l_c = 2.94$ m, using (a) full-aspect array and (b) limited-aspect array configurations.

technique does not require backpropagation of time-reversed waves in the imaging domain. Once the steering vectors (Green’s function) of the imaging domain are computed and stored, they can be reused to construct images of different targets in the same domain, for different interrogating signal parameters (central frequency and bandwidth). It was shown that, for point-like targets in a uniform background medium, the imaged target location is effectively invariant with respect to the range
of interrogating signal parameters considered. In addition, the resolution of time-reversal MUSIC images is effectively invariant with respect to the signal parameters as well. It was also observed that when using full-aspect arrays, the resolution of time-reversal DORT images is hardly affected by the bandwidths considered here. In the case of random medium backgrounds, the image can be considered as a random functional itself, whose variance is related to the covariance matrix of the constituting frequency components. The more pronounced is the frequency decorrelation in the random medium, the faster is the decrease of the image variance with increasing bandwidth. A Monte Carlo study of the statistical stability of the resulting images was carried out under different combinations of interrogating signal parameters, random medium parameters, and sensor array geometries. This work can be further extended to analyze the statistical stability of differential TR [73] techniques and to examine scenarios involving time-varying background media.
Chapter 3: Experimental Demonstration of Statistical Stability in Ultrawideband Time-Reversal Imaging

In Chapter 2, we presented a numerical study on the statistical stability of TR imaging in inhomogeneous media modeled as continuous random permittivity fluctuations. Recently, an algorithm for imaging of vegetation, based on filtered back-projection, has been presented in [85]. It combines, coherently, data from different view angles and frequencies to obtain tomographic images of sparse, low contrast vegetation, and metallic targets hidden therein. Although not explicitly discussed, the robust results in [85] indicate that frequency decorrelation can be exploited by coherent imaging algorithms to obtain stable images of deterministic features in stochastic clutter.

In this chapter, we experimentally verify the statistical stability of UWB TR imaging using electromagnetic waves. We carry out measurements using UWB time-domain radar that provides useful bandwidth up to 40 GHz [86, 70]. The intervening random medium is comprised of discrete dielectric scatterers (wooden sticks). This experiment is a downscaled model that can mimic, under certain limits, radar imaging in forests or vegetation [85]. We consider imaging in both “source mode” (active sources) and “echo mode” (passive targets). Although this experimental verification opens up the possibility that statistical stability may also occur in other scenarios,
we only consider spatially stationary random media, i.e. those whose statistical properties do not vary in space.

### 3.1 Experimental Setup

A schematic diagram and a photograph of the experimental setup are shown in Fig. 3.1(a) and (b), respectively. The transmitting antenna is an omnidirectional UWB conical antenna fed against a ground plane by a short monocycle generated.
by a pulse generator followed by two impulse forming networks. The generator provides UWB spectrum covering from near DC up to 40 GHz, as shown in Fig. 3.1(c). A sensor array is constructed synthetically by recording received signal at each of the indicated receiving antenna locations. The receiver locations are distributed uniformly along a 90° circular arc centered at the transmitting antenna. The receiving antenna is a TEM horn designed to acquire direct time-domain measurements of the ultrafast transient electromagnetic fields without distortion [70]. Measurements are conducted directly in the time-domain, where received signals are recorded by a 50 GHz digital storage oscilloscope. The size of the measurement setup is limited by the available power, and hence the signal-to-noise ratio. In general, the co-range of the experiment could be increased if more power is available. For the setup shown in Fig. 3.1(a), the average signal-to-noise ratio at the receiver locations is around 26 dB. The intervening medium between the source and the receivers is occupied by randomly deployed dielectric scatterers. The scatterers are bamboo sticks with diameter of 0.5 cm, and approximate dielectric constant of 2.2. The sticks are uniformly distributed in both directions on 9 cm × 9 cm tiles of styrofoam, which has a dielectric constant of approximately 1. Different realizations of the random medium are produced by rearranging the styrofoam tiles. We consider three types of discrete scatterers: (i) dry scatterers with low fractional volume (25 sticks per tile, fractional volume= 6%), (ii) dry scatterers with high fractional volume (50 sticks per tile, fractional volume= 12%), and (iii) wet scatterers with high fractional volume (50 sticks per tile, fractional volume= 12%). The moisture content of wet sticks is approximately 25%. This estimation is done by measuring the resistance between two points along the wet sticks.
and comparing with measured data for the average electric resistance of wood at different levels of moisture content [87]. At 10 GHz, this moisture content corresponds to dielectric constant of approximately 3 and a loss tangent of approximately 0.2 [88]. The signal-to-clutter ratio SCR is used to quantify the amount of clutter introduced by each type of random media. The SCR is defined as

\[
\text{SCR} = \frac{\sum_{n=1}^{N} \int_{0}^{T} |R_n^H(t)|^2 dt}{\sum_{n=1}^{N} \int_{0}^{T} |R_n^R(t) - R_n^H(t)|^2 dt}
\]  

(3.1)

where \(R_n^H(t)\) and \(R_n^R(t)\) are the signals measured by the \(n^{th}\) receiver in homogeneous (free-space) and random media, respectively. \(N\) is the number of receivers and \(T\) is the total duration of the signal. In random media, the denominator of (3.1) is averaged over many realizations. For the three types of discrete scatterers described above, the SCRs are 2.0, 0.6 and -1.3 dB, respectively. Note that depolarization occurs due to inclined bamboo sticks; nevertheless, we will show that the vertical (co-)polarization, recorded by the receiving antenna, is sufficient to produce statistically stable images.

### 3.2 Statistical Stability of TR Imaging

In TR beamforming imaging, time-reversed signals are backpropagated in a synthetic imaging medium and the image is captured at a reference time instant \(t = 0\). This approach is not applicable in cases where the time reference of recorded signals is unknown or there exists significant mismatch in the average speed of propagation between forward and imaging domains. A frequency synthesized imaging algorithm was developed in [71] to circumvent this difficulty. An alternative imaging algorithm, that provides a focused image automatically, is the TR-based multiple-signal-classification (MUSIC) algorithm. In addition, TR MUSIC images in homogeneous media are weakly dependent on the signal bandwidth and center frequency, unlike
their TR beamforming counterparts [71]. This makes MUSIC images more suitable for comparing stability of images constructed employing different bandwidths and center frequencies. Finally, the null-subspace projection (which is the reciprocal of MUSIC image) [32, 49, 48, 71] can be written as a sum of a covariance matrix (see below), which provides a better insight into the link between frequency decorrelation and statistical stability. For the above reasons, we adopt TR MUSIC as our basic imaging algorithm in this study.

### 3.2.1 TR-based MUSIC Imaging

The signal vector received by an $N$-elements sensor array, in the frequency-domain, is given by $r(\omega) = S(\omega)\tilde{g}_s(\omega) + w(\omega)$ where $S(\omega)$ is the source spectrum, and $\tilde{g}_s(\omega)$ is the steering vector between the source location and array elements, given by $\tilde{g}_s(\omega) = [\tilde{G}_{(s,1)}(\omega), ..., \tilde{G}_{(s,N)}(\omega)]^T$ where $\tilde{G}_{(s,n)}(\omega)$ is the Green’s function between the source location $s$ and the $n^{th}$ element of the array, in the presence of random scatterers. Note that, since the random medium is not known deterministically, $\tilde{G}_{(s,n)}(\omega)$ is treated as a random variable, and hence $r(\omega)$ becomes a random vector. Finally, $w(\omega)$ is a random vector of additive noise.

In TR-based MUSIC imaging, steering vectors of the synthetic imaging medium $g_p(\omega) = [G_{(p,1)}(\omega), ..., G_{(p,N)}(\omega)]^T$ are projected on the null-subspace of the received signal vector, where $G_{(p,n)}(\omega)$ is the Green’s function between (pixel) point $p$ in the imaging domain and the $n^{th}$ element of the array. The null-subspace projection vector is given by

$$n_p(\omega) = \frac{g_p(\omega)}{\|g_p(\omega)\|} - \frac{\left[\frac{g_p(\omega)}{\|g_p(\omega)\|}, \frac{r(\omega)}{\|r(\omega)\|}\right]}{\|r(\omega)\|} r(\omega)$$  \hspace{1cm} (3.2)
where \( \langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{b}^\dagger \mathbf{a} \) denotes the inner product between \( \mathbf{a} \) and \( \mathbf{b} \), and \( \dagger \) denotes conjugate transpose (note that complex conjugation in frequency-domain is equivalent to time-reversal in time-domain), and \( \| \mathbf{a} \| = \sqrt{\langle \mathbf{a}, \mathbf{a} \rangle} \).

The \( N_f \) frequency components, constituting the source spectrum, are combined to form the null-subspace projection as follows

\[
N_p = \frac{1}{N_f} \sum_{i=1}^{N_f} \| \mathbf{n}_p(\omega_i) \| \tag{3.3}
\]

The focusing location \( p_f \) is the minimizer of the null-subspace projection, i.e., \( p_f = \arg \min_p N_p \).

The null-subspace projection is a random variable whose variance can be written as the sum of the autocovariance matrix (\( \mathbf{C} \)) of the random vector \( \mathbf{\bar{n}}_p \) stacking the \( N_f \) frequency components of \( \| \mathbf{n}_p(\omega) \| \) [71], that is

\[
\text{var}(N_p) = \frac{1}{N_f^2} \sum_{i=1}^{N_f} \sum_{j=1}^{N_f} [\mathbf{C}(\mathbf{\bar{n}}_p, \mathbf{\bar{n}}_p)]_{ij} \tag{3.4}
\]

where \( \mathbf{\bar{n}}_p = [\| \mathbf{n}_p(\omega_1) \|, \ldots, \| \mathbf{n}_p(\omega_{N_f}) \|]^T \).

### 3.2.2 Measurements Results and Discussion

We assume the imaging domain as a homogeneous medium with relative permittivity equal to the mean effective permittivity of the actual random medium. We do not use the exact, pointwise properties of the actual medium because, in scenarios of practical interest, these are typically not known (at least in a deterministic sense). The steering vectors in the imaging domain \( \mathbf{g}_p(\omega) \) are computed analytically using the homogeneous medium Green’s function. The receiving horn antenna properties are taken into account through a probe compensation procedure.

Null-subspace projection images \( N_p \) are considered in Fig. 3.2. These images are produced from 15 realizations of random medium composed of wet scatterers.
Figure 3.2: Null-subspace images $N_p$ from 15 realizations of wet scatterers (ensemble-averaged over the realizations). The source location is indicated by a circle, and the focusing locations, determined from different realizations, are indicated by ‘x’s. Colorbar is in log scale. (a) Narrowband imaging: $f_c = 9$ GHz, $B = 1.67$ GHz. (b) UWB imaging: $f_c = 9$ GHz, $B = 13.67$ GHz. (c) Lower frequency imaging: $f_c = 3.3$ GHz, $B = 2.33$ GHz. (d) Higher frequency imaging: $f_c = 14.67$ GHz, $B = 2.33$ GHz.

and constructed using different sets of bandwidths $B$ and center frequencies $f_c$ as indicated in Fig. 3.2 caption. We plot the (ensemble) average of the 15 images and indicate the focusing location of each individual image by a cross. By considering the spatial spread of the latter, one can assess the stability of imaging carried out using different bandwidths and center frequencies. Note that in practical imaging scenario,
Figure 3.3: (a) Variance of individual frequency components (diagonal elements of the covariance matrix) versus number $N$ of array elements, fractional volume (f.v.) of scatterers, and wet/dry conditions. Dashed lines delimit useful frequency band used for comparing the effect of scatterers parameters. (b) r.m.s. error in source localization versus bandwidth, for $N = 5$ and $f_c = 9$ GHz. (c) Null-subspace image variance, averaged over imaging domain pixels, versus bandwidth for $f_c = 9$ GHz. (d) Null-subspace covariance matrix for the case of wet scatterers and $N = 5$.

we will have only one realization of the random medium and will only obtain the single corresponding image.

The imaging domain, as indicated in Fig. 3.1(a), consists of $180 \times 180$ pixels with pixel size of 1.16 mm. Throughout this work, we assume a rectangular spectrum, such that $B = f_{\text{max}} - f_{\text{min}}$. It is clear that UWB imaging (Fig. 3.2(b)) is more
Figure 3.4: (a) Null-subspace covariance matrix, averaged over image pixels, in homogeneous medium (free-space). (b) Fractional decrease in image variance with bandwidth relative to the variance using bandwidth of 1 GHz and $f_c = 8$ GHz.

stable (detected source location varies only slightly from one realization to the other) vis-à-vis narrowband imaging (Fig. 3.2(a)). Moreover, Fig. 3.2(c) and (d) show that, for the same $B$, increasing $f_c$ results in less stable images, since higher frequencies are more vulnerable to distortion by the discrete scatterers, as evident by the variance of individual frequency components plotted in Fig. 3.3(a).

In order to be able to investigate imaging stability versus bandwidth under different number of array elements and scatterer configurations, we employ two measures to quantify stability. The first measure is the r.m.s. error in source localization, measured from the true source location, as shown in Fig. 3.3(b) for a five-elements array. The second measure is the image variance averaged over image pixels, i.e. $\text{avg}_p(\text{var}(N_p))$, as shown in Fig. 3.3(c) for five- and three-elements arrays. Both measures are consistent in showing that, as the scatterers fractional volume and/or contrast increase, more bandwidth is required to ensure image stability (i.e., error or
variance below certain threshold). The second measure, however, exhibits a smoother variation with bandwidth, since it involves averaging over the entire imaging domain.

To obtain better insight into the relationship between bandwidth and image stability, we plot the covariance matrix (in (3.4)), averaged over image pixels, for the case of wet scatterers, in Fig. 3.3(d). The covariance matrix shows that sufficiently apart frequency components become decorrelated; therefore, summing up $N_f$ frequency components to construct the image approximates an ensemble average over $N_f$ independent realizations. As the bandwidth increases, the image variance decays asymptotically as $\sim 1/N_f$. A significant increase in image variance with a reduction on the number of array elements is evident in Fig. 3.3(c). This can be explained by a similar argument. The norm in (3.3) involves summation over the $N$ receivers; therefore, the variance at a given frequency is proportional to the sum of the covariance matrix of the null-subspace projection vector at this frequency. In the presence of random medium, signals recorded by sufficiently apart receivers become more decorrelated; therefore, combining signals from different receivers mimics ensemble averaging as well. For comparison, we plot the covariance matrix of a measurement in free-space in Fig. 3.4(a). In this case, the only residual source of randomness is additive noise. Comparing Fig. 3.3(d) with Fig. 3.4(a) shows that the covariance matrix from the free-space case exhibits slower frequency decorrelation, and hence it yields smaller fractional decrease in image variance with bandwidth, as shown in Fig. 3.4(b).

### 3.3 Statistical Stability of Differential TR

_Differential_ TR was introduced in [73] as a technique for detecting and tracking moving target(s) in clutter. It will be discussed in details in the next chapter. In the
Figure 3.5: Imaging passive metallic target moving across high fractional volume dry scatterers using differential TR. (a) Problem setup: Moving target locations at two consecutive snapshots (1 and 2) are indicated by circles, and the direction of motion is indicated by an arrow. (b) Narrowband imaging: $f_c = 9$ GHz, $B = 1.67$ GHz. (c) UWB imaging: $f_c = 9$ GHz, $B = 13.67$ GHz.

next part of the experiment, our goal is to locate a passive target (metallic cylinder with diameter of 1.8 cm) moving across a random medium with high fractional volume of dry scatterers, as depicted in Fig. 3.5(a). Backscattered signals corresponding to the two indicated target locations are recorded and subtracted, producing a differential signal. Subtraction eliminates direct (first-order) scattering from any stationary object. The differential signal vector now plays the role of the received signal vector $\mathbf{r}$.
in (3.2), and is used in the imaging process. Resulting images, averaged over five realizations of the random medium, using narrowband and wideband signals are shown in Fig. 3.5(b) and (c), respectively. This result shows that differential TR imaging is statistically stable under wideband operation.

3.4 Conclusion

In summary, we have demonstrated experimentally how UWB TR-based imaging can exploit frequency decorrelation to yield self-averaging in random media, hence producing images that are stable w.r.t. the particular realization. We considered imaging in both source and echo modes. This work, along with that in chapter 2, serve to establish a framework for quantifying statistical stability of TR imaging and understanding the various factors affecting it. Results presented in this chapter can be extended to the case of multiple sources or scatterers as long as active sources are uncorrelated or passive scatterers are well-resolved, that is, multiple scattering effects among scatterers can be neglected in comparison with direct returns from each scatterer.
Chapter 4: Imaging and Tracking of Targets in Clutter
Using Differential Time-Reversal Techniques

Imaging and tracking of obscured targets are of interest in many applications including law-enforcement and search-and-rescue operations [13]. Many techniques have been developed for moving targets detection and localization, among which is Doppler radar [89]. In its basic form, Doppler radar is based upon a continuous sinusoidal signal transmitted from a single transmitter; the range of moving targets can be determined by inspecting the backscattering spectrogram. So, in a sense, it is merely a 1-D motion detector [13]. An array of transmitters can be used and backprojection can be combined with Doppler processing to provide 2-D and 3-D localization. This type of strategy has been employed in [90], where combined Doppler processing and spatial beamforming was used to track humans through walls. The sensitivity of Doppler radars increases with the frequency of operation. However, the attenuation offered by walls and other obstacles to electromagnetic waves increases rapidly with frequency [91] and this puts an upper bound to the radar sensitivity. Another technique, for through-wall motion detection, was developed in [92, 93, 94]. It utilizes a motion detection filter (which is a high pass filter in essence) to filter out stationary clutter. Recently, UWB TR has been applied in through-wall imaging.
and differential TR for through-wall moving targets localization was briefly considered in [96, 97].

The objective of this chapter is to introduce and assess two new techniques for imaging and tracking targets in clutter using UWB TR. We first develop an algorithm for classifying existing scatterers into stationary vs. moving targets. Stationary targets may represent, for example, pillars, furniture, tree trunks, or any fixed clutter. Moving targets may represent moving persons or vehicles. Even though, in principle, estimation of targets’ materials and electrical sizes can also be made from the available data, we are limiting ourselves to point-like scatterers here. A brief consideration on extended targets (e.g. walls) characterization [49, 98, 99] is presented in Appendix A.

Next, we develop a tracking algorithm based on differential TR. The proposed algorithm has outstanding clutter rejection performance since it is a differential technique. It is fast, simple, and requires minimal processing as compared with Doppler processing. In addition, it exploits beamforming to focus on each of the moving targets individually. Thus, it has potential applicability in wireless communication covertness in cluttered environments [46, 100] with selective beams automatically tracking respective moving users. It can also be integrated with micro-Doppler processing [101] to further analyze the motion characteristics of each moving target. We present the mathematical formulation of the algorithm and study factors affecting its tracking sensitivity. Performance in the presence of clutter from discrete secondary scatterers and continuous random inhomogeneous backgrounds (“discrete” and “continuous” clutter) is evaluated via numerical simulations. The algorithm is shown to
Figure 4.1: Problem layout: Two stationary targets (squares) and two moving targets (circles) at ten consecutive time instants. Left stationary: Metallic box with side length $= 6\Delta_s$. Right stationary: Dielectric box with $\epsilon_r = 6$ and side length $= 8\Delta_s$. Left moving: Dielectric box with $\epsilon_r = 6$ and side length $= 8\Delta_s$. Right moving: Metallic box with side length $= 6\Delta_s$. The directions of motion are indicated by black arrows and the positions of the 14 antenna array elements are indicated by black triangles.

possess both superresolution and statistical stability properties of UWB TR. Finally, the algorithm is applied to track multiple targets.

