Geographic Routing in the Presence of Location Errors [1]

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Abstract

In this paper, we propose a new geographic routing algorithm that alleviates the effect of location errors on routing in wireless ad hoc networks. In most previous work, geographic routing has been studied assuming perfect location information. However, in practice there could be significant errors in obtaining location estimates, even when nodes use GPS. Hence, existing geographic routing schemes will need to be appropriately modified. We investigate how such location errors affect the performance of geographic routing strategies. We incorporate location errors into our objective function by considering both transmission failures and backward progress. Each node then forwards packets to the node that maximizes this objective function. We call this strategy "Maximum Expectation within transmission Range" (MER). Simulation results with MER show that accounting for location errors significantly improves the performance of geographic routing. Our analysis also shows that our algorithm works well up to a critical threshold of error. We also show that MER is robust to the location error model and model parameters. Further, via simulations, we show that in a mobile environment MER performs better than existing approaches.

Key words: wireless ad hoc networks, geographic routing, location errors, simulations.

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1 Introduction

Geographic routing for multi-hop wireless networks has become an active area of study over the last few years (e.g., see [1–4] and the references therein). Geographic routing is appealing because of its simplicity and scalability. However, most work in this area has implicitly assumed that location information available at each node is perfect, while in practice only a rough estimate of this information is available. In this paper, we will show that imperfect location information can lead to substantial degradation in the performance of geographic routing. We will also develop a routing scheme that accounts for location errors, and whose performance is robust to such errors.

In geographic routing, each node determines its own location by using either the Global Positioning System (GPS) [5,6] or the location sensing techniques [7–10]. It then broadcasts its location information to other nodes proactively and periodically. Packet forwarding is accomplished based on the neighbors' location information stored in each node's database (DB) and the destination's location information contained in the packet. Packets are typically forwarded using what is commonly referred to as the *greedy mode*, in which nodes use local information to forward packets towards their destination. If the greedy mode is not successful (i.e., the destination node is not available in the local databases, or a greedy mode forwarding results in a failure), a special routine called a *recovery mode* is initiated through the entire network to find an appropriate route to the destination. Since the greedy mode uses local information and most packets are forwarded in this mode [11], geographic routing is generally considered to be scalable and applicable to large networks.

Several forwarding schemes have been proposed for use in the greedy mode [12–15]. Figure 1 provides an illustration of such schemes when a node S with transmission range R has a packet to send to some node D. Arcs $\widehat{ab}$ and $\widehat{pq}$ are centered at node D and with radii DS and DG, respectively. When the nodes in the wireless network have a fixed transmission range, the Most Forward within Radius (MFR) scheme [12] and the Greedy Routing Scheme (GRS) [13] have been proposed to minimize the hop count and the energy consumption. MFR forwards a packet to the neighbor (node M in Figure 1) that is the farthest from the source in the direction of the destination within the transmission range. GRS selects the closest neighbor (node G in Figure 1) to the destination among neighbors. Since in most cases MFR and GRS provide the same path to the destination [16], we only consider GRS in this paper. When nodes have the ability to control the transmission ranges, the Nearest Forward Progress (NFP) algorithm [14] has been proposed to reduce energy power consumption. NFP chooses the closest neighbor (node N in Figure 1) to the sender within the forward region. Yet another scheme called compass routing [15] is used to select that neighbor (node C in Figure 1),

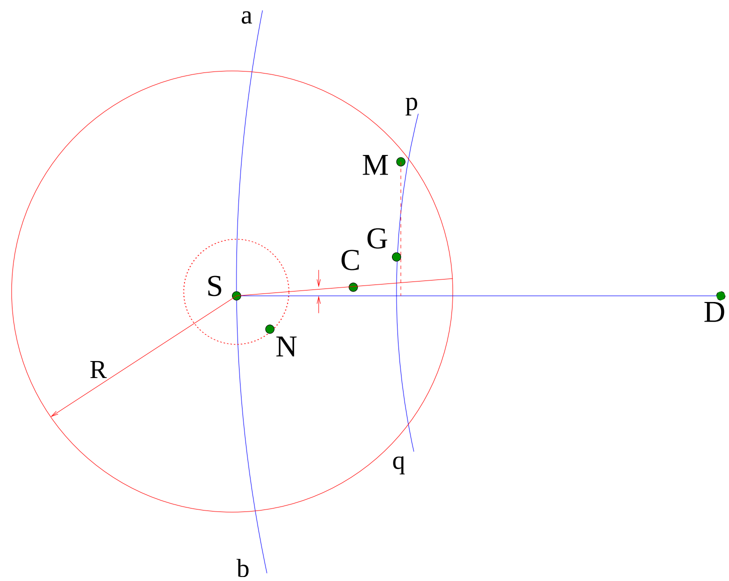

Fig. 1. The examples of forwarding schemes when node S with transmission range R have a packet to send to node D : compass routing(C), GRS(G), MFR(M) and NFP(N)

which has a minimum angle with respect to the line between the sender and the destination.

Most work on geographic routing assumes that location information received from GPS (or other techniques) is perfect [1,16–20]. Effective algorithms in the recovery mode have been addressed in [1,16–19]. Flooding based algorithms have been proposed for finding alternative paths in [16,19]. Face routing (perimeter routing) has also been studied for route recovery in [1,17]. In [20], the authors consider the information discrepancy due to mobility and propose a mobility prediction scheme based on reported location information. However, the location information in [20] is assumed to be measured perfectly by GPS.

In practice, the received location information is not perfect [5,10,21] and the error in the location information degrades the performance of geographic routing [22]. This inaccuracy is caused even when nodes use GPS because of GPS's inherent error in location estimation [6]. This location error could induce *transmission failures* and *backward progress* in the greedy mode. A transmission failure happens when the selected node is out of the transmission range. Backward progress occurs when the chosen node is located farther from the destination than the sender and could cause looping. Loops occur when the selected node is one of the previous senders in the route. Further, it could also result in a local minimum, i.e., there exist solutions but there are no nodes in the forwarding direction. Such failures produce unnecessary transitions from the greedy mode to the recovery mode, which in turn results in an inefficient routing solution [16,17].

In this paper, we study the impact of these location errors on the performance of geographic routing. We further propose a new routing scheme to improve the performance of geographic routing. We focus on the case when the transmission ranges of the nodes are fixed. Using numerical simulations we verify the performance of the proposed algorithm and the robustness of the location error models and model parameters. Unless otherwise stated, the term "location error" means "location error due to measurement" through this paper.

The rest of the paper is organized as follows. In Section 2, we study how location errors in geographic routing arise and evaluate the impact of location errors on the performance of the geographic routing. In Section 3, we propose a new algorithm for geographic routing in an environment with location errors and analyze properties of the algorithm. In Section 4, we use simulation to compare the performance of our scheme with that of known schemes. Section 5 concludes this paper.

2 The Impact of Location