Numerical simulations of the forward problem are carried out here using the finite-difference time-domain (FDTD) method [84]. The computational domain is truncated with perfectly matched layer (PML) [102] defined through stretched coordinates [103, 104, 105, 106] to emulate an open space. All simulation setups are two-dimensional and the grid utilized is uniform with spatial cell size $\Delta_s = 2.5$ cm.
Figure 4.2: Spectrum of the eigenvalues of the TRO of the tenth snapshot. (a) Eigenvalues spectrum. (b) Normalized eigenvalues spectrum. Eigenvalues in (b) are normalized with respect to the input pulse spectrum. The spectrum of the first four eigenvalues is distinguishable from the remaining, especially from the behavior at the low-frequency end of the spectrum as indicated in (a). This reflects the presence of four point-like scatterers in the domain under investigation.

Figure 4.3: Fluxogram of the hierarchy for the discrimination algorithm with its two branches: stationary and moving targets detection.
4.1 Stationary vs. moving targets discrimination algorithm

4.1.1 Problem scenario

Consider the setup shown in Fig. 4.1. Two point-like targets are moving upwards in the presence of two point-like stationary scatterers. The locations of the moving targets at ten consecutive instants are indicated. The background is homogeneous with relative permittivity $\varepsilon_r = 2$. The relative permittivity is chosen to coincide with the mean permittivity of scenarios involving continuously random background media, as considered later on (in practice, these scenarios correspond to imaging under fog, smoke, or heavy dust conditions, for example). A 14-element linear antenna array of $y$-polarized line sources is used. Consecutive array elements are separated by one half-wavelength.

Multistatic data matrices (MDMs) corresponding to each of the indicated locations are recorded. Recall that a MDM is obtained by successively firing a short pulse by each array element and recording the received signal by all elements [80, 30]. In our case, we use the first derivative of the Blackmann-Harris (BH) pulse [107] which has a center frequency $f_c = 400$ MHz and time duration $1.55/f_c$. It is assumed that the acquisition time required to record a MDM is much shorter than the other time scales of the problem. In particular, each moving target can be assumed still during an individual MDM acquisition. We call this acquisition a snapshot. The time interval between snapshots also needs to be sufficiently small so that the maximum displacement of moving targets (within the interval) is much shorter than the minimum wavelength of operation (in practice this criterion is also easily met with conventional electronics). This condition is necessary for accurately detecting moving targets as discussed in Section 4.2 below.
To analyze the contents of each snapshot, TR operators (TROs) of the recorded MDMs are computed for a discrete set of frequencies in the spectral window of the input signal. The TRO at frequency $\omega$, $T(\omega)$, is obtained from the MDM $K(\omega)$ by $T(\omega) = K^\dagger(\omega)K(\omega)$, where $\dagger$ denotes conjugate transpose [30]. The “time-reversal operator decomposition” (DORT) method [76, 77, 78, 79, 39] can be then applied to the TRO of each snapshot to obtain its eigenvalue/vector structure. As an example, the eigenvalue spectrum of the TRO of the tenth snapshot of Fig. 4.1 is shown in Fig. 4.2(a). This spectrum reflects the presence of four significant eigenvalues, corresponding to four well-resolved point-like targets. This can also be deduced from the normalized spectrum shown in Fig. 4.2(b).

The hierarchy of the discrimination algorithm is shown in Fig. 4.3. Having recorded several snapshots, the algorithm splits then into two separate processes: stationary target detection and moving target detection. Each process is described in detail below.

### 4.1.2 Stationary targets detection

The first step in stationary targets detection is to average out the recorded MDMs, then apply the DORT to the TRO of this (time-)averaged MDM. Since different MDMs capture moving targets at different locations, the contribution of moving targets to the eigenspectrum of the TRO of the averaged MDM is “washed out” after sufficient number of acquisitions. This enhances the relative contribution of stationary targets versus moving targets in the averaged MDM. Hence, the spectrum of the averaged TRO reflects the presence of only two significant scatterers in this case, which are the stationary targets. This is shown in Fig. 4.4(a) and (b).
Figure 4.4: Spectrum of the eigenvalues of the TRO of the averaged MDM. (a) Eigenvalues spectrum. (b) Normalized eigenvalues spectrum. The spectrum of the first two eigenvalues is distinguishable from that of the rest, indicating the presence of two stationary scatterers. The vertical green-dashed lines in (b) point out the locations of the valleys in the dielectric target’s response.

Figure 4.5: DORT images of the averaged MDM. (a) Image of the first eigenvector corresponding to the left stationary target. (b) Image of the second eigenvector corresponding to the right stationary target.

Images and locations of the stationary targets are obtained by projecting the eigenvectors associated with the significant eigenvalues of the average TRO on a synthesized imaging domain. Here we choose the imaging domain to be a homogeneous
medium having the same permittivity as the actual background. The projection of the eigenvector \( \mathbf{q} \) on point \( p \) in the imaging domain at frequency \( \omega \) is given by

\[
P_p(\omega) = \left\langle \frac{\mathbf{g}_p(\omega)}{\|\mathbf{g}_p(\omega)\|}, \mathbf{q}(\omega) \right\rangle
\]

(4.1)

where \( \mathbf{g}_p(\omega) \) is the steering vector of point \( p \), given by

\[
\mathbf{g}_p(\omega) = [G(p,1)(\omega), ..., G(p,N)(\omega)]^T
\]

(4.2)

where \( G(p,i)(\omega) \) is the scalar Green’s function between location \( p \) and the \( i^{th} \) element of the array, and \( N \) is the number of elements in the array. In addition, \( \langle \mathbf{g}_k, \mathbf{q} \rangle = \mathbf{q}^\dagger \mathbf{g}_k \) denotes the inner product between \( \mathbf{g}_k \) and \( \mathbf{g}_l \), where \(^\dagger\) refers to the conjugate transpose, and \( \|\mathbf{g}_p(\omega)\| \) is the norm of \( \mathbf{g}_p \) given by \( \|\mathbf{g}_p\| = \sqrt{\langle \mathbf{g}_p, \mathbf{g}_p \rangle} \).

To combine all frequencies, the projection in the time domain is computed by taking the inverse Fourier transform of \( P_p(\omega) \), that is

\[
P_p(t) = \mathcal{F}^{-1}(P_p(\omega))
\]

(4.3)

The focusing time \( t_f \) is defined as the time instant at which the peak of \( P_p(t) \) over all points in the imaging domain occurs. Finally, the image at point \( p \) is given as

\[
I_p = \left| \frac{P_p(t_f)}{\max_{p,t}(P_p(t))} \right|
\]

(4.4)

The obtained images are shown in Fig. 4.5. It is apparent that the images coincide well with the actual targets locations.

Further information about the targets’ size and/or composition can be extracted from their eigenvalue spectrum. By inspecting the spectrum we can infer whether the target is metallic or dielectric and, in the latter case, we can estimate its electrical size. The reflectivity of metallic scatterers with sufficiently high conductivity is almost
frequency independent within the spectral range considered here. So, the eigenvalue associated with a metallic scatterer will have same spectrum as the input pulse. This can be seen in the spectrum of the first eigenvalue in Fig. 4.4(a), which is basically the spectrum of the first derivative of the BH pulse used as input. On the other hand, dielectric targets are penetrable so may act as imperfect resonators. Their reflectivity will possess periodic “peaks” and “valleys” experienced by such resonators. In the second eigenvalue in Fig. 4.4, we can observe the presence of a first valley at \( f_v \approx 330 \) MHz. From this value we can estimate the electrical length of the target along the co-range at the center frequency \( l_z/\lambda \) as \( f_c/2f_v \), where \( l_z \) is the dimension of the target in the co-range direction, \( \lambda \) is the wavelength inside the dielectric target, \( f_c \) is the center frequency, and \( f_v \) is the frequency of the first valley. The previous relation follows directly from the simple null-frequencies formula of one-dimensional resonators. In our case, \( f_v \approx 330 \) MHz and \( f_c = 400 \) MHz, so the estimated electrical length \( l_z/\lambda = 0.6 \) agrees reasonably well with the actual value 0.65. A more detailed consideration of target characterization using DORT is presented in Appendix A.

Having estimated the locations/compositions of stationary targets, we can incorporate this information into a synthetic background for tracking of moving targets with increased accuracy [13]. This can be done as follows: a forward problem scenario is constructed where synthetic targets are positioned at the locations of the fixed targets with compositions and sizes chosen in virtue of estimated values. The forward problem is next solved numerically, from which the Green’s function between each point in the domain and each array element can be computed and stored for use in the imaging of moving targets.

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4.1.3 Moving targets detection

As opposed to the stationary targets detection algorithm, the goal of the moving targets detection algorithm is to enhance the contribution of moving targets in the eigenspectrum while suppressing that of stationary targets. Therefore, a reasonable procedure would be to compute the differential MDM i.e., the difference between the last two recorded MDMs. Since the displacement of moving targets between differenced snapshots is assumed much shorter than the wavelength, most of the contribution from stationary targets and multiple scatterings among moving and stationary targets cancel out under differencing. Therefore the eigenspectrum of the differential TRO will better correspond to the moving targets as shown in Fig. 4.6, where the two significant eigenvalues pertain to the two moving targets.

Backprojecting the respective eigenvectors of the differential TRO results in images that are located at some intermediate target locations between those of the last two snapshots. A mathematical analysis of this is presented in the next section on differential tracking. The resulting images are shown in Fig. 4.7. A similar analysis to the one used in Section 4.1.2 can be made to estimate the composition of the moving targets using their eigenvalues spectrum.

4.2 Time-reversal algorithms for tracking of moving targets

In this section, we present the basic theory and discuss the performance of the proposed TR tracking algorithms. We start by presenting mathematical formulations for both non-differential and differential processings as well as contrasting their properties. It will be shown that non-differential processing is hampered by the presence of secondary scatterers. On the contrary, differential processing provides stationary
Figure 4.6: Spectrum of the eigenvalues of the TRO of the differential MDM. (a) Eigenvalues spectrum. (b) Normalized eigenvalues spectrum. The spectrum of the first two eigenvalues is distinguishable from that of the rest, indicating the presence of two moving scatterers. Again, the vertical green-dashed lines in (b) indicate the locations of the valleys in the dielectric target’s response.

Figure 4.7: DORT images of the differential MDM. (a) Image of the first eigenvector corresponding to the moving metallic target. (b) Image of the second eigenvector corresponding to the moving dielectric target. The images are centered at intermediate locations between the tenth and the ninth snapshots.
clutter subtraction therefore enables higher tracking accuracy of obscured targets in rich scattering backgrounds. The performance of differential tracking in both discrete and continuous clutter is evaluated and the problem of tracking multiple targets is addressed.

4.2.1 Theory

Consider the setup shown in Fig. 4.8. A target is moving diagonally upwards as indicated by the arrow. Our goal is to track the motion of the target. Apparently the tracking algorithm has to be fast enough to respond to the target motion. The main limitation on the tracking speed comes from the processing time required to compute the target location after each snapshot acquisition. This is in fact the main deficiency of the discrimination algorithm presented in Section 4.1.3, since it involves the relatively expensive singular value decomposition of the acquired MDMs; therefore, it is less suited for real-time tracking. Nevertheless, it can be used to determine the initial
Figure 4.9: Actual (red) and detected (blue) traces for (a) non-differential tracking. (b) differential tracking. In both cases the target is moving ‘upwards’.

locations of moving targets at some particular instant of time. From that point on, the tracking algorithm takes over and continues tracking each of the moving targets.

The idea behind the proposed tracking algorithm is to simultaneously excite array elements so as to provide a beam that selectively focuses on the moving target, rather than exciting each array element separately by the same input signal as done in the conventional MDM acquisition. Backscattering is recorded and processed as described below to yield the updated target location and the process is repeated. In cases where there exists more than one moving target, selective beams corresponding to each of the targets are fired successively. The acquisition time will be less than that of the MDM as long as the number of moving targets is less than the number of array elements.
Figure 4.10: Non-differential processing projections, in the frequency domain, at locations \( p \) and \( p + 1 \) indicated in Fig. 4.9(a). (a) The magnitude. (b) The phase.

**Non-differential processing**

Having determined the location of the target at a particular instant indicated by \( p \) in Fig. 4.8, a selective beam is launched in the physical medium. The input to the array is basically the time-reversed (phase conjugated in the frequency domain) steering vector of point \( p \), \( \mathbf{g}_p \), given in (4.2). The incident beam illuminates the target at location \( p + 1 \), and the backscattering at frequency \( \omega \) is proportional to the electric field received by the antenna array \( \mathbf{s}_{p+1} \), given by

\[
\mathbf{s}_{p+1} = \langle \mathbf{g}_{p+1}, \mathbf{g}_p \rangle \tau(\omega) \mathbf{g}_{p+1} + \mathbf{c}_{p+1} \tag{4.5}
\]

where \( \tau(\omega) = |\tau(\omega)| e^{j\Phi(\tau(\omega))} \) is the scattering coefficient of the target at frequency \( \omega \); and \( \mathbf{c}_{p+1} \) is the backscattering collected by the array elements from the clutter when the target is at location \( p + 1 \). In order to guarantee that the target at location \( p + 1 \) is illuminated by the focused beam initially transmitted to focus at location \( p \), namely \( \mathbf{g}_p \), the time interval \( \delta t \) between successive illuminations has again to be
short enough\(^1\). In scenarios where direct scattering from ambient clutter is negligible compared with the scattering from the target, the \(c_{p+1}\) term in (4.5) can be neglected.

This is an important assumption for non-differential processing to be able to trace the motion.

In order to detect the target’s image and location, the normalized phase conjugated scattering vector is projected onto the normalized steering vectors of a synthesized imaging domain. In the absence of any information about the imaging scenario, a possible choice of the imaging domain in this case would be a homogeneous medium with the same (estimated) average permittivity as the background of the forward problem setup. The projection onto location \(I\) in the imaging domain is given by

\[
P_I = \frac{\langle g_I, s_{p+1} \rangle}{\|g_I\| \|s_{p+1}\|}
\]

where \(g_I\) is the steering vector corresponding to location \(I\) in imaging domain. The above projection can be expanded as

\[
P_I = \sum_{i=1}^{N} G_{(I,i)} S^*_{(p+1,i)}
\]

where \(S_{(p+1,i)}\) is the \(i^{th}\) component of \(s_{p+1}\) and the * denotes complex conjugation.

The phase of the \(i^{th}\) term of the above sum can be written as

\[
\Phi_{P_{(I,i)}} = -\alpha_{(p+1,p)} - \Phi_{\tau(\omega)} - \theta_{(p+1,i)} + \theta_{(I,i)}
\]

where \(\langle g_k, g_l \rangle = |\langle g_k, g_l \rangle| e^{j\alpha_{k,l}}\) and \(G_{(m,i)} = |G_{(m,i)}| e^{j\theta_{(m,i)}}\).

The maximum value for the projection occurs when all of its components add constructively. This requires \(\Phi_{P_{(I,i)}}\) to be independent of \(i\). As seen from (4.8), this

\(^1\)Suppose, for example, a (conservative) 3 dB margin such that location \(p + 1\) lies within the half-power beamwidth \(\Omega_H\) of the beam (from \(g_p\)). In this case, \(\delta t\) has to be less than \(\Omega_H/\dot{\theta}_{max}\), where \(\dot{\theta}_{max}\) is the maximum angular velocity of the target in cross-range.
occurs only when \( \theta_{(I,i)} = \theta_{(p+1,i)} \). In other words, the projection peaks at the location of the target when the latter is “hit” by the pulse initially emitted to focus on \( p \), and this is how the tracking operates. The target location is updated and the process is repeated to detect the successive locations. To verify the previous analysis, non-differential processing is applied to track a target moving in homogeneous background. Actual and detected traces are shown in Fig. 4.9(a). It is clear from this figure that detected locations coincide with locations \( p + 1 \) of each \( (p, p + 1) \) pair.

Magnitudes and phases of the projections on locations \( p \) and \( p + 1 \), indicated in Fig. 4.9(a), are plotted in Fig. 4.10(a) and (b) respectively. It is clear that \( |P_I| \) at \( p + 1 \) has a value of one over all frequencies. This is the maximum value \( |P_I| \) can take according to (4.6). Note that the projection \( P_I \) is almost coherent, i.e. it has almost linear phase progression with frequency. Some deviations from the linear behavior, especially at low frequencies, are observed. These deviations are due to the phase response of the complex scattering coefficient \( \tau(\omega) \). In case of coherent (or near-coherent) projection, we can find the location of the maximum by looking for the point in the imaging domain that maximizes the magnitude of each frequency component as discussed above.

**Differential processing**

Consider the same setup as the previous section, where now the focused beam \( g_p \) is transmitted twice at two closely separated time instants. The first transmission illuminates the target at location \( p + 1 \) and the backscattering is given by (4.5). Similarly, the second transmission illuminates the target at location \( p + 2 \) as shown.
Figure 4.11: Phases of the 14 terms composing the projections of the differential vector on the imaging domain at the center frequency of the considered bandwidth.

Figure 4.12: Differential processing projections, in the frequency domain, at locations $p+1$, $p+2$ and the intermediate focusing location indicated in Fig. 4.9(b). (a) The magnitude. (b) The phase. Highlighted frequency bands will be used later to study the traceability of the tracking algorithm.

In Fig. 4.8 and $s_{p+2}$ can be written as

$$s_{p+2} = \langle g_{p+2}, g_p \rangle \tau(\omega)g_{p+2} + c_{p+2}$$  \hspace{1cm} (4.9)
If the acquisition speed is chosen sufficiently faster than the target speed, we can safely assume that the distance between location \( p + 1 \) and location \( p + 2 \) is small. Thus the difference in the backscatterings from the clutter corresponding to the two locations is negligible. That is

\[
c_{p+1} \approx c_{p+2} \tag{4.10}
\]

Moreover, from the properties of the Green’s functions, we can assume that the difference in the Green’s functions affects mostly the relative phase, so that

\[
\left| G_{(p+1,i)} \right| \approx \left| G_{(p+2,i)} \right| \tag{4.11}
\]

and

\[
\left| \langle g_{p+1}, g_p \rangle \right| \approx \left| \langle g_{p+2}, g_p \rangle \right| \tag{4.12}
\]

We define the differential backscattering vector \( d_{(p+2,p+1)} \) as

\[
d_{(p+2,p+1)} = s_{p+2} - s_{p+1} \tag{4.13}
\]

Substituting with (4.5) and (4.9) and using (4.10), (4.11) and (4.12), the \( i^{th} \) component of \( d_{(p+2,p+1)} \) can be written as

\[
D_{i}^{(p+2,p+1)} \approx \left| \langle g_{p+2}, g_p \rangle \right| \tau(\omega) \left| G_{(p+2,i)} \right| (e^{j\alpha_{(p+2,p)}}e^{j\theta_{(p+2,i)}} - e^{j\alpha_{(p+1,p)}}e^{j\theta_{(p+1,i)}}) \tag{4.14}
\]

Note that the clutter components canceled out upon subtraction and this is one of the important advantages of differential processing. After some phase manipulation, the above equation can be written as

\[
D_{i}^{(p+2,p+1)} \approx \left| \langle g_{p+2}, g_p \rangle \right| \tau(\omega) \left| G_{(p+2,i)} \right| e^{j\frac{\alpha_{(p+2,p)}}{2}} e^{j\frac{\alpha_{(p+1,p)}}{2}} e^{j\left(\frac{\theta_{(p+2,i)} - \theta_{(p+1,i)}}{2}\right)} (\frac{s_{p+2} - s_{p+1}}{2}) \tag{4.15}
\]

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As we saw earlier, $\tau(\omega)$ has, in general, nonlinear phase response with frequency. Further processing can be done to eliminate its unknown phase whereby the differential vector $d^{(p+2,p+1)}$ is normalized by the inner product between $d^{(p+2,p+1)}$ itself and $g_p$. In this way, the phase of the $i^{th}$ component of $d^{(p+2,p+1)}$ becomes

$$-rac{\alpha_{(p+2,p)}}{2} - \frac{\alpha_{(p+1,p)}}{2} + \frac{\theta_{(p+2,i)} + \theta_{(p+1,i)}}{2}$$

(4.16)

where $\Phi_{\tau(\omega)}$ is absent.

Finally, to detect the target’s new location, $d^{(p+2,p+1)}$ is projected onto the imaging domain. The projection at location $I$ in the imaging domain is given by

$$P_I = \left\langle \frac{g_I}{\|g_I\|}, \frac{d^{(p+2,p+1)}}{\|d^{(p+2,p+1)}\|} \right\rangle$$

(4.17)

A similar expression can be written for $P_I$ as in (4.7), where the phase of the $i^{th}$ term is given by

$$\Phi_{P_{(i,o)}} = \frac{\alpha_{(p+2,p)}}{2} + \frac{\alpha_{(p+1,p)}}{2} - \frac{\theta_{(p+2,i)} + \theta_{(p+1,i)}}{2} + \theta_{(I,i)}$$

(4.18)

To verify the approach, differential processing is applied to track a target moving in homogeneous background. Actual and detected traces are shown in Fig. 4.9(b). Consider the projection at each of the following locations indicated in Fig. 4.9(b).

Note that, without loss of generality, it is assumed that the initial target’s location $p$ coincides with location $p + 1$ as shown in the figure.

(a) Location $p + 1$:

$$\theta_{(I,i)} = \theta_{(p+1,i)}$$

(4.19)

From (4.18), $\Phi_{P_{(p+1,i)}}$ becomes

$$\Phi_{P_{(p+1,i)}} = \frac{\alpha_{(p+2,p)}}{2} + \frac{\alpha_{(p+1,p)}}{2} - \frac{\theta_{(p+2,i)} - \theta_{(p+1,i)}}{2}$$

(4.20)
which has $-(\theta_{(p+2,i)} - \theta_{(p+1,i)})/2$ variation with $i$ as shown in Fig. 4.11.

(b) Location $p + 2$:

$$\theta_{(I,i)} = \theta_{(p+2,i)}$$

(4.21)

and from (4.18), $\Phi_{P_{(p+2,i)}}$ is given by

$$\Phi_{P_{(p+2,i)}} = \frac{\alpha_{(p+2,p)}}{2} + \frac{\alpha_{(p+1,p)}}{2} + \frac{\theta_{(p+2,i)} - \theta_{(p+1,i)}}{2}$$

(4.22)

which has $(\theta_{(p+2,i)} - \theta_{(p+1,i)})/2$ variation with $i$ as shown in Fig. 4.11.

(c) Focusing location:

Assume that a location $I$ exists such that

$$\theta_{(I,i)} = \frac{\theta_{(p+2,i)} + \theta_{(p+1,i)}}{2}$$

(4.23)

then, from (4.18), $\Phi_{P_{(I,i)}}$ is given by

$$\Phi_{P_{(I,i)}} = \frac{\alpha_{(p+2,p)}}{2} + \frac{\alpha_{(p+1,p)}}{2}$$

(4.24)

which is independent of $i$, i.e. all components of the inner product of $P_I$ add in phase. We call this location the “focusing location”. At the focusing location, $|P_I|$ assumes a maximum value of

$$|P_I| = \sum_{i=1}^{N} \left| \frac{G_{(I,i)}}{\|g_i\| \|u_i\|} \right|$$

(4.25)

where

$$u_i = \sin \left( \frac{\alpha_{(p+2,p)} - \alpha_{(p+1,p)}}{2} + \frac{\theta_{(p+2,i)} - \theta_{(p+1,i)}}{2} \right) \left| G_{(p+2,i)} \right|$$

(4.26)

is the $i^{th}$ component of $u$. However, there is no guarantee that there exists a location satisfying the focusing condition (4.23) for all frequencies. So it would be more accurate to determine the focusing location by considering the projection in the time domain $P_I(t)$, obtained by taking the inverse Fourier transform of the projection in
the frequency domain. The location at which \( P_I(t) \) peaks is the one that best satisfies (4.23) over the bandwidth of interest. Note that the time instant at which the peak of \( P_I(t) \) occurs is irrelevant to determining the focusing location, and is determined by the phase of \( P_I \) shown in Fig. 4.12(b).

Focusing (detected) locations in Fig. 4.9(b) are determined using the projection in the time-domain as described above. The detected locations lie somewhere in between locations \( p + 1 \) and \( p + 2 \) as expected from the ideal focusing condition (4.23). Fig. 4.11 shows that the components of the inner product of \( P_I \) at the focusing location are almost in phase, which agrees well with our assumption.

The magnitude of the projections at the above three locations are shown in Fig. 4.12(a). It is clear that the focusing location has the maximum projection over the entire bandwidth. Note also that this maximum is close to one as long as
\[
\sin \left( \frac{\alpha(p+2,p) - \alpha(p+1,p)}{2} + \frac{\theta(p+2,j) - \theta(p+1,j)}{2} \right)
\] varies only slightly with \( i \), as can be seen from (4.25) and (4.26).

Determining the focusing location from the time-domain projection is also useful in case of incoherent projection, where the frequency components have random phase relationship among them. This situation occurs in the case of tracking under the presence of clutter, where different frequency components “respond” to the clutter differently. More discussion on this is presented in Section 4.2.3.

**Traceability**

In this subsection we discuss the effect of the operating frequency on the ability of the differential tracking algorithm to track small motions of the target. This characterizes the sensitivity of the algorithm. The ability of the algorithm to detect (focus on) the new location rather than the prior location depends on how large
Figure 4.13: Variation of $\theta_{(p+2, i)} - \theta_{(p+1, i)}$ from one array element to the other for the two frequency bands indicated in Fig. 12(a).

Figure 4.14: Actual (red) and detected (yellow) traces for (a) the lower frequency band. (b) the upper frequency band. Higher center frequency results in better tracking sensitivity. The array elements are indicated by white triangles.

the projection at the focusing location (which is biased towards the new location) is compared with that at the prior location. In other words, the more “defocused” the
projection at the prior location is, the more sensitive the algorithm will respond to the motion.

As discussed in the last subsection, the phases of the constituting terms of the projection on location \( p + 1 \) \( (\Phi_{P_{p+1,i}}) \) vary with \( i \) as \(- \left( \theta_{(p+2,i)} - \theta_{(p+1,i)} \right) / 2\). So the larger the variation of \( (\theta_{(p+2,i)} - \theta_{(p+1,i)} ) \) with \( i \), the more defocused the projection will be at location \( p + 1 \). The variation of \( (\theta_{(p+2,i)} - \theta_{(p+1,i)} ) \) with respect to the antenna index \( i \) for the two highlighted lower and upper frequency bands in Fig. 4.12(a), are shown in Fig. 4.13. This figure shows that, as the center frequency \( f_c \) increases, the phase difference variation increases and the algorithm becomes more sensitive to the movement. Note that a similar feature is presented by Doppler radars. Detected traces corresponding to the aforementioned frequency bands are shown in Fig. 4.14. It is apparent that the detected movement trace utilizing higher \( f_c \) agrees better with the actual movement trace.

### 4.2.2 Performance in clutter

In this section, we present simulation results of the differential tracking algorithm applied to moving target in discrete clutter (secondary scatterers). The setup is shown in Fig. 4.15. A point-like metallic target is moving (towards the antenna array location) in a room enclosed by four walls with \( \epsilon_r = 8 \) and in the presence of four other secondary discrete scatterers, e.g. pillars. All pillars have \( \epsilon_r = 14 \). Fig. 4.15(a) shows the actual and detected traces when the synthesized imaging domain used for backprojection takes the clutter into account (that is, the Green’s function used for the inverse problem (backprojection) incorporates the presence of the walls and the pillars). Very good agreement between the actual and the constructed traces.
Figure 4.15: Performance of differential tracking in discrete clutter. Actual trace (red) and detected traces (yellow). Moving target is surrounded by four walls ($\varepsilon_r = 8$) and in the presence of four pillars ($\varepsilon_r = 14$) (a) Imaging domain takes into account the walls and the pillars. (b) Imaging domain takes into account the front wall only. (c) Imaging domain is a homogeneous medium. The array elements are indicated by white triangles. The wall thickness is purposely different for the front-, lateral-, and back-walls.

Figure 4.16: Moving target’s images along its trace. (a) The target is moving in homogeneous background. (b) The target is moving in the presence of discrete clutter.

is achieved. Information about the location, size, and composition of the point-like stationary scatterers can be provided by the discrimination algorithm explained in the first part of the chapter.
Fig. 4.15(b) shows the performance when only the front wall properties are known to the inverse problem. Some deviations in tracking occur due to the unaccounted-for multipath in the utilized (estimated) Green’s function. Finally, Fig. 4.15(c) shows the performance when backprojection occurs on homogeneous medium, that is, when the utilized Green’s function does not incorporate any secondary scatterers. The algorithm is still capable of tracking the motion well. However, the detected trace is slightly shifted upwards due to the (unaccounted) delay caused by the front wall permittivity.

Images of the moving target at different locations along its trace are shown in Fig. 4.16. Fig. 4.16(a) shows the images when the target is moving in a homogeneous medium, whereas Fig. 4.16(b) shows the images when the same target is moving in the presence of four walls and four pillars as depicted in Fig. 4.15. It can be noticed that the images in the presence of clutter are more focused along the cross-range direction than in the absence of clutter; this is the superresolution effect offered by
Figure 4.18: Performance of differential tracking in continuous random clutter: bandwidth effect. A different random medium realization is used for backprojection. A center frequency of 507 MHz is used with different bandwidths. (a) Bandwidth = 253 MHz, average deviation = 16.8\(\Delta_s\). (b) Bandwidth = 507 MHz, average deviation = 8.5\(\Delta_s\). (c) Bandwidth = 760 MHz, average deviation = 9\(\Delta_s\).

The cross-range resolution is proportional to \(\lambda L/a_e\), where \(L\) is the distance between the target and the array and \(a_e\) is the effective aperture of the array [30]. The normalized cross-range profiles of the images at the first (lowermost) location are plotted in Fig. 4.17. A reduction of about 30% in the half maximum width is achieved in the presence of clutter. This means that the multipath offered by the clutter effectively increased the antenna aperture by approximately 30% in this particular example. We can also notice, in Fig. 4.16, successive improvement in the resolution as the target moves closer to the array, as expected.

4.2.3 Statistical stability

Statistical stability is another distinctive feature of UWB TR techniques. Under certain conditions, different frequency components of the interrogating signal “perceive” the random medium differently than other (sufficiently well separated) frequency components. This implies a “frequency averaging” effect that mimics an
Figure 4.19: Performance of differential tracking in continuous random clutter; center frequency effect. A different random medium realization is used for backprojection. A bandwidth of 253 MHz is used with different center frequencies. (a) $f_c = 253$ MHz, average deviation = 11.4$\Delta_s$. (b) $f_c = 507$ MHz, average deviation = 16.8$\Delta_s$. (c) $f_c = 760$ MHz, average deviation = 28.6$\Delta_s$.

Figure 4.20: Performance of differential tracking in continuous random clutter; background effect. $f_c = 507$ MHz and bandwidth = 760 MHz with different imaging backgrounds. (a) Imaging in the same random realization, average deviation = 3.38$\Delta_s$. (b) Imaging in a different random realization, average deviation = 9$\Delta_s$. (c) Imaging in homogeneous medium, average deviation = 6.3$\Delta_s$.

ensemble averaging, making the image independent of the particular random medium realization and instead depending only on the medium statistics [37, 40]. In this section, we examine the statistical stability of the tracking algorithm as a function of the bandwidth, the center frequency of operation, and the background medium.
properties. In this context, we quantify statistical stability by the average deviation in the detected trace from the actual one; that is, the less the deviation, the more stable the tracking is.

In this case, the background is assumed as a realization of a continuous random medium with permittivity fluctuations following a clipped Gaussian distribution with mean permittivity $\epsilon_{rm} = 2$, standard deviation $\sigma = 0.3\epsilon_{rm}$ and correlation length $l_c = 10\Delta_s$, $\Delta_s = 2.5$ cm [82, 83]. A different realization having the same statistical properties is used for backprojection. The bandwidth effect is examined in Fig. 4.18, where traces constructed using successively increasing bandwidths are plotted. It is obvious from this figure that as the bandwidth increases, the tracing quality is improved.

The center frequency effect is considered in Fig. 4.19. Traces obtained using increasingly higher center frequencies (while maintaining constant bandwidth) are shown. It can be noticed that as the center frequency decreases, the trace becomes more stable. This is because of the fact that as the wavelength increases with respect to the correlation length, the random medium behaves more like an effectively homogeneous medium. Note that the effect of increasing the center frequency on statistical stability counteracts its effect on tracking sensitivity. A compromise has to be made to select the optimum operating frequency given the expected medium statistics.

Finally, for a given bandwidth and center frequency the effect of backprojecting on different media is considered in Fig. 4.20. In Fig. 4.20(a) the exact random realization is assumed to be known and used for projection. Perfect tracking is obtained, as expected. In Fig. 4.20(b) and (c) a different realization with the same properties and a homogeneous medium are used, respectively, for projection. Comparable tracking
Figure 4.21: Two-targets tracking with non-intersecting paths. Red and green dots respectively indicate actual and detected traces of the first target moving upwards. Yellow and black diamonds respectively indicate actual and detected traces of the second target moving downwards. Four stationary targets (pillars) are present and the entire domain is surrounded by walls. Array elements are indicated by white triangles. (a) Linear 14-elements antenna array. (b) Full aspect 14-elements array.

is obtained in both cases, with part (b) performing better in the first part of the trace but getting stuck at some point. A more comprehensive study incorporating all factors affecting statistical stability is under development and will be presented elsewhere.

4.2.4 Multiple targets tracking

The performance of the algorithm when used to track multiple targets is examined in this section. As mentioned before, the algorithm is fed with the initial locations of each of the moving targets and starts transmitting focused beams (which are the time-reversed steering vectors of the targets’ locations) alternatively to each of those locations. Differential processing is applied to determine the updated location of each target and the process is repeated. Fig. 4.21(a) shows the case of tracking two
Figure 4.22: Two-targets tracking with intersecting paths. Red and green dots respectively indicate actual and detected traces of the first target moving to the right. Yellow and black diamonds respectively indicate actual and detected traces of the second target moving to the left. Array elements are indicated by white triangles. (a) Linear 14-elements antenna array. (b) Full aspect 14-elements array.

targets using a linear 14-elements array. Tracking is successful as long as the targets do not cross through the wave paths between the array and other targets. When the latter situation occurs, ambiguity ensues; both beams start tracking the dominant (e.g. closer) target to the array. This ambiguity can be mitigated, for example, by using a full aspect array as shown in Fig. 4.21(b). A full aspect array guarantees the presence of uninterrupted wave paths between each target and at least some of the array elements.

Another situation is shown in Fig. 4.22, where the traces of the two moving targets intersect. At this point, the two focused beams will be pointing to the same location, hence there will be no guarantee that the beams will continue on tracking their respective targets. The beams may be swapped or both of them can track one and the same target resulting in the loss of tracking of the other, as shown in the
Fig 4.22(a). Even the use of a full aspect array does not resolve the ambiguity in this case, as shown in Fig. 4.22(b). It becomes necessary to record MDMs of two snapshots and apply DORT on the differential MDM to determine the new locations of the targets.

4.3 Conclusion

Two algorithms for identifying and tracking moving targets in clutter based on TR techniques have been developed. The first algorithm classifies targets into stationary and moving ones by applying the DORT to the averaged and differential MDMs acquired at consecutive time instants. The number of stationary and moving targets, as well as their locations and images can be determined. Moreover, electrical lengths of dielectric targets can be estimated from their eigenspectra. The second algorithm provides real-time tracking of moving targets using differential TR processing. The performance of the algorithm in the presence of discrete and continuous random clutter is evaluated. It was shown to have outstanding clutter rejection performance thanks to differential processing. The algorithm is capable of tracking multiple targets as long as their paths do not intersect. The tracking algorithm is an efficient algorithm in terms of processing time and memory resources. It also utilizes selective beams focused at each moving target rather than illuminating the entire domain; this suggests possible applications in communication covertness for example.
Chapter 5: Bayesian Compressive Sensing for Ultrawideband
Inverse Scattering in Random Media

In Bayesian-based inversion, both target’s parameters and clutter are modeled as random variables with certain probability density functions (PDFs). The inversion algorithm combines (any) a priori information on the target’s parameters, with physics-based forward-problem PDFs, and array acquisitions to produce a posteriori PDFs of the unknowns [108, 109, 110, 111, 112, 9, 1, 2]. This approach provides means for measuring the confidence interval of the inversion and adaptively optimizing subsequent measurement(s) [113, 114]. Bayesian inference applied to compressive sensing was presented in [114, 115], where sparsity priors were imposed on a compressible (sparse) set of unknowns. That problem was solved efficiently using the relevance vector machine (RVM) technique [116, 117]. Recently, Bayesian compressive sensing has been applied in microwave imaging of sparse discrete scatterers using single frequency data, and the contrast source formulation [111], or the first order Born approximation [112]. Bayesian compressive sensing, combined with signal subspace methods, for imaging discrete targets has been presented in [118, 119, 120].

In non-Bayesian inverse scattering techniques, a cost function, to be iteratively minimized, is defined, and optimization techniques, such as the conjugate gradient method [121], are used to guide the iterations. This approach is computationally
costly, since it requires forward problem solution to compute the cost function and check convergence at each iteration step [3, 4, 122, 123, 124, 125]. Bayesian inversion alleviate the need for that since it has a ‘built-in’ measure for accuracy through the confidence level it provides. In addition, Bayesian compressive sensing (BCS) solved using the RVM, as presented in [116, 114], provides an elegant, closed-form solution for the posterior PDF; and therefore, there is no need for (costly) numerical computations of higher-order integrations that are done otherwise using Markov Chain Monte Carlo and Gibbs sampling [126, 127], as in [1, 2, 9, 24] for example. Finally, BCS with RVM does not require inversion of the projection matrix relating measurements to model parameters. Note that, this matrix may not be square, where for example the number of measurements is less than the number of unknowns, and it can be ill-conditioned, making it highly sensitive to noise.

In this chapter, we develop ultrawideband (UWB) inverse scattering techniques for continuous random media based on BCS. We exploit frequency decorrelation of the UWB interrogating signal to produce a statistically stable inversion; that is, an inversion that does not depend on the particular realization of the clutter but only on its statistics. We start by presenting a summary of BCS as applied to a linear regression model following [116, 114, 111]. Then, we apply that model to the electromagnetic inverse scattering problem under a first-order Born approximation, as in [112], but we extend the method to incorporate, in a single inversion, multifrequency multistatic measurements, and apply it to continuous random media, rather than to (sparse) discrete scatterers. We present several examples based on numerical simulations to assess the performance of the proposed technique under different scenarios. Next, we present an adaptive sensing approach for optimizing the location of the successive
measurement so as to maximize the differential information gain. After that, a new technique denoted as time-reversal-assisted localized-inversion (TRALI) is presented to reduce the computational cost of the algorithm by focusing the inversion effort onto sub-domains of interest. The method is then extended to high contrast media (i.e., those that do not conform with the first-order Born approximation) by introducing the Bayesian Distorted-Born Iterative Method (BDBIM). Finally, some applications of practical interest are presented, such as: underground imaging of layered media, breast imaging for cancer detection, and rough surface reconstruction.

In this work, the term compressive sensing is used in its broad sense, as adopted in [111, 112], to refer to problems where the unknown function (electrical properties of targets in our case) can be expressed as a sparse set of weights w.r.t. some expansion bases. This does not necessarily conform with the more formal usage of compressive sensing that refers to recovering certain signals from sparse acquisitions (fewer samples or measurements than those dictated by Nyquist rate) [128, 129]. In particular, in this work as well as the work presented in [111, 112], the number of measurements can be larger than the number of unknowns. Perhaps a better terminology for this method would be ‘Bayesian inversion using sparsity priors’.

5.1 Bayesian Compressive Sensing using the Relevance Vector Machine

Consider a linear regression model, where a vector $\mathbf{y}$ of $N$ noisy measurements is related to a vector $\mathbf{w}$ of $M$ (unknown) weights through the linear relationship

$$\mathbf{y} = \mathbf{Bw} + \mathbf{n}$$

(5.1)
where $\mathbf{B}$ is the projection matrix, and $\mathbf{n}$ is a vector of additive noise. We are seeking maximum a posteriori (MAP) estimates for the weights as follows

$$
\mathbf{w} = \arg \max_{\mathbf{w}} (p(\mathbf{w} | \mathbf{y}))
$$

(5.2)

The main challenge is trying to avoid ‘over-fitting’ the noisy measurements [116]. From Bayes’ rule, the posterior PDF is given by

$$
p(\mathbf{w} | \mathbf{y}) = \frac{p(\mathbf{y} | \mathbf{w})p(\mathbf{w})}{p(\mathbf{y})}
$$

(5.3)

Let’s consider each term of the above PDF; we start with the prior $p(\mathbf{w})$. If we have a priori knowledge that the weights vector is sparse, meaning that only few number of weights are non-zero, then a reasonable choice for the prior will be a sparsity prior such as the Laplace PDF. However, using such prior, a closed form solution for the posterior PDF cannot be obtained [114]. An alternative approach, introduced in [116], is to use hierarchical priors by defining $p(\mathbf{w})$ through a vector of hyperparameters $\mathbf{\alpha}$ as follows

$$
p(\mathbf{w}) = \int p(\mathbf{w} | \mathbf{\alpha})p(\mathbf{\alpha})d\mathbf{\alpha}
$$

(5.4)

where the conditional PDF is defined as

$$
p(\mathbf{w} | \mathbf{\alpha}) = \prod_{i=1}^{M} \mathcal{N}(\mathbf{w}_i | 0, \alpha_i^{-1})
$$

(5.5)

in which the hyperparameters are the reciprocals of the variances of the zero-mean normal distributions. The hyperparameters themselves are assumed to be distributed according to the following Gamma distribution

$$
p(\mathbf{\alpha}) = \prod_{i=1}^{M} \Gamma(\alpha_i | a, b)
$$

(5.6)
with $a$ and $b$ being the scale parameters of the Gamma distribution. The resulting prior in (5.4) is a student-$t$ distribution that, with appropriate choice of scale parameters, is highly peaked at zero, thus favoring sparsity [116] 2. Now, consider the likelihood $p(y | w)$. Assuming independent, zero-mean, Gaussian noise with variance $\sigma^2_n$, the likelihood can be written as

$$p(y | w) = \int p(y | w, \sigma^2_n)p(\sigma^2_n)d\sigma^2_n$$

where

$$p(y | w, \sigma^2_n) = (2\pi \sigma^2_n)^{-N/2} \exp\left(-\frac{1}{2\sigma^2_n} \|y - Bw\|^2\right)$$

and the reciprocal of the noise variance is distributed according to the following Gamma distribution with parameters $c$ and $d$

$$p(\sigma^2_n) = \Gamma(\sigma^{-2}_n | c, d)$$

Combining (5.4), (5.7) and (5.8), the posterior becomes

$$p(w | y) = \int \int p(y | w, \sigma^2_n)p(\sigma^2_n)p(w | \alpha)p(\alpha)p(y) d\alpha d\sigma^2_n$$

which can be simplified to

$$p(w | y) = \int \int p(w, \alpha, \sigma^2_n | y)d\alpha d\sigma^2_n$$

$p(w, \alpha, \sigma^2_n | y)$ is the joint posterior PDF of all unknowns, and can be factorized as follows

$$p(w, \alpha, \sigma^2_n | y) = p(y | w, \alpha, \sigma^2_n)p(\alpha, \sigma^2_n | y)$$

The first term in the r.h.s. of (5.12) can be expanded as

$$p(w | y, \alpha, \sigma^2_n) = \frac{p(y | w, \sigma^2_n)p(w | \alpha)}{p(y | \alpha, \sigma^2_n)}$$

2A reasonable choice, adopted in [116], is to set scale parameters $a$ and $b$ to zero. In this case, $p(\ln(\alpha))$ is uniform, i.e. the hyperparameters become scale invariant.
where
\[
p(y | \alpha, \sigma_n^2) = \int p(y | w, \sigma_n^2) p(w | \alpha) dw
\]
\[
= \exp \left[ -\frac{1}{2} (y^T C^{-1} y) \right] \frac{(2\pi)^{N/2}}{\sqrt{|C|}}
\] (5.14)

with \( C := \sigma_n^2 I + B \alpha^2 A^{-1} B^T \), and \( A := \text{diag}(\alpha) \). Using (5.8), (5.5) and (5.14) in (5.13),
\[
p(w | y, \alpha, \sigma_n^2) = (2\pi)^{-\frac{N+1}{2}} |\Sigma|^{-1/2} \exp \left( -\frac{1}{2} (w - \mu)^T \Sigma^{-1} (w - \mu) \right)
\] (5.15)

where \( \Sigma = (\sigma_n^{-2} B^T B + A)^{-1} \) and \( \mu = \sigma_n^{-2} \Sigma B^T y \). This is the sought posterior PDF of the weights once the hyperparameters \( \alpha \), that are embedded in \( A \), and the noise variance \( \sigma_n^2 \) are determined. Towards this end, the second term in the r.h.s. of (5.12) can be approximated by an impulse centered around the most probable (MP) values of \( \alpha \) and \( \sigma_n^2 \), as follows
\[
p(\alpha, \sigma_n^2 | y) \approx \delta(\alpha - \{\alpha\}_{MP}, \sigma_n^2 - \{\sigma_n^2\}_{MP})
\] (5.16)

Note that \( p(\alpha, \sigma_n^2 | y) \propto p(y | \alpha, \sigma_n^2) p(\alpha) p(\sigma_n^2) \), and by properly adjusting the scale parameters \( a, b, c \) and \( d \), \( \{\alpha\}_{MP} \) and \( \{\sigma_n^2\}_{MP} \) can be obtained by only maximizing the marginal likelihood \( p(y | \alpha, \sigma_n^2) \), i.e.
\[
(\{\alpha\}_{MP}, \{\sigma_n^2\}_{MP}) = \arg \max_{\alpha, \sigma_n^2} \left\{ \exp \left[ -\frac{1}{2} (y^T C^{-1} y) \right] \right\} \frac{(2\pi)^{N/2}}{\sqrt{|C|}}
\] (5.17)

This is known as type-II maximum-likelihood process, and can be efficiently solved using the fast relevance vector machine (RVM) presented in [117]. A detailed description of the fast RVM is presented in Appendix B. Finally, from (5.15), the MAP estimates and the corresponding covariance matrix are given by
\[
\hat{w} = \mu | \{\alpha\}_{MP}, \{\sigma_n^2\}_{MP} \}
\] (5.18)

and
\[
\text{cov}(w) = \Sigma | \{\alpha\}_{MP}, \{\sigma_n^2\}_{MP} \}
\] (5.19)
5.2 Ultrawideband Electromagnetic Inverse Scattering based on the First-Order Born Approximation

So far, we have discussed a Bayesian compressive sensing (BCS) solution for a generic linear regression model relating noisy measurements to sparse weights. To apply this model to the EM inverse scattering problem, we first have to be able to write the scattered field as a linear combination of the model weights. One way to do this, is by using the contrast source formulation presented in [111]. Another way, is by using the first order Born approximation as presented in [112]. In this work, we choose to use the latter approach for two reasons that will be clarified shortly.

Under the first order Born approximation, the scattered field at spatial location $\mathbf{r}$ and frequency $\omega_k$ resulting from incident field $E_{\text{inc}}^t$ generated by transmitter $t$, is given by

$$E_t^s(\mathbf{r}, \omega_k) = \int_D \tau(\mathbf{r}', \omega_k)E_{\text{inc}}^t(\mathbf{r}', \omega_k)G(\mathbf{r}, \mathbf{r}', \omega_k)\,d\mathbf{r}'$$  \hspace{1cm} (5.20)

where $G$ is the 2-D scalar Green’s function, and $D$ is the support of the scattering object. $\tau$ is the complex contrast function given by

$$\tau(\mathbf{r}, \omega_k) = [\varepsilon_r(\mathbf{r}) - \langle \varepsilon_r \rangle] - j \left[ \frac{\sigma(\mathbf{r}) - \langle \sigma \rangle}{\omega_k \epsilon_0} \right] = \Delta \varepsilon_r(\mathbf{r}) - j \frac{1}{\omega_k \epsilon_0} \Delta \sigma(\mathbf{r})$$  \hspace{1cm} (5.21)

with $\varepsilon_r$ and $\sigma$ being the relative permittivity and conductivity, respectively, and $\langle \varepsilon_r \rangle$ and $\langle \sigma \rangle$ the corresponding mean values of the background medium. Real and imaginary parts of the scattered field, recorded at $N_s$ sensors, can be stacked in a column vector as follows

$$\mathbf{e}_{t,k}^s = \begin{bmatrix} \text{Re}\{E_t^s(\mathbf{r}_1, \omega_k)\} \\ \vdots \\ \text{Re}\{E_t^s(\mathbf{r}_{N_s}, \omega_k)\} \\ \text{Im}\{E_t^s(\mathbf{r}_1, \omega_k)\} \\ \vdots \\ \text{Im}\{E_t^s(\mathbf{r}_{N_s}, \omega_k)\} \end{bmatrix}$$  \hspace{1cm} (5.22)
Discretizing the domain of investigation into \( N_p \) pixels, with pixel size \( D_p \), and assuming pulse-basis function expansion for the contrast [112], the projection matrix can be constructed as follows

\[
G_{t,k} = \begin{bmatrix}
\text{Re}\{g_1(r_1, \omega_k)\} & \cdots & \text{Re}\{g_{N_p}(r_1, \omega_k)\} & -\text{Im}\{g_1(r_1, \omega_k)\} \times \left(\frac{1}{\omega_k} \right) & \cdots & -\text{Im}\{g_{N_p}(r_1, \omega_k)\} \times \left(\frac{1}{\omega_k} \right)
\vdots & \ddots & \vdots & \cdot & \ddots & \cdot
\text{Re}\{g_1(r_N, \omega_k)\} & \text{Im}\{g_1(r_1, \omega_k)\} & \cdots & \text{Re}\{g_{N_p}(r_1, \omega_k)\} & \text{Im}\{g_{N_p}(r_1, \omega_k)\} \times \left(\frac{1}{\omega_k} \right)
\vdots & \cdots & \vdots & \cdot & \ddots & \cdot
\text{Im}\{g_1(r_N, \omega_k)\}
\end{bmatrix}
\]  

(5.23)

where

\[
g_{p,t}(r_n, \omega_k) = \int_{D_p} E_{t}^{inc}(r', \omega_k) G(r_n, r') dr'
\]  

(5.24)

Equation (5.20) can now be written in matrix form as follows

\[
e_{t,k}^s = G_{t,k} (F^{-1} t) + n_{t,k}
\]  

(5.25)

where \( F^{-1} t \) is the real contrast vector, given by

\[
F^{-1} t = \begin{bmatrix}
\Delta \epsilon_1(r_1) \\
\vdots \\
\Delta \epsilon_{N_p}(r_{N_p}) \\
\Delta \sigma_1(r_1) \\
\vdots \\
\Delta \sigma_{N_p}(r_{N_p})
\end{bmatrix}
\]  

(5.26)

\( F^{-1} \) is the inverse Fourier transform matrix, and \( t \) is the vector of the spatial harmonics of the real contrast function. \( G_{t,k}F^{-1} \) can now be perceived as the projection matrix \( B \) in (5.1) and the unknown weights are the spatial harmonics \( t \). Once the covariance matrix of \( t \) is solved for by the RVM, the covariance matrix of the real contrast vector can be computed as \( \text{cov}(F^{-1} t) = F^{-1} \text{cov}(t)(F^{-1})^T \).

Formulating the problem in terms of the spatial harmonics, rather than the contrast function itself, has the following advantages: 1. Spatial harmonics conform
better with the sparsity requirement of the model, since the solution is likely to possess an amount of spatial correlation that would make the contrast function more sparse in the spatial harmonics domain than in the spatial domain. 2. Spatial harmonics provide better regularization for the solution; for example, one can choose to solve for a subset of the spatial harmonics according to the problem specifics and/or the available resources.

Measurements corresponding to illuminations from different transmitters can be stacked in one inversion and, assuming negligible dispersion over the utilized frequency band, multifrequency measurements can be stacked in the same inversion as well (since $t$ is frequency independent under this assumption). This yields what we call multistatic UWB Bayesian inversion. In UWB inversion, low frequencies are more sensitive to lower spatial harmonics, whereas high frequencies are more sensitive to higher spatial harmonics. The highest spatial harmonic that can be resolved depends on the maximum frequency that can be used without violating the Born approximation. Increasing the number of uncorrelated measurements, whether from sufficiently spaced sensors and/or frequency samples, makes the inversion statistically more stable against random noise and/or clutter, as discussed in Chapter 2 and 3.

Being able to define transmitter- and frequency-independent unknowns is a consequence of the adopted Born approximation. In the contrast source formulation in [111], the unknowns are the equivalent currents of the contrast function, which vary with frequency and incident field. For high contrast media, iterative inversion approaches can be used, as discussed in Section 5.5 ahead.
5.3 Simulation Results

The UWB BCS inversion process is summarized in Fig. 5.1. The pointwise distribution of the medium can be characterized by some parameters such as the average electrical properties, the correlation length and the contrast level of the fluctuations. Another parameter dictated by the problem is the signal-to-noise ratio (SNR) of the measurements. A priori knowledge of any of the medium parameters can be used to select the user-controlled parameters. Those include the domain of interest (DOI) pixelization, the utilized frequency band, and the number and location of sensors. The UWB BCS inversion is then invoked, and the output contrast level and confidence level are used to decide whether more iterations are needed. In that case, posterior information can be fed back to refine the user-controlled parameters of the next iteration.

An example of the permittivity contrast of a continuous random medium is shown in Fig. 5.2(a). This distribution is a realization of a Gaussian random process with
zero-mean, standard deviation of 0.064, and a Gaussian correlation function with correlation length $l_c = 1.25$ m. The spatial spectrum is shown in Fig. 5.2(b). For simplicity, we consider 2-D models throughout this work, but evidently the same analysis can be easily extended to 3-D cases. The background medium is assumed to have mean permittivity of 3 and mean conductivity of 0.15 mS/m. This example may correspond to underground imaging of dry soil [3]. The interrogating frequency band ranges from 5-250 MHz with 50 samples. Note that the maximum wavenumber of the interrogating signal $k_{max} = 1.48 \times 2\pi$ rad/m is larger than the maximum spatial harmonic of the medium ($= 1.1 \times 2\pi$ rad/m computed across the diagonal).

Forward problem simulations are carried out using the finite-difference time-domain method [84].

In the inverse problem, the DOI is discretized uniformly into 20×20 pixels, and the 2-D Green’s function is computed analytically assuming known average medium
properties. We use $N_s = 15$ multistatic sensors deployed either in full-aspect (FA) circular geometry as shown in Fig. 5.3(a), crosshole (CH) geometry as in Fig 5.3(d), or borehole (BH) geometry as in Fig. 5.3(g). Transmitters are point sources in 2-D (infinite line source) radiating TM$_z$ polarization. For the particular application of underground imaging, the $x - y$ plane in the FA case can be perceived as the horizontal plane, with the shown distribution being a horizontal cross-section in the formation, and the sensors are deployed in circularly distributed wells. For the BH and CH cases, the shown distribution is a vertical cross-section, and the sensors are deployed in one or two wells, respectively. The SNR is assumed to be 10 dB for all measurements performed using different sensors and frequencies. Reconstructed profiles for the three geometries are shown in Fig. 5.3(b), (e) and (h), and the estimated standard deviations (which determine the confidence level of the inversions) are shown in Fig. 5.3(c), (f) and (i), respectively. Reconstructed images are interpolated to a finer grid for the sake of visualization. Comparing actual and reconstructed profiles, we note that FA and CH outperform BH. Also the estimated standard deviation provides a reasonably good measure for the inversion accuracy, this more obvious in the BH case, where the reduced-accuracy inversion in the right half of the investigation domain (farther from the array) is associated with higher standard deviation. Roughly speaking, the inversion accuracy and the reciprocal of standard deviation at a certain point are proportional to the spatial resolution offered by the sensors array at that point.
Figure 5.3: Forward problem permittivity distribution, reconstructed profiles and estimated standard deviation for different sensor array geometries. Sensors are indicated with ‘x’s. \( N_s = 15 \) and SNR=10 dB. (a)-(c) Full-aspect. (d)-(f) Crosshole. (g)-(i) Borehole.

### 5.3.1 Performance analysis

In this section, we provide some qualitative measures for assessing inversion accuracy and efficiency. We first define the actual r.m.s. error as

\[
\text{Actual r.m.s. error} = \sqrt{\text{avg}_D [\epsilon_r(r) - \epsilon_r^0]^2}
\]  

(5.27)
where $\hat{\varepsilon}_r$ is the estimated permittivity. The (average) estimated standard deviation can be defined as

$$\text{Estimated std. deviation} = \sqrt{\text{avg} \left[ \text{diag}(\text{cov}(\mathbf{F}^{-1}\mathbf{t})) \right]} \quad (5.28)$$

Both actual and estimated errors are plotted in Fig. 5.4(a) for the three previously discussed geometries and two SNRs. Percentile error is defined as the ratio of the absolute error to the r.m.s. of the actual contrast function ($\sqrt{\text{avg}_D |\varepsilon_r(\mathbf{r})|^2}$). This plot shows that estimated error follows pretty well the actual error, with the latter being always larger. This makes perfect sense, since the estimated error only accounts for errors due to additive noise, whereas actual error encloses, in addition to noise, errors due to the adopted Born approximation and discretization error. Estimated SNRs are shown in Fig. 5.4(b). They are below their actual values by 1-2 dB, which indicates that the noise variance was over-estimated by the RVM solver.

Errors and processing times for a FA array with uniformly distributed increasing number of sensors are tabulated in Table 5.1. Errors decrease monotonically with increasing the number of sensors at the expense of increasing the processing time, as expected. Listed times are those required for solving the fast RVM, using non-optimized Matlab code, running on a machine with average CPU speed of 2.7 G.cycle/s. They are very short times (almost real-times) w.r.t. the size and the number of measurements of the considered problem. Note that there are costs associated with computing the Green’s function and constructing the projection matrix, but those are considered as pre-processing costs.

Another measure for quantifying the confidence level of the inversion is the differential entropy (DE) [130]. Referring to (5.1), the DE of the posterior multivariate
Figure 5.4: (a) Actual and estimated errors for different array geometries and SNRs. Percentile error is the ratio of the absolute error to the r.m.s. of the actual contrast function. (b) Estimated SNR.

Table 5.1: Actual error, estimated standard deviation, and processing time using a FA array with increasing number of sensors and SNR=5 dB.

<table>
<thead>
<tr>
<th>Number of sensors</th>
<th>Actual r.m.s. error</th>
<th>Estimated std. dev.</th>
<th>Processing time (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.0267</td>
<td>0.0196</td>
<td>5.4</td>
</tr>
<tr>
<td>15</td>
<td>0.02</td>
<td>0.01176</td>
<td>10.6</td>
</tr>
<tr>
<td>30</td>
<td>0.0114</td>
<td>0.005</td>
<td>36</td>
</tr>
</tbody>
</table>

Gaussian PDF is given by

\[ h(p) = - \int p(w | y) \ln (p(w | y)) \, dw \]

\[ = \frac{1}{2} \ln \left( (2\pi e)^M |\Sigma| \right) \]  

The DE given by the above equation is in information units (nats). It can be divided by \( \ln(2) \) to give the DE in bits. DE measures randomness -random variables with PDF concentrated on a small interval yields smaller DE. For continuous random
Table 5.2: Differential entropy (in kb) for the setups in Fig. 5.3.

<table>
<thead>
<tr>
<th>Array geometry</th>
<th>SNR=10 dB</th>
<th>SNR=5 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>FA</td>
<td>-3.34</td>
<td>-3.09</td>
</tr>
<tr>
<td>CH</td>
<td>-3.33</td>
<td>-3.03</td>
</tr>
<tr>
<td>BH</td>
<td>-3.32</td>
<td>-2.8</td>
</tr>
</tbody>
</table>

variables, DE can be negative (as opposed to the entropy of discrete random variables which is always positive). Differential entropies for the setups of Fig. 5.4(a) are summarized in Table 5.2. Individual values of DE do not give much information about the randomness of the PDF; however, comparing DEs of two setups gives an idea about the accuracy gained or lost (measured in units of information) on going from one setup to the other. This measure agrees well with the behavior described in Fig. 5.4(a).

5.3.2 Adaptive sensing

Our goal in this section is to develop a systematic procedure for optimizing the location(s) of subsequent measurement(s), such that the information gain from each measurement is maximized [113, 114]. The DE, after adding the \((N+1)\)th measurement, can be written in terms of the DE of \(N\) measurements as follows [114]

\[
h(p_{\text{new}}) = h(p) - \frac{1}{2} \ln \left( 1 + \sigma_n^{-2} r_{B,N+1}^T \Sigma r_{B,N+1} \right) \tag{5.30}
\]

where \(r_{B,N+1}^T\) is the new row added to the projection matrix \(B\) associated with the \((N+1)\)th measurement. To maximize information gain, the absolute value of the second term in the r.h.s. of (5.30) should be maximized, which implies that \(r_{B,N+1}^T\)
should be chosen such that

\[ r_{B,N+1}^T \Sigma r_{B,N+1} = \text{var} (y_{N+1}) \]  \hspace{1cm} (5.31)

is maximized. In other words, we choose to place the next sensor where we expect highest uncertainty in the measurement, in this way, the information gain is maximized [114]. The above equation is maximized by choosing \( r_{B,N+1} \) to be the eigenvector of \( \Sigma \) corresponding to the largest eigenvalue [114].

Referring to our case study, we apply the adaptive scheme to place new sensors in a ‘myopic’ sense (one sensor in each step) as shown in Fig. 5.5. Suppose that the locations of five sensors in step 1 are pre-determined, the goal is to optimally place five more sensors. Also, suppose that sensors can only be deployed on a circle with 7.5 m radius. The figure shows the reconstructed profile from each step, the location of the utilized sensors, the estimated standard deviation, and the optimized projection vector. The new sensor has to be placed such that the field pattern produced from it best matches the optimized projection vector. Note that the standard deviation distribution is not enough to determine the location of the new sensor without computing its eigenvalue decomposition. Actual and estimated errors as well as the DE of each step are summarized in Table 5.3. For comparison, a non-adaptive scenario is shown in Fig. 5.6, where the same five sensors are pre-determined and the other five sensors are uniformly distributed as shown. Corresponding performance parameters are shown in Table 5.3, as well. From this comparison, it is obvious how adaptive optimized sensing yields more accurate inversion given the same number of sensors, or in other words, adaptive sensing can achieve a given inversion accuracy with less number of sensors.
Figure 5.5: Reconstructed profiles, estimated standard deviations, and adaptive projection vectors for six steps of an adaptive sensing scenario. Sensors used in each step are indicated with ‘x’ s. Sensors locations of the first step are pre-determined, and a new sensor is added adaptively in each step. SNR=10 dB.
Figure 5.6: Non-adaptive sensing scenario. $N_s=10$ and SNR=10 dB.

Table 5.3: Summary of the performance parameters of adaptive and non-adaptive sensing scenarios.

<table>
<thead>
<tr>
<th></th>
<th>Step</th>
<th>Actual Error</th>
<th>Estimated Error</th>
<th>Differential Entropy (kb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adaptive Sensing</td>
<td>1</td>
<td>0.117</td>
<td>0.108</td>
<td>-2.32</td>
</tr>
<tr>
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<td>-3.12</td>
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<td>Non-Adaptive Sensing</td>
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</tbody>
</table>

5.4 Time-Reversal-Assisted Localized-Inversion

If our interest is to reconstruct only a localized region of the investigation domain that can change dynamically, time-reversal (TR) focusing [29, 48, 73, 75] can be used
Figure 5.7: Time-reversal-assisted localized-inversion. (a)-(b) $N_s=30$ without TR. (c)-(d) $N_s=30$ with TR focused on one hundred pixels within the upper right quarter of the investigation domain. (e)-(f) $N_s=10$ without TR. SNR=5 dB.
to achieve accurate localized inversion with significantly shorter processing time. We call this technique 'Time-Reversal-Assisted Localized-Inversion' (TRALI). In TRALI, measurements from different sensors are linearly combined as follows

\[ E_{TR}(r_p, \omega_k) = \sum_{r=1}^{N_s} \sum_{t=1}^{N_s} G^*(r_p, r_r, \omega_k) G^*(r_p, r_t, \omega_k) E^*_t(r_r, \omega_k) \]  

(5.32)

where \( G^*(r_p, r, \omega_k) \) is the complex conjugated Green's function between pixel \( p \), in the region of interest, and location \( r \). Note that complex conjugation in the frequency domain is equivalent to TR. Assuming multistatic acquisition, the above equation is equivalent to simultaneously firing all transmitters to illuminate the DOI by a beam focused at location \( r_p \), backscattering is then recorded by all receivers, time-reversed and projected on pixel \( p \). In this way, \( E_{TR}(r_p, \omega_k) \) will be most sensitive to the contrast of pixel \( p \), and consequently, using \( E_{TR}(r_p, \omega_k) \) in place of \( E^*_t(r, \omega_k) \) in the linear regression model (5.25), yields accurate localized inversion. Of course, the rows of the projection matrix need to undergo the same linear combination in (5.32). An example is shown in Fig. 5.7. A thirty-transceivers FA array is used to obtain very accurate inversion of the entire DOI as shown in Fig. 5.7(a) and (b). Using the same array, TRALI is applied to obtain localized inversion of one hundred pixels in the upper right quarter of the DOI, as shown in Fig. 5.7(c) and (d). Note that the local inversion sub-domain does not need to be static or contiguous, also it can be extended to encompass the entire DOI. To further assess the performance of TRALI, a ten-transceivers FA array (which has the same data points and requires the same processing time as TRALI) is used in Fig. 5.7(e) and (f). Corresponding total error, local error (of the upper right quarter), and processing time are summarized in Table 5.4. TRALI is shown to produce local inversion with almost the same accuracy as the full multistatic acquisition, but with much less processing time. This comes at
Table 5.4: Summary of the performance parameters for the setups in Fig. 5.7.

<table>
<thead>
<tr>
<th>Setup</th>
<th>Total r.m.s. error</th>
<th>Local r.m.s. error</th>
<th>Processing time (sec.)</th>
</tr>
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<td>0.01122</td>
<td>36</td>
</tr>
<tr>
<td>$N_s=30$ w/ TR</td>
<td>0.0306</td>
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<td>$N_s=10$ w/o TR</td>
<td>0.0267</td>
<td>0.0315</td>
<td>5.4</td>
</tr>
</tbody>
</table>

the expense of sacrificing the accuracy elsewhere outside the local domain of interest. Using the same number of multistatic acquisitions as the TR focusing pixels results in a larger local error, but less overall error.

5.5 Bayesian Distorted-Born Iterative Method

So far, we considered the application of the proposed UWB BCS inversion to low contrast media obeying the first order Born approximation. In this section, we extend the applicability of the method to high contrast continuous media. The proposed Bayesian inversion scheme can be applied iteratively, yielding what we call ‘Bayesian Distorted-Born Iterative Method’ (BDBIM). In conventional DBIM [122, 123, 124, 125, 131, 132], a cost function is defined, usually as the $L_2$ norm between measured scattered field and synthetic scattered field computed from the reconstructed profile, and the method proceeds iteratively to minimize that cost function. Reconstructed profile from each iteration is used to compute the synthetic scattered field as well as the Green’s function used in the next iteration. The method converges when the cost function gets below a certain pre-determined threshold. BDBIM proceeds the same way; however, instead of explicitly defining a cost function on the scattered fields, the estimated standard deviation from the Bayesian solver can be
Figure 5.8: Bayesian DBIM. (a) Forward permittivity distribution. (b)-(f) Reconstructed profiles from five iterations. $N_s=30$ and SNR=10 dB.

used as stopping criterion. To illustrate that, consider the example in Fig. 5.8. In the first iteration, a uniform homogeneous background is used in BCS inversion. The reconstructed profile is shown in Fig. 5.8(b). Actual error and estimated standard deviations are shown in Fig. 5.9(a) and (b), respectively. The percentile error shown in Fig. 5.9(b) is the ratio of the estimated standard deviation to the r.m.s. of the contrast function contributed from each iteration. The reconstructed profile from the first iteration is plugged into a forward problem numerical solver, and used to compute the synthetic scattered field and the Green’s function to be used in the following iteration. The synthetic scattered field is subtracted from the (noisy) measurements, and that differential signal is used as the measurements vector in the second iteration. The reconstructed profile from the second iteration (refereed to as iteration 2
<table>
<thead>
<tr>
<th>Iteration</th>
<th>Actual error</th>
<th>Cumulative estimated std. dev.</th>
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</thead>
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<tr>
<td>5</td>
<td>0.005</td>
<td>0.05</td>
</tr>
</tbody>
</table>

(a) Actual total error and cumulative standard deviation after each iteration.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Estimated std. dev. of each iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.03</td>
</tr>
<tr>
<td>2</td>
<td>0.02</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
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<tr>
<td>4</td>
<td>0.005</td>
</tr>
<tr>
<td>5</td>
<td>0.001</td>
</tr>
</tbody>
</table>

(b) Estimated standard deviation of individual contributions from each iteration. Percentile error is the ratio of the estimated standard deviation to the r.m.s. of the differential contrast function contributed from each iteration.

Figure 5.9: Error analysis of Bayesian DBIM. (a) Actual total error and cumulative standard deviation after each iteration. (b) Estimated standard deviation of individual contributions from each iteration. Percentile error is the ratio of the estimated standard deviation to the r.m.s. of the differential contrast function contributed from each iteration.

Contribution) is added to the reconstructed profile from the first iteration to yield the overall profile of iteration 2 shown in Fig. 5.8(c). The process is then repeated. Assuming that reconstructed contributions from different iterations are independent random variables, covariance matrices form all iterations can be added up, yielding the cumulative estimated standard deviation plotted in Fig. 5.9(a). There are several interesting points to note here. In the early iterations, the cumulative estimated error is not an accurate measure for the actual error; this is because of the deficiency of the underlying Born approximation to precisely model the scattered field as these stages. With increasing iterations, the discrepancy between actual and estimated errors gets smaller. As the method proceeds, reconstructed contribution gets smaller and smaller, and so does the associated estimated standard deviation. However, the standard deviation decreases at a slower rate, because the SNR of each inversion
also decreases, this explains the increase in the percentile error shown in Fig. 5.9(b) with iterations. The percentile error is inversely proportional to the confidence level, therefore, a maximum threshold can be set on the former to determine when to stop. Intuitively, the higher SNR we have, the further we can go on with iterations, and the more accurate the inversion will be for a given (desired) confidence level.

5.6 More Applications

5.6.1 Layered media

Two examples of layered media are shown in Fig. 5.10. In Fig 5.10(a), CH sensors are used to reconstruct a slanted layered medium with abrupt changes in permittivity. The problem is solved as a 2-D problem and the reconstructed profile is shown in Fig. 5.10(b). Fig. 5.10(c) shows a quasi-horizontally layered medium. It is a realization of an anisotropic continuous random Gaussian medium with $l_{cx} = 125$ m along $x$-direction and $l_{cy} = 1.25$ m along $y$-direction. This is a good model for layered Earth formations encountered in geophysical exploration [41, 42, 43]. Prior knowledge of the layered nature of problem can simplify the inversion significantly by solving the problem as 1-D inversion problem (i.e. restricting the unknowns to spatial harmonics along $y$-direction), as shown in Fig. 5.10(d) for a BH scenario. The linear array shown in Fig. 5.10(c) can be deployed horizontally along $x$-direction as a surface controlled source electromagnetic (CSEM) array. In that case, the problem becomes 1-D along a direction normal to the array. Shown results are for a single iteration inversion. For high contrast media, BDBIM can be invoked.
Figure 5.10: Forward problem and reconstructed permittivity profiles for layered media. (a)-(b) Slanted layers imaged by $N_s=30$ CH sensors. (c)-(d) (Quasi)-horizontally layered medium imaged by $N_s=15$ BH sensors. This setup is efficiently solved as 1-D inverse problem. SNR=10 dB.

5.6.2 Breast tissue imaging and early cancer detection

Microwave imaging of breast tissue, with the aim of detecting and locating malignant tumors, has been an active topic of research recently [7, 8, 133, 53, 134, 50, 51, 11, 12]. The difference in electrical properties of tumors from those of healthy breast tissues enables tumor detection through electromagnetic inversion techniques.
Figure 5.11: Breast modeling. (a) MRI image. (b) Permittivity distribution. (c) Conductivity distribution. A conformal array of fifteen sensors is indicated with ‘x’s. (d) Pixelated permittivity distribution.
Microwave breast imaging provides a cheaper and safer alternative to existing imaging techniques such as X-ray mammography [7, 8]. The main challenge to accurate tumor localization lies in the absence of precise knowledge of the pointwise properties of the background tissues, and consequently, the inaccurate computation of the Green’s function used in imaging. In [53] and [133], a detection algorithm based on time-reversal adaptive interference canceling (TRAIC) was presented. However, that algorithm requires the acquisition and storage of a baseline measurement of the healthy breast, which is, of course, not attainable in many cases.

We propose applying the BCS inversion to reconstruct an image of the electrical properties of the breast from which tumors can be easily detected. Our approach requires only knowledge of the average permittivity and conductivity of breast tissue and approximate knowledge of skin and chest wall properties. To construct a 2-D model of the breast permittivity, we follow the same procedure outlined in [50, 133, 53, 134]. We first take a magnetic resonance image (MRI) of the breast as shown in Fig. 5.11(a), then take it logarithm, adjust its mean to have $\epsilon_r=9$, and scale it to
have variability of 16%. Finally, the outer skin and the chest wall are added with permittivities of 30 and 50, respectively, as shown in Fig. 5.11(b). We follow the same procedure for the conductivity, except that the mean conductivity is set to 0.4 S/m, and the skin and the chest wall have the same conductivity of 0.4, as shown in Fig. 5.11(c). The tumor is inserted as a circle with radius of 5 mm, $\varepsilon_r=50$ and $\sigma=1$ S/m. More accurate breast and tumor models can be found in [135, 136].

Fifteen sensors are deployed conformal to the skin as shown in Fig. 5.11(b). The interrogating signal has frequency range of 1-5 GHz with 33 steps. Scattered field form the breast tissue is computed by subtracting a synthetic scattered field, computed numerically from a model that takes into account the skin and the chest wall and assumes uniform tissue properties ($\varepsilon_r=9$ and $\sigma=0.4$ S/m), from the (noisy) measurements. For the inverse problem, the DOI, which is the breast tissue confined between the skin and the chest wall, is discretized using a conformal non-uniform polar grid as shown in Fig. 5.11(d). The shown grid has 30 pixels along the $\rho$-direction and 30 pixels along the $\phi$-direction. Such conformal grid confines the unknown weights to the DOI, and hence avoids encountering abrupt changes in the weights that would have been experienced had a rectangular grid been used. The discretized permittivity profile versus the pixel index along $\rho$- and $\phi$-directions is shown in Fig. 5.11(e). Note that this image is a distorted version of the actual breast profile; yet, it still exhibits sufficient degree of smoothness that makes the spatial spectrum sparse as shown in Fig. 5.11(f). Now, we can invoke the UWB BCS to solve for the spatial harmonics. The projection matrix is computed numerically using a model that accounts for the skin and the chest wall, and assumes uniform tissue properties. Reconstructed permittivities of a healthy breast, same breast with one
and two tumors are shown in Fig. 5.12. The results show the ability of the method to locate tumors accurately, even by using only one iteration, and in the absence of pointwise knowledge of tissue properties.

5.6.3 Rough surface reconstruction under the Small Perturbation Approximation

In this section, we further extend the applicability of the proposed BCS inversion to rough surface reconstruction [137, 138, 139] under the small perturbation approximation (SPA). SPA is a linear formulation that relates scattered fields, from a PEC half space with rough surface, to the rough surface profile [140]. The SPA assumes that the rough surface height deviations are much smaller than the interrogating wavelength, and the height slopes are small [140]. Many problems of interest can be modeled this way, such as wet soils and sea surfaces. More specifically, referring to Fig. 5.13, a 1-D rough surface is illuminated with a plane wave of the form

\[
E^{inc}_z(r) = 1.e^{-jk_xx}e^{+jk_yy} \tag{5.33}
\]

where \( k^2_x + k^2_y = k^2 \). The scattered field can be written as the a linear summation of plane waves as follows

\[
E^s_z(r) = \sum_{m=-\infty}^{\infty} e^{-j2\pi x/L}e^{-jk_xx}e^{-jbeta_m y}B_m \tag{5.34}
\]

where \( L \) is the length of the rough surface, \( B_m \) is the weight of the \( m^{th} \) scattered plane wave, and

\[
b_m = \sqrt{k^2 - (k_x + 2\pi m/L)^2} \tag{5.35}
\]
Matching the boundary condition at the interface, the first order approximation for $B_m$ can be written as

$$B_m^{(1)} = -\frac{2jky}{L} \int_0^L e^{j2\pi mx/L} \zeta(x) dx = -2jky P(m) \quad (5.36)$$

where $\zeta(x)$ is the rough surface profile, and $P(m)$ is the $m^{th}$ spatial harmonic of the rough surface. From (5.34) and (5.36), we see that, under the first order SPA, scattered field is linked to the spatial harmonics of the rough surface through a linear relationship, which conforms perfectly well with the linear regression model defined in (5.1). Consequently, the previously detailed BCS inversion technique can be used to solve for $P(m)$, from which the sought rough surface profile can be computed as

$$\zeta(x) = \sum_{m=-\infty}^{\infty} e^{-j2\pi mx/L} P(m) \quad (5.37)$$

In practice, we may have point sources rather than incident plane waves as shown in Fig. 5.13(a). Luckily enough, point sources can be written as an infinite summation of plane waves using the spectral representation of sources formulation [141]. For point sources in 2-D, we have

$$E_z^{inc}(\mathbf{r}, \mathbf{r}_s) = (-j\omega\mu) \times \frac{-j}{4\pi} \int_{-\infty}^{\infty} \frac{e^{-jk_y(x-x_s)}e^{-jk_y|y-y_s|}}{k_y} dk_x$$

\[ (5.38) \]
Figure 5.13: Rough surface reconstruction. (a) Forward problem setup: The rough surface is a realization of a Gaussian random process with Gaussian correlation function, with height standard deviation=12.5 cm and $l_c=0.75$ m. Sensors are indicated by ‘x’'s. (b) Reconstructed profile and error bars using SAR configuration. (c) Reconstructed profile and error bars using MDM configuration. $N_s=15$ and SNR=10 dB.

Reconstructed profiles, using a synthetic aperture array (SAR) and a multistatic data matrix (MDM), with $N_s=15$ and frequency range of 5-250 MHz, are shown in Fig. 5.13(b) and 5.13(c), respectively. The error bars ($\pm$ one standard deviation)
provided by the Bayesian solver are also shown. Actual and estimated r.m.s. errors are computed in Table 5.5. MDM shows satisfactory reconstruction that captures all important features of the rough surface.

5.7 Conclusion

An information-theoretic approach based on Bayesian compressive sensing was developed for ultrawideband multistatic inverse scattering in continuous random media. Simulation results showed that not only does UWB BCS provide accurate reconstruction in different problem scenarios, but it also provides means for assessing the accuracy of the inversion. In addition, it allows for a systematic way of determining the optimal location of the next measurement so that the information gain is maximized. Time-reversal focusing was combined with UWB BCS for efficient localized inversion. The proposed technique was successfully applied to a number of problems including medical imaging, underground geophysical imaging, and conductive rough surfaces reconstruction.
Chapter 6: Time-Reversal Techniques for MISO and MIMO Wireless Communication Systems

In this chapter, we study the application of novel TR techniques to wireless communication systems. We start with a brief review on the basics of TR communication and its performance metrics. Then, we consider multiple-input single-output (MISO) configurations, where there exists one intended receiver and possibly one or more eavesdroppers. We introduce and study the applicability of three TR techniques, namely, (i) equalized TR beamforming, (ii) TR beamforming with multiple-signal-classification (MUSIC), and (iii) differential TR, in both free-space and rich scattering environments. Conventional TR beamforming provides spatial focusing at the intended receiver; however, it does not yield perfect channel equalization. Equalized TR beamforming yields perfect channel equalization at the expense of spatial focusing. TR beamforming combined with MUSIC produces null fields at eavesdroppers locations for increased physical layer covertness. Differential TR is used to extract the pilot signal of passive moving receivers (scatterers) from array acquisitions [73]. These passive receivers may represent, for example, receivers in one-way communication links, sensors such as passive RFIDs or intercepting eavesdroppers. We highlight relative strengths and limitations of these different techniques, and compare their bit
error rate performances under high and low data rates operations. We also compare the performance of equalized TR beamforming in rich scattering scenarios with that of conventional (non-equalized) TR and conventional (non TR-based) beamforming. After that, we consider multiple-input multiple-output (MIMO) configurations, where TR beamforming with MUSIC is shown to significantly reduce undesired interuser interference. Finally, a recent linear precoding technique, known as interference alignment (IA), is briefly discussed, and contrasted with TR techniques for wireless communications.

6.1 Time-Reversal Communications and Performance Metrics

Consider a MISO communication link between an $N$ elements transmitter array and a receiver (let us call it receiver $A$). In TR-based communication systems, to start the link, the receiver transmits a signal that is recorded by the transmitter array elements. These recorded $N$ signals constitute the steering vector (column vector of Green’s functions or equivalently impulse responses) between receiver $A$ and each element in the array. The steering vector (or processed versions of it, as discussed later) is used as a pilot for information signal transmission from the array to receiver $A$. From the time-reversal invariance of the wave equation, when the $N$ signals of the pilot are time-reversed and simultaneously transmitted, they tend to automatically focus at the intended receiver location (regardless of the intervening medium which is, in general, not known to the transmitter array) and produce a compressed pulse in time as well [55, 54]. After time-reversed backpropagation, the received signal, in
the frequency domain, at location \( i \) is given by the following inner product

\[
H_i(\omega) = \langle g_i(\omega), p_A(\omega) \rangle
\]  

(6.1)

where \( p_A(\omega) \) is the pilot vector of receiver \( A \), \( g_i(\omega) = [G(i,1)(\omega), ..., G(i,N)(\omega)]^T \) is the steering vector of location \( i \), where \( G(i,n)(\omega) \) is the Green’s function between location \( i \) and the \( n^{th} \) element of the array, and \( \langle a, b \rangle = b^\dagger a \) denotes the inner product between \( a \) and \( b \) where \( ^\dagger \) represents a conjugate transpose.

The received signal in the time domain is obtained by taking the inverse Fourier transformation \( h_i(t) = \mathcal{F}^{-1}\{H_i(\omega)\} \). We refer to \( h_i(t) \) as the equivalent channel impulse response after TR, or simply the channel impulse response (CIR). The CIR peak at location \( i \) is defined as \( \eta_i = \max_t |h_i(t)| \). This will be used as a measure for the spatial distribution of energy [142].

The intersymbol interference (ISI) is defined as the ratio of the sum of values of the CIR, offset by integer multiples of \( T \) from the peak, to the peak value of CIR [143], as follows

\[
\text{ISI}_i = \frac{\sum_{n=-N_1}^{N_2} |h_i(\tau_i + nT)|}{|h_i(\tau_i)|}
\]  

(6.2)

where \( T \) is the symbol duration, \( \tau_i \) is the time delay of the CIR peak at location \( i \), \( N_1 = [\tau_i/T] \), and \( N_2 = [(\tau - \tau_i)/T] \), where \( \tau \) is the duration of the CIR.

An information signal \( s(t) \) is convolved with the time-reversed pilots. The transmitted signal vector intended for receiver \( A \) is given by \( t_A(t) = s(t) \ast_t p_A(-t) \), where \( \ast_t \) denotes convolution. The received signal can be written as \( r_A(t) = s(t) \ast_t h_A(t) + n(t) \), where \( n(t) \) is the additive noise at the receiver.
6.2 TR Techniques for Wireless Communications

In this section, the exact steering vector of the receiver, also known as the channel state information (CSI) between the array and the receiver, is assumed to be perfectly known to the transmitters. This implies that the time period that the transmitter array takes to record the receiver’s steering vector is long enough to capture (mostly) all the multiple scattering in the medium. In addition, the sampling rate is equal to or higher than the Nyquist sampling rate corresponding to the bandwidth of operation. In other words, we are assuming that TR does not add intrinsic bandwidth limitation, i.e. if the original communication system (before using TR) is capable of generating signals with certain bandwidth $B$, we assume that it will also be capable of generating and sampling the TR pilot over the same bandwidth. Deviations from ideality due to some hardware limitations are discussed in Section 6.3.4.

6.2.1 TR Beamforming

Conventional TR (without equalization)

The simplest form of the pilot is to coincide with the steering vector of the intended receiver [56], that is

$$\mathbf{p}_A(\omega) = \mathbf{g}_A(\omega)$$  \hspace{1cm} (6.3)

This choice exhibits high spatial focusing performance that is further enhanced in rich scattering scenarios, as will be shown in the next section. However, the equivalent CIR at the intended receiver is proportional to $\|\mathbf{g}_A(\omega)\|^2$, where $\|\mathbf{g}_A(\omega)\|$ is the norm of $\mathbf{g}_A$ given by $\|\mathbf{g}_A\| = \sqrt{\langle \mathbf{g}_A, \mathbf{g}_A \rangle}$. This means that the CIR is not flat with frequency, i.e. the equivalent channel transfer function is not perfectly equalized. This gives rise
to undesired intersymbol interference which hampers performance in case of high data rates.

**Equalized TR**

In order to achieve perfect equalization at the receiver, the steering vector can be normalized by the square of its norm as follows

\[ p_A(\omega) = \frac{g_A(\omega)}{\|g_A(\omega)\|^2} \]  

This guarantees a flat CIR at the receiver, as can be easily deduced from (6.1). This choice, however, implies inferior spatial focusing performance as compared with conventional TR, as shown later. We note that temporal side lobes suppression by iterative TR [65] and spatial linear inversion based on TR [144] are interesting techniques which do not require any inversion, and are probably less sensitive to noise. However, these techniques require precise weighting of the gain factors in each iteration to guarantee convergence. Also, they require extra processing at the user to subtract, time-reverse, and transmit the differential signal in each iteration, rather than sending the user’s pilot only once at the beginning.

It is interesting to contrast TR techniques with the waterfilling power allocation scheme in which the transmitter allocates more power to stronger sub-carriers, and less or even no power to weaker ones [145]. In that case, the goal is to maximize the signal to (external) noise ratio at the receiver, regardless of the equalization of the received signal or ISI. This is analogous to what conventional TR (or a matched filter) does. On the other hand, equalized TR basically does the reverse: the gain assigned to each frequency is inversely proportional to the channel response at that frequency,
as described in (6.4). The goal here is to have equalized (flat) channel response at the receiver after backpropagation.

6.2.2 Conventional Beamforming

In conventional (non TR-based) beamforming, the information available to the array about the receiver can be its direction (which can be estimated using some direction of arrival (DOA) algorithms [146]), or, in the best-case scenario, its position with respect to the array. So if we assume that the array knows the receiver location, it can generate an approximate pilot, which is the steering vector of the receiver location based on free-space assumption. Note that although the array may know the receiver’s location, it does not know its exact CSI. Following the above discussion, the pilot vector of conventional beamforming can be written as

$$p_{A}(\omega) = \frac{\tilde{g}_{A}(\omega)}{\|\tilde{g}_{A}(\omega)\|^{2}}$$

(6.5)

where $\tilde{g}_{A}$ is the steering vector of receiver $A$ based on some background medium assumption that may not correspond to the actual one.

6.2.3 TR Beamforming with Nulling at Eavesdroppers

If one of the concerns of the communication system is to minimize information leakage to eavesdroppers (or interuser interference in case of MIMO configurations), TR beamforming can be combined with MUSIC algorithm to produce null fields at eavesdroppers. Assume the presence of $M$ eavesdroppers $M < N$, whose steering vectors are known to the array. The pilot of receiver $A$ can be orthogonalized to each and everyone of the steering vectors of the $M$ eavesdroppers as follows

$$p_{A}(\omega) = \frac{g_{A}(\omega)}{\|g_{A}(\omega)\|^{2}} - \sum_{i=1}^{M} \left\langle \frac{g_{A}(\omega)}{\|g_{A}(\omega)\|^{2}}, \hat{g}_{i}(\omega) \right\rangle \hat{g}_{i}(\omega)$$

(6.6)
where

\[ \hat{g}_i(\omega) = \frac{g_i(\omega)}{\|g_i(\omega)\|} \]  \hspace{1cm} (6.7)

For the above processing to produce ideal nulling at the \(M\) locations, the \(M\) steering vectors \((g_i(\omega), i = 1, \ldots, M)\) must be mutually orthogonal. The pilot given by (6.6) does not produce perfectly equalized CIR at the intended receiver. For perfect equalization, the pilot is normalized as follows

\[ \bar{P}_A(\omega) = \frac{P_A(\omega)}{\langle g_A(\omega), P_A(\omega) \rangle^*} \] \hspace{1cm} (6.8)

### 6.2.4 Extracting Pilot Signals using Differential TR

For scenarios involving receivers that are unable to transmit pilots, like, for example, passive RFIDs or receivers in one-way communication links, differential TR techniques [73] can be used to extract an approximate version of the receiver’s pilot. Assuming that the initial location of the receiver (denoted by location 0) is known to the array, differential TR proceeds as following: the array transmits a beam \(g_0(\omega)\) twice at two close time instants. The beam illuminates the moving receiver (target) at two adjacent locations (locations 1 and 2), and backscatterings from both illuminations \((s_1(\omega)\) and \(s_2(\omega)\)) are recorded and subtracted to yield the differential backscattering vector, written as \(d(\omega) = s_2(\omega) - s_1(\omega)\). It was shown in [73] that when \(d(\omega)\) is time-reversed and backpropagated, it focuses in the vicinity of the moving target. Therefore, \(d(\omega)\) can be used as a pilot for communication. To achieve better equalization at the receiver, \(d(\omega)\) can be normalized as follows

\[ P_A(\omega) = \frac{d(\omega)}{\langle g_0(\omega), d(\omega) \rangle^*} \] \hspace{1cm} (6.9)
6.3 MISO Configuration

6.3.1 Problem Setup

In this section, we assess and contrast the performances of the proposed TR techniques applied to MISO configurations. MISO here refers to the configuration where a multiple-input antenna array is communicating with one single intended receiver. To assess the covertness property offered by TR, we assume the presence of one eavesdropper, and compare the performance of the wireless channel at both the intended receiver and the eavesdropper. Our goal is to maximize the signal “quality” at the receiver, while achieving maximum covertness (low probability of intercept (LPI)) at the eavesdropper [66].

The simulation domain is either extended free-space or a 3.6 m × 3.6 m room surrounded by 60 cm thick walls with relative permittivity of 4. The transceiver array consists of eight point sources in 2-D. The simulation is therefore a 2-D domain with electric field transverse to the domain (scalar case). Two array deployments are considered: (i) A dense linear array with inter-element spacing of 15 cm and total aperture of 105 cm. (ii) A sparse square-shaped full-aspect array. Pilots are extracted from UWB pulses generated as the first derivative of Blackmann Harris (BH) pulse [107] with center frequency of 1.25 GHz and useful bandwidth $B$ covering from DC up to 2.5 GHz. All simulations are carried out using the finite-difference time-domain (FDTD) method [84].
Table 6.1: Summary of the considered MISO setups.

<table>
<thead>
<tr>
<th>Acronym</th>
<th>TR technique</th>
<th>Array configuration</th>
<th>Background</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETRLFS</td>
<td>Equalized TR beamforming</td>
<td>Linear</td>
<td>Free-space</td>
</tr>
<tr>
<td>ETRLW</td>
<td>Equalized TR beamforming</td>
<td>Linear</td>
<td>with walls</td>
</tr>
<tr>
<td>ETRFAFS</td>
<td>Equalized TR beamforming</td>
<td>Full-aspect</td>
<td>Free-space</td>
</tr>
<tr>
<td>ConvLW</td>
<td>Conventional beamforming</td>
<td>Linear</td>
<td>with walls</td>
</tr>
<tr>
<td>TRLFS</td>
<td>Conventional TR beamforming</td>
<td>Linear</td>
<td>Free-space</td>
</tr>
<tr>
<td>TRLW</td>
<td>Conventional TR beamforming</td>
<td>Linear</td>
<td>with walls</td>
</tr>
<tr>
<td>MLFS</td>
<td>TR MUSIC w/o equalization</td>
<td>Linear</td>
<td>Free-space</td>
</tr>
<tr>
<td>EMLFS</td>
<td>TR MUSIC w/ equalization</td>
<td>Linear</td>
<td>Free-space</td>
</tr>
<tr>
<td>DTRLFS</td>
<td>Differential TR beamforming</td>
<td>Linear</td>
<td>Free-space</td>
</tr>
<tr>
<td>DMLFS</td>
<td>Differential TR MUSIC w/o equalization</td>
<td>Linear</td>
<td>Free-space</td>
</tr>
</tbody>
</table>

6.3.2 Spatial Focusing and Temporal Compression

The different setups are summarized in Table 6.1. Pilots of all setups are normalized to have equal (unity) total energies as follows

$$p_A(\omega) = \frac{p_A(\omega)}{\sqrt{\int_\omega \|p_A(\omega)\|^2 d\omega}} \quad (6.10)$$

CIR peak values and ISI at the intended receiver and eavesdropper for all setups are plotted in Fig. 6.1 for comparison. To compute the ISI using (6.2), $T$ is chosen equal to $1/(2B)$. This corresponds to the case of UWB modulation using Nyquist pulses discussed later.

We start with equalized TR beamforming in free-space and use it as a reference for comparison with other setups. Spatial distributions of the peak value of the CIR and the ISI are plotted in Fig. 6.2. It is obvious from the figure that the spatial
Figure 6.1: Performance comparison of the proposed MISO setups under equal input power assumption. Acronyms are summarized in Table 6.1. (a) CIR peak and (b) ISI, at the intended receiver and the eavesdropper.

focusing of this setup is not so good. The power is distributed over a wide region of space, and the eavesdropper (indicated by “x” in the figure) is receiving a significant amount of power. Also, the ISI at the eavesdropper is comparable to that at the receiver. Consequently, the eavesdropper will be receiving a high quality signal that allows it to intercept the signal.
Figure 6.2: Equalized TR beamforming in free-space using linear array (indicated by white circles). The intended receiver is indicated by “o” and the eavesdropper is indicated by “x”. (a) Normalized CIR peak distribution (in dB). (b) ISI (in dB).

Figure 6.3: Same as Fig. 6.2, but in the presence of surrounding walls with relative permittivity equal to 4 and thickness equal to 60 cm.

Better spatial focusing is obtained when the same linear array is operating in the presence of rich scattering environment, such as including the surrounding walls shown in Fig. 6.3. This is a result of the way TR exploits multipathing, where frequency components of the CIR add (only) incoherently at all locations in space.
except at the receiver’s location where different frequency components add \textit{coherently} [31, 64, 55, 54]. This is known as the superresolution property of UWB TR.

It is interesting to observe that, despite the spatial focusing in the presence of walls evident in Fig. 6.3 (higher ratio between CIR peak at the intended receiver compared to other locations), the absolute value of the CIR peak at the receiver is actually less than that in free-space. From the input energy normalization in (6.10), the ratio between the CIR peak at the receiver in the presence of walls to that in free-space can be written as \( \sqrt{\int_\omega 1/ \| \mathbf{g}_{A,f}(\omega) \|^2 d\omega/ \int_\omega 1/ \| \mathbf{g}_{A,w}(\omega) \|^2 d\omega} \), where subscripts \( f \) and \( w \) refer to free-space and walls respectively. This ratio is arbitrary and depends on the scattering medium.

Full-aspect deployment is considered in Fig. 6.4. Such deployment is convenient for indoors communication systems, where fixed transmitters can be mounted on the surrounding walls. Comparing the performance of the full-aspect array with that of linear array with the same number of elements, Fig. 6.4 shows that better spatial and temporal focusing around the receiver are achievable using full-aspect arrays.
Fig. 6.1 shows that conventional (non-equalized) TR provides for higher CIR peaks at the expense of increased ISI, as compared with equalized TR. For conventional TR, the ratio between the CIR peak in the presence of walls to that in free-space is given by $\sqrt{\int \omega \| g_{A,w}(\omega) \|^2 d\omega / \int \omega \| g_{A,f}(\omega) \|^2 d\omega}$, which is always larger than unity. Although multipath serves to increase the CIR peak to temporal sidelobe ratio at the intended receiver [55], it increases the temporal span of the sidelobes, which increases the overall ISI as shown in Fig. 6.1(b) [63].

At this point, it is useful to compare the performance of TR-based beamforming with that of conventional beamforming. Conventional beamforming, despite estimating the receiver’s DOA or even its location, is unable to compensate for delays due to multipathing and hence unable to achieve satisfactory equalization. Therefore it yields higher ISI than the equalized TR case as shown in Fig. 6.1(b).

The CIR peak images of TR MUSIC without and with equalization for the case of linear array in free-space are shown in Fig. 6.5(a) and (b), respectively. It is interesting to compare these images with Fig. 6.2(a) to see how MUSIC produces null field at the eavesdropper while delivering power to the intended receiver. This is a common feature to both equalized and non-equalized cases. Equalization, however, spreads out the power into larger region of space. This means that, given equal input powers, equalization results in less power reaching the receiver as shown in Fig. 6.1(a).

As explained in the previous section, differential TR can be used to extract the pilot signal of passive moving receivers, as that shown in Fig. 6.6(a). The resulting

---

3From physical reasoning, multiple scatterings, such as those offered by walls, confine more energy in the medium and yield longer impulse responses between any transmitter/receiver pair. Therefore the energy of the impulse responses in the presence of walls is larger than that in free-space.
CIR peak image is very close to that obtained using TR beamforming in Fig. 6.2(a). Differential TR can also be used to extract the steering vectors of moving eavesdroppers to be used in the MUSIC algorithm, as shown in Fig. 6.6(b). Note that the null in this case is not perfect because differential TR provides only an approximate version of the steering vector.

6.3.3 Bit Error Rate Performance

In this section, we compare the bit error rate (BER) performance of the discussed techniques under high and low data rates operations. In the high data rate case, bits are represented by Nyquist pulses whose spectrum covers the entire useful bandwidth of operation (UWB pulses) as shown in Fig. 6.7(a) and (b). In this particular example, a maximum bit rate of 5 Gbps is accommodated, which is equal to twice the bandwidth. For the low data rate case, we use binary phase shift keying (BPSK)
Figure 6.6: Normalized CIR peak distribution of differential TR in free-space using a linear array. (a) Differential TR beamforming. The intended receiver is moving along the path indicated by the arrow. (b) Differential TR MUSIC. The eavesdropper is moving along the path indicated by the arrow.

Received bit streams at the intended receiver and at the eavesdropper are obtained by convolving the input stream with the corresponding equivalent CIR. Additive white Gaussian noise (AWGN) is added, and the noisy stream is decoded. Assuming perfect synchronization at the decoder, Nyquist pulses stream is decoded by simply sampling the stream at the bit rate, whereas BPSK stream is decoded using coherent detection [147]. Finally, Monte Carlo simulation is used to compute the BER of the decoded stream. For the computed BER to represent a statistically stable measure for the probability of error, the process is repeated over a large number of noise realizations and the results are averaged. In our simulations, we generate streams of 1000 bits and average over 100 noise realizations.
To make comparison easy, considered setups are divided into four groups. The first group includes equalized TR techniques as shown in Fig. 6.8. Note that to provide a consistent measure for comparing different setups, the bit energy $E_b$ in the abscissa of the BER curves represents the pilots energy at the transmitter array (which is equal for all setups) rather than the bit energy at the receiver (which varies from one setup to the other). Since these setups are equalized (have negligible ISI), the BER performance at the receiver depends on the CIR peak value at the receiver.
That is why the full-aspect case exhibits better performance. At the eavesdropper, however, the CIR is not perfectly equalized. Therefore, the performance at high data rate is hampered by the internal noise due to ISI, especially for low external noise levels (high $E_b/N_0$), as evident from the BER saturation in Fig. 6.8(b). At low data rates, the effect of ISI is less pronounced, especially in free-space, which allows the eavesdropper to intercept the signal.

The second group compares conventional TR and conventional beamforming with equalized TR as shown in Fig. 6.9. At high data rates, the performance of equalized TR is superior to other techniques. On the contrary, at low data rate the performance is controlled by the power level at the receiver. Therefore, when ISI at the receiver is not the limiting factor, conventional TR operating in rich scattering environments yields the best performance since it provides the highest CIR peak at the receiver and the highest ISI at the eavesdropper.

The third group compares TR beamforming with MUSIC techniques with equalized TR as shown in Fig. 6.10. Using MUSIC significantly increases the BER at the eavesdropper (thus increases covention) especially at low data rates. However, because of the reduced CIR peak provided by equalized MUSIC, it yields unsatisfactory high BER at the receiver for both high and low rates. Only at very high $E_b/N_0$ (beyond the limits of Fig. 6.10(a)), equalized MUSIC performance can outperform that of non-equalized MUSIC, which saturates at high $E_b/N_0$ due to ISI.

Finally, the fourth group compares differential TR techniques with equalized TR. As shown in Fig. 6.11(a) and (c), both differential TR techniques yield satisfactory
performance at the receiver, comparable with that of equalized TR. At the eavesdropper, differential TR MUSIC serves to increase the covertness by increasing the BER, but with slightly less efficiency than TR MUSIC in Fig. 6.10, as expected.

6.3.4 Effect of Hardware Limitations

So far, we have studied the performance of different techniques without considering any practical limitations that may be imposed by the utilized hardware. In practice, the time allocated for recording the steering vector of the receiver might not be
long enough to capture all multiple scatterings in rich scattering media [148]. This produces truncated pilots, and therefore imperfectly equalized equivalent CIR. To demonstrate this effect, consider for example the case of equalized TR technique using linear array operating in the presence of surrounding walls. A typical response at the receiver array for a BH pulse derivative transmitted by the receiver in shown in Fig. 6.12(a). The recorded signal is truncated at 34 ns which corresponds to 90% of the total energy. The CIRs in the frequency domain for both truncated and full
Figure 6.10: BER performance of TR MUSIC. For high data rate: (a) at the intended receiver, (b) at the eavesdropper. For low data rate: (c) at the intended receiver, (d) at the eavesdropper.

responses are plotted in Fig. 6.12(b). Imperfect equalization is evident as a result of truncation. This increases the ISI by 100%.

Another limitation may arise from the dynamic range of the array transceivers. Pilots energy distribution among array elements for different setups are plotted in Fig. 6.13. Most techniques result in almost uniform energy distribution among array elements with dynamic range less than 5 dB. Equalized TR MUSIC, however, requires
Figure 6.11: BER performance of differential TR beamforming and MUSIC. For high data rate: (a) at the intended receiver, (b) at the eavesdropper. For low data rate: (c) at the intended receiver, (d) at the eavesdropper.

A stringent power distribution with dynamic range of 25 dB. This problem can be mitigated by muting array elements whose energies fall below a certain threshold.

### 6.4 Multiuser MIMO Configuration

In multiuser MIMO configurations, the array attempts to communicate with multiple receivers simultaneously. If the interuser interference (IUI) among receivers is adequately mitigated, communication with different users can be achieved in the
Figure 6.12: Effect of receiver channel response truncation at the array. (a) A typical response at the receiver array for a BH pulse derivative transmitted by the receiver in the presence of walls. (b) CIR in the frequency domain at the intended receiver.

Figure 6.13: Pilots energy distribution among array elements for different setups

same time and frequency slots, which is the principle of space division multiplexing. The capacity of the MIMO system is mainly limited by the amount of co-channel interuser interference (IUI) [57, 149, 59, 150, 67]. In this section, we will show how TR beamforming with MUSIC can be used to mitigate IUI.
Consider the presence of two receivers $A$ and $B$, the transmitted signal vector in the frequency domain can be written as

$$
t(\omega) = S_A(\omega)p_A^*(\omega) + S_B(\omega)p_B^*(\omega) \quad (6.11)
$$

where $S_A(\omega)$ and $S_B(\omega)$ are the information signals to be transmitted to receiver $A$ and $B$ respectively. The received signal at receiver $A$ is given by

$$r_A(t) = s_A(t) * t h_{AA}(t) + s_B(t) * t h_{AB}(t) + n(t) \quad (6.12)$$

where $h_{AA}(t)$ is the CIR of receiver $A$ computed at receiver $A$, whereas $h_{AB}(t)$ is the CIR of receiver $B$ computed at receiver $A$. Similarly, the received signal at $B$ can be written as

$$r_B(t) = s_B(t) * t h_{BB}(t) + s_A(t) * t h_{BA}(t) + n(t) \quad (6.13)$$

Obviously, the second terms in the r.h.s. of the above two equations represent undesired interuser interference. TR beamforming with MUSIC can be used to deliver the signal to the intended receiver while imposing null on the other receiver, therefore, it sets the cross terms $h_{AB}(t)$ and $h_{BA}(t)$ to zero, and the self terms become the CIR of the TR with MUSIC technique. As an example, consider the setup in Fig. 6.2, where the eavesdropper now represents receiver $B$. The BER performance of non-equalized TR with MUSIC is compared with that of equalized TR in Fig. 6.14. Note that at high data rate both techniques suffer from saturation with $E_b/N_0$. Equalized TR saturates because of IUI, whereas TR with MUSIC saturates because of ISI. Nevertheless, TR MUSIC still shows better performance especially at low data rates where IUI is stronger than ISI.
6.5 Time-Reversal versus Interference Alignment for Wireless Communication Networks

Interference alignment (IA) is a linear precoding strategy that has the potential of increasing the capacity of wireless networks. It is useful in distributed networks with multiple independent transmitter-receiver pairs sharing the same channel resources. When the strength of interference is comparable to that of the desired signal, multiple users can coordinate their transmissions so that they produce overlapping interference at unintended receivers, while producing distinguishable signal components at
intended receivers [151]. Hence, desired signal can be extracted, and interference can be canceled out using simple linear detection. It was shown in [151] that using interference alignment, the sum capacity of $K$-user time-varying interference channel is $K/2$ times the capacity of one interference-free link, given high signal-to-noise ratio (SNR). The basic idea behind interference alignment can be summarized as follows: transmitters precode their signals by beamforming over multiple symbol extensions. Symbols may represent time slots, carrier frequencies, or antennas. The extension of the symbol determines the dimensionality of the signal space. To achieve the aforementioned outrebound on the capacity, each receiver must be able to partition its observed signal space into two equal subspaces, one for the desired signal and the other for (aligned) interference signals. This was found to be an over-constrained problem, for which a nontrivial solution does not exist. However, an approximate solution can be achieved that works at maximizing the overlap between interfering signals at all receivers (rather than perfectly aligning them). This approximate solution asymptotically tends to the outrebound when sufficiently large number of symbols is used [151].

The calculation of IA precoders requires the channel state information (CSI), between each transmitter-receiver pair, to be known at each transmitter. The overhead caused by sharing the global CSI among transmitters is one of the main challenges of IA. One way to allow distributed users to align interference, with knowledge of only local CSI at each node, is by iterative IA [151, 152]. In full-duplex channels, reciprocity enables transmitters to infer the IA precoders by observing the interference they cause at the receivers. This technique has the drawback of large overhead since it includes recurring pilot transmissions to compute the precoders over the air.
Other challenges of IA along with suggested solutions can be summarized as follows: (i) Dimensionality of the signal space: as the number of users increase, larger and larger signal space is required to reach the IA capacity gain. This requires a prohibitively large number of carrier frequencies or time slots. Such requirement becomes less severe using multiple antenna nodes. This favors the application of IA in MIMO systems [153]. (ii) SNR: IA in its basic form described above, is only effective at high SNR, whereas its performance drops significantly below the expected theoretical limit in moderate SNR conditions. This is because IA focuses on aligning the interference, unaware of desired signal level [153]. Some techniques were devised to mitigate this limitation including for example maximum signal-to-interface-plus-noise ratio (Max-SINR) [154], and ergodic IA that was shown to achieve the optimal performance at any SNR [152, 154]. (iii) Channel estimation overhead and robustness to CSI distortion especially in fast fading channels: As mentioned before, the burden of estimating and sharing global CSI can be alleviated by using iterative IA, or even further, as proposed in [155], exploiting channel correlations to achieve IA without any knowledge of channel coefficients at transmitters (blind IA). (iv) Synchronization: since IA is a coherent algorithm, it requires tight synchronization among all transmitters both in frequency and time. This can be achieved, for example, using GPS satellite signals [153]. (v) Network organization: In large scale networks, required resources and CSI sharing overhead blow up, and strict application of IA becomes unfeasible. However, grouping users into smaller clusters and using hybrid IA/TDMA strategy seem to be promising approaches in such networks [156].

On the other hand, as seen in this chapter, TR beamforming only requires local CSI information about the intended receiver, and does not require coordination
nor synchronization between transmitting users. This yields much simpler processing. Multiple-input (e.g. multiple-antenna) users can combine TR beamforming with MUSIC to produce null fields at unintended receivers, given that the number of the latters is less than the number of array elements. In this case, global knowledge of CSI is required at transmitters. Similar to IA, TR suffers CSI estimation overhead and robustness to distortion, especially in fast varying channels. In contrast with IA, TR does not suffer SNR limitation, since it inherently tends to focus the signal at the intended receiver. In addition to interuser interference reduction, TR also allows for simpler receiver design thanks to the temporal compression feature. In large scale networks, the idea of users clustering can be used in conjunction with TR techniques; TR MUSIC can be used to null the field only at close receivers, leaving interference reduction at far users to the spatial focusing property of TR beamforming.

6.6 Conclusion

We have extended existing TR-based wireless communication strategies by introducing three techniques that satisfy different performance criteria. We have applied them to both MISO and MIMO configurations using both linear and full-aspect arrays. BER performances of different techniques under various operational scenarios were compared for high and low data rates. Two main factors affect the BER: (i) ISI (internal noise), and (ii) received power level relative to external noise. Equalized TR beamforming was introduced to eliminate the ISI that limits the performance at high data rates; however, it was shown to possess inferior power focusing compared with conventional TR. Focusing resolution was shown to depend significantly on the
array configuration, where full-aspect arrays were capable of providing better focusing than linear arrays. TR beamforming using linear arrays typically does not allow for sufficient degree of covertness, and especially when an eavesdropper is closer to the array than the receiver. In this case, TR beamforming combined with MUSIC becomes very beneficial in producing null field at the eavesdropper location. TR beamforming with MUSIC is also useful in reducing the IUI in MIMO configurations, and therefore increasing the system capacity. In case of passive receivers that can not send pilots, approximate versions of the pilots can be obtained from sequential array acquisitions. Some effects of hardware limitations on the performance, such as truncated impulse response and transmitters dynamic range, were also considered. As for potential future works, we plan to extend these techniques for their applications in dynamically changing and possibly lossy environments. For example, the existence of (lossy) walls further degrades the electromagnetic propagation as well as breaks the TR invariance. Under such conditions, the techniques presented here should be appropriately modified to achieve the desired performance.
Chapter 7: Conclusions and Future Work

We devised novel TR-based signal processing techniques for imaging and tracking of targets embedded in rich scattering environments, and applied those techniques to through-wall tracking, UWB inverse scattering in continuous random media based on Bayesian compressive sensing, in addition to MISO and MIMO wireless communications in indoor scenarios. We verified the proposed techniques with numerical simulations and, whenever possible, experimental measurements. First, we carried out a numerical study on the statistical stability of UWB TR imaging in random media under different combinations of random medium parameters and interrogating signal properties. We presented a new frequency-synthesized technique for UWB time-reversal-based imaging. This technique was employed to construct DORT and MUSIC images using either linear or full-aspect transceiver array configurations. The proposed technique was shown to automatically provide the best images of desired target(s), in terms of focusing resolution, without any need for (synthetic) propagation of time-reversed signals and ad hoc determination of the optimal focusing time instant. Second, we experimentally verified the statistical stability of UWB TR-based imaging of targets in discrete random media. The measurements were carried out using a time-domain radar system that provided useful bandwidth of up to 40 GHz. We studied the effect of the excitation bandwidth, discrete scatterers permittivity
and fractional volume, and radar aperture size on the image stability. Third, we introduced two TR algorithms for identifying, imaging, and tracking moving targets in clutter. The first algorithm classified existing scatterers into stationary vs. moving targets. SVD of average and differential MDMs provided information on stationary and moving targets, respectively. The second algorithm yielded real-time selective tracking of each moving target by means of differential TR. It required minimal processing and memory resources, and was shown to exploit distinctive features of TR such as statistical stability and superresolution. Numerical simulations were used to illustrate the capabilities of the proposed algorithms in different scenarios involving clutter from discrete secondary scatterers, and from inhomogeneous random medium backgrounds. After that, we developed UWB inverse scattering techniques for continuous random media, based on Bayesian compressive sensing. We applied TR focusing to increase the efficiency of the inversion. In addition, we proposed a technique for determining the optimal data acquisition features (sensors locations and frequency band) so as to maximize the differential information gain. Finally, we considered the application of different TR signal processing and beamforming techniques to MISO and MIMO wireless communication systems. Time-reversed pilot was normalized to provide perfect equalization at the expense of power level. This equalization is particularly important for high data rates where the bit error rate performance is dominated by internal noise due to ISI. To increase physical layer covertness, TR beamforming was combined with the MUSIC technique to produce null fields at eavesdroppers. This technique was also applied to MIMO setups to eliminate interuser interference, and hence increase system capacity. Differential TR was used to obtain and update
pilot signals for passive moving receivers, i.e. those that cannot (or do not) transmit pilot signals.

7.1 Future Work

Some extensions of this work, and research topics of potential interest, may include:

7.1.1 Combining BCS with TR subspace methods

TR-based subspace methods, such as DORT and MUSIC, can be combined with the proposed UWB BCS inversion to focus inversion on the target(s) of interest and enhance clutter rejection. Relevant works include, coupling a nonlinear monochromatic inversion method with DORT for generating incident fields that selectively focus on each target [5], and combining Bayesian compressive sensing with signal subspace methods for imaging discrete targets [118, 119, 120]. In addition, compensation techniques of UWB TR [44] can be used to ameliorate inversion sensitivity in dispersive lossy media, given sufficiently high SNR.

7.1.2 Incorporating dispersion in the UWB BCS inversion technique

The analysis presented in Chapter 5 assumed negligible dispersion over the frequency band of interest. However, dispersion can be crucial in UWB applications involving, for example, wet soils or biological tissues. Dispersion can be incorporated in the proposed technique by dividing the overall frequency band into narrow sub-bands, and solving for the electrical properties of the medium in each sub-band, using the multitask compressive sensing approach proposed in [115]. Another possible
way to tackle dispersion, is by developing linearized model for dispersion as the one presented in [157].

### 7.1.3 Applying BCS to brain imaging

The application of the proposed UWB BCS inversion for breast imaging can be extended to other medical applications such as brain imaging [158, 159]. An example of electromagnetic imaging of brain using Bayesian approach was presented in [9]. The brain has mean permittivity of 40 and mean conductivity of 1 S/m over a frequency range of 1-5 GHz, and can be imaged using a full-aspect tomographic array.

### 7.1.4 Phaseless BCS in random media

A recent study on compressive sensing inversion of point-like dielectric targets using phaseless (intensity-only) data is presented in [160]. Perhaps, that method can be adapted to develop phaseless BCS in continuous random media.

### 7.1.5 Addressing practical challenges of TR for wireless communications

Despite its promising advantages, practical implementation of TR in wireless communication still faces some challenges. We can identify some of them as follows:

- *Extracting pilots of multiple users from the eigenvalue decomposition of the array correlation matrix:* In multiple users networks, pilot signals recorded by the transceiver array can be a weighted sum of the steering vectors between each user and the array. The autocorrelation matrix of this received signal vector is referred to as the array correlation matrix. In multipath environments, the
number of significant eigenvalues of the array correlation matrix equals the number of uncorrelated paths between the users and the array. This is in essence how conventional direction-of-arrival (DOA) estimation algorithms work. To exploit the full capability of TR in harnessing multipath and achieving super-resolution focusing, all eigenvectors pertaining to each user should be combined (with their respective weights) to constitute the pilot (steering vector) to be used to communicate with that user. Effective techniques for assigning different eigenvectors to their respective users need to be developed.

- **Incorporating antennas gain pattern:** In many TR studies, array elements as well as users antennas are modeled as omnidirectional sources or sinks. Consequently, antennas are capable of collecting multipath signals from all directions, and superresolution focusing performance is assessed based upon this assumption. In practice, however, antennas with directive gain patterns are commonly used. This limits the angular visibility range of the antenna. The effect of antennas gain patterns on spatial and temporal focusing performance of TR techniques can be studied.

- **Effect of pilot noise on the TR focusing performance:** In the TR communications studies carried out in Chapter 6, additive noise on user pilot at the array side was neglected, and bit-error-rate (BER) performance was studied by adding noise at the user after TR re-transmission. The sensitivity of TR communications to noisy pilots can be studied.

- **Effect of time-varying background on the pilot update rate:** TR techniques are based upon the wave equation invariance under TR, assuming time-invariant
background media. However, the assumption of time-invariant media does not always hold true due to moving users and/or time-varying surroundings. In that case, periodic pilot update is necessary to guarantee satisfactory TR performance. The relationship between the speed of moving users and the required pilot update rate, and its effect on the throughput of the system can be studied.

7.1.6 Time-reversing layer

Pendry drew attention to the relationship between negative refraction and TR in [161]. He observed that a wave travelling through a material with negative refractive index is equivalent to a wave propagating in a material with oscillating material properties with twice the frequency of the wavefield. In the latter case, the medium appears (to the travelling wave) to be oscillating with negative frequency. This is indeed equivalent to negative time, and hence the wave will be time-reversed. This idea can be used to design a time-reversing layer (TRL); when a wave impinges on such layer, it reflects back on itself, retraces its path and focuses at its original source location. Phase conjugating metamaterial, composed of non-linearly-loaded split-ring resonators, was implemented in [162, 163], and its spectral response was characterized. A challenge still remains in implementing such TRL over a wide range of frequencies. Perhaps, one possible way to realize this is by stacking layers of phase conjugating unit cells, oscillating with different frequencies covering the bandwidth of interest, to constitute an equivalent wideband TRL. Those unit cells can be implemented using lumped elements, or electrically tunable materials [164].
Appendix A: On some heuristics for target characterization using DORT

In this appendix, we present some heuristics for characterization of scatterers using the eigenvalue/vector decomposition provided by the DORT. Using this information, we can determine the composition of the scatterer being either metallic or dielectric, and plot the image of the scatterer showing its cross-range extension. We can also estimate the co-range electrical length in case of dielectric scatterers.

We start by explaining the approach, followed by a presentation of results for different metallic and dielectric targets.

A.1 Approach

The MDM of a point-like scatterer, located at point \( p \), has one significant singular value/vector. The singular value of the MDM (which equals the square root of the eigenvalue of the corresponding TRO) at frequency \( \omega \) is given by

\[
\mu(\omega) = |s(\omega)| |\tau(\omega)| \|g_p\|^2
\]  

(A.1)

where \( s(\omega) \) is the spectrum of the input pulse and \( \tau(\omega) \) is the target’s scattering coefficient. The singular vector (which equals the eigenvector of the corresponding
TRO) is given by

$$v(\omega) = \frac{g_p^*}{\|g_p\|}$$  \hspace{1cm} (A.2)

So, in principle, singular values carry information on the scattering coefficients as functions of frequency and singular vectors carry information on the locations of the targets.

The singular value spectrum can be used to determine whether the target is metallic or dielectric. Metallic targets, with sufficiently high conductivity, have almost “flat” scattering coefficient spectrum in the frequency band of interest (microwave range assumed here). On the other hand, dielectric targets act as imperfect resonators. So their singular values spectra have periodic “peaks” and “valleys” corresponding to constructive and destructive interferences. The first valley frequency $f_v$ can be approximately described by the 1-D resonator relation

$$f_v = \frac{f_c \lambda}{2l_z}$$  \hspace{1cm} (A.3)

where $f_c$ is the operating frequency , $\lambda$ is the wavelength inside the dielectric target and $l_z$ is the dimension of the target along the co-range direction.

For a given operating frequency, $f_v$ depends only on the electrical length of the target. Thus, valley frequencies can be used to estimate the co-range electrical length of the target.

Extended targets across the co-range are also associated with one significant singular value/vector. The image of the singular vector corresponds to the side of the target facing the array. The co-range extension can be estimated through the singular value spectrum as described before.
Figure A.1: Spectra of the first three singular values (SVs) of two metallic targets. The singular values are normalized with respect to the input pulse spectrum.

Extended targets across the cross-range are associated with *multiple* significant singular values depending on their size. If the composition of the target is uniform, all these singular values have similar spectrum. The associated singular vectors point to different scattering centers on the target. Therefore, to estimate the cross-range extension, we can, for example, construct the image of the *sum* of the significant singular vectors weighted by the respective singular values.

A.2 Simulation results

A.2.1 Metallic targets

The spectra of the first (most significant) three singular values of two metallic targets are shown in Fig. A.1. The first target is point-like with $l_x = 5\Delta_s$ and $l_z = 5\Delta_s$, $\Delta_s = 2.5$ cm. The second target is extended along the cross-range direction with $l_x = 40\Delta_s$ and $l_z = 5\Delta_s$. The singular values are normalized with respect to
Figure A.2: Images of the sum of the significant singular vectors of metallic targets: (a) point-like target \((l_x = 5\Delta_s)\). (b) extended target \((l_x = 40\Delta_s)\). The actual targets are indicated by white rectangles.

the input pulse spectrum, so that the plotted spectra directly express the targets’ scattering coefficients.

It can be noticed that the point-like target has only one significant singular value, whereas, the extended target has three significant values (those that have comparable magnitudes). Also, we can notice that, all significant singular values have similar spectra. This spectrum is almost flat with frequency with some linear dependence because of the \(||g_p||^2\) factor in \(\mu\).

The image of the significant singular vector for the point-like target is shown in Fig. A.2(a), and the image of the sum of the three singular vectors of the extended target is shown in Fig. A.2(b). Both images indicate the shape of the corresponding targets reasonably well. Of course, for an extended target, the image of the sum of the significant singular vectors is more indicative of the target’s shape than those of any individual singular vectors.
A.2.2 Dielectric targets

(a) Permittivity effect:

Fig. A.3(a) and (b) show the spectra of the first and second singular values, respectively, of three near point-like dielectric targets with different permittivities. The targets’ sizes are $5\Delta_s \times 5\Delta_s$ and have relative permittivities $\epsilon_r = 3, 6$ and $9$. We
Table A.1: Actual and estimated electrical lengths of point-like dielectric targets with different permittivities

<table>
<thead>
<tr>
<th>$\epsilon_r$</th>
<th>3</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>actual $l_z/\lambda$</td>
<td>0.29</td>
<td>0.408</td>
<td>0.5</td>
</tr>
<tr>
<td>estimated $l_z/\lambda$</td>
<td>0.307</td>
<td>0.412</td>
<td>0.49</td>
</tr>
<tr>
<td>% error</td>
<td>5.8</td>
<td>0.9</td>
<td>2</td>
</tr>
</tbody>
</table>

can see that the spectra of the first singular values have the peaks/valleys response of dielectric resonators. The valleys frequencies are used to estimate the electrical sizes of the targets along the co-range direction. Actual and estimated electrical lengths are summarized in Table A.1. Very accurate estimation is achieved. The second singular values do not possess the same spectra as those of the first. This indicates that each target is associated with only one significant value; this is expected since they are near point-like targets.

The images of the first singular vectors are shown in Fig. A.4, expressing the targets shapes well.

(b) Co-range extension effect:

Fig. A.5(a) and (b) show the spectra of the first and second singular values, respectively, of three dielectric targets with different co-range dimensions. The targets have the same relative permittivity $\epsilon_r = 3$ and the same cross-range length $l_x = 5\Delta_s$. The co-range lengths are 5, 10 and $20\Delta_s$. Again, the valleys frequencies can be used to estimate the co-range electrical length. Actual and estimated values are summarized in Table A.2.
It is interesting to note that the spectra of the second singular values are “complementary” to the first singular values, in the sense that they have the spectra of the transmission coefficients through the dielectric targets. The images of the first singular vectors are shown in Fig. A.6. Note that they are all located at the top faces of the targets (those facing the array).
Table A.2: Actual and estimated electrical lengths of extended dielectric targets with different co-range lengths

<table>
<thead>
<tr>
<th>$l_z/\Delta_s$</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>actual $l_z/\lambda$</td>
<td>0.29</td>
<td>0.58</td>
<td>1.15</td>
</tr>
<tr>
<td>estimated $l_z/\lambda$</td>
<td>0.289</td>
<td>0.5</td>
<td>0.96</td>
</tr>
<tr>
<td>% error</td>
<td>0.3</td>
<td>13</td>
<td>16</td>
</tr>
</tbody>
</table>

(c) Cross-range extension effect:

Fig. A.7 shows the spectra of the first four singular values of three dielectric targets with different cross-range dimensions. The targets have same relative permittivity $\epsilon_r = 3$ and the same co-range length $l_z = 10\Delta_s$. The cross-range lengths are 20, 40 and 80$\Delta_s$, respectively. It can be observed that the number of significant singular values depends on the cross-range extension. The first target has two significant singular values, the second has three, and the third has four. All significant singular values have similar spectrum, from which the co-range length can be estimated.

The images of the sum of the significant singular vectors of each target are shown in Fig. A.8. They span the shapes of the respective targets reasonably well.
Figure A.7: Singular values spectra of cross-range extended dielectric targets with similar permittivities and $l_x = 10$. (a) First (b) Second (c) Third (d) Fourth singular values.

Figure A.8: Images of the sum of the significant singular vectors of cross-range extended dielectric targets. (a) $l_x = 20\Delta_s$. (b) $l_x = 40\Delta_s$. (c) $l_x = 80\Delta_s$. The actual targets are indicated by white rectangles.
Appendix B: The Fast Relevance Vector Machine

The goal of the RVM is to determine $\alpha$ and $\sigma_n^2$ that maximize the marginal likelihood $p(y|\alpha, \sigma_n^2)$. The original RVM, presented in [116], is an iterative re-estimation process for $\alpha$ and $\sigma_n^2$. During the re-estimation, some $\alpha_i$s tend to infinity, and hence $p(w_i)$ becomes highly peaked at zero, indicating that the corresponding weight should be set to zero (i.e., its value is determined by the prior rather than the measured data). For other weights that well-fit measurements, $\alpha_i$ is finite, indicating that the weight has a probability of being non-zero, and its posterior value is updated during the iterative process. Therefore, the RVM basically selects only those weights whose corresponding basis vectors (columns of $B$) are 'relevant' for fitting the data. The original RVM is, however, computationally costly and requires matrices inversion, which makes it slow for models with large number of unknowns $M$. A fast RVM was developed in [117]. It starts with an empty model, and enables a principled and efficient sequential addition and deletion of candidate basis vectors to monotonically maximize the marginal likelihood. It proceeds until all the $M_r$ relevant basis vectors (for which the associated weights are nonzero) have been included. Thus, the complexity of the algorithm is more related to $M_r$ than $M$ (for sparse models, $M_r \ll M$)

\[4\]This appendix borrows heavily from Tipping’s and Faul’s paper entitled ‘Fast marginal likelihood maximisation for sparse Bayesian models’ [117].
Here, we summarize the main steps in the fast RVM algorithm as presented in [117].

The log marginal likelihood, to be maximized, is given by

$$\mathcal{L}(\alpha, \sigma^2_n) = \ln(p(y | \alpha, \sigma^2_n)) = -\frac{1}{2}[N \ln(2\pi) + \ln |C| + y^T C^{-1} y]$$  \hspace{1cm} (B.1)

The dependence of $\mathcal{L}(\alpha, \sigma^2_n)$ on the $i^{th}$ hyperparameter can be separated by decomposing $C$ as follows

$$C = \sigma^2_n I + \sum_{m \neq i} \alpha_m^{-1} b_m b_m^T + \alpha_i^{-1} b_i b_i^T = C_{-i} + \alpha_i^{-1} b_i b_i^T$$  \hspace{1cm} (B.2)

where $C_{-i}$ is $C$ with the contribution of basis vector $i$ removed. Terms of interest in $\mathcal{L}(\alpha, \sigma^2_n)$ can be written as

$$|C| = |C_{-i}| |1 + \alpha_i^{-1} b_i^T C_{-i}^{-1} b_i|$$  \hspace{1cm} (B.3)

and

$$C^{-1} = C_{-i}^{-1} - \frac{C_{-i}^{-1} b_i b_i^T C_{-i}^{-1}}{\alpha_i + b_i^T C_{-i}^{-1} b_i}$$  \hspace{1cm} (B.4)

Now, $\mathcal{L}(\alpha, \sigma^2_n)$ can be written as

$$\mathcal{L}(\alpha, \sigma^2_n) = -\frac{1}{2}[N \ln(2\pi) + \ln |C_{-i}| + y^T C_{-i}^{-1} y - \ln(\alpha_i)$$

$$- \ln(\alpha_i + b_i^T C_{-i}^{-1} b_i) - \frac{(b_i^T C_{-i}^{-1} y)^2}{\alpha_i + b_i^T C_{-i}^{-1} b_i}]$$

$$= \mathcal{L}(\alpha_{-i}, \sigma^2_n) + \frac{1}{2}\ln(\alpha_i) - \ln(\alpha_i + s_i) + \frac{q_i^2}{\alpha_i + s_i}$$

$$= \mathcal{L}(\alpha_{-i}, \sigma^2_n) + l(\alpha_i, \sigma^2_n)$$  \hspace{1cm} (B.5)

where $s_i := b_i^T C_{-i}^{-1} b_i$, and $q_i := b_i^T C_{-i}^{-1} y$. From analysis of $l(\alpha_i)$, $\mathcal{L}(\alpha, \sigma^2_n)$ has a unique maximum w.r.t. $\alpha_i$, at

$$\alpha_i = \begin{cases} 
  s_i^2 / (q_i^2 - s_i), & \text{if } q_i^2 > s_i \\
  \infty, & \text{if } q_i^2 \leq s_i
\end{cases}$$  \hspace{1cm} (B.6)
Tipping-Faul RVM makes use of this observation to constructively add and delete candidate basis vectors as follows [117]:

1. Initialize $\sigma_n^2$.

2. Initialize the model by adding a single basis vector $b_i$, and computing its hyperparameter as follows

$$
\alpha_i = \frac{\|b_i\|^2}{\|b_i^T y\|^2 / \|b_i\|^2 - \sigma_n^2}
$$

and all other $\alpha_m$ are set to infinity.

3. Compute $s_m$ and $q_m$ for all $M$ bases.

4. Select a candidate basis vector $b_i$ from the set of $M$ bases.

5. Compute $\theta_i := q_i^2 - s_i$.

6. According to (B.6), if $\theta_i > 0$ and $\alpha_i < \infty$ (i.e. $b_i$ is in the model), re-estimate $\alpha_i$.

7. If $\theta_i > 0$ and $\alpha_i = \infty$ (i.e. $b_i$ is not included in the model), add $b_i$ with updated $\alpha_i$.

8. If $\theta_i \leq 0$ and $\alpha_i < \infty$ (i.e. $b_i$ is included in the model), delete $b_i$ and set $\alpha_i = \infty$.

9. Re-estimate the noise variance as follows

$$
\sigma_n^2 = \frac{\|y - B\mu\|^2}{N - M_r + \sum_{M_r} \alpha_m \Sigma_{mm}}
$$

where $M_r$ is the number of relevant basis vector (those that are already included in the model).
10. If converged terminate, otherwise go to step 3. Convergence is achieved when
    the changes in $\alpha_i$ in step 6, for all bases in the model, are below certain threshold
    (e.g. $\ln(\alpha_i) < 10^{-6}$ as chosen in [117]), and $\theta_i \leq 0$ for all others (i.e. no more
    bases need to be added in this iteration).
Bibliography


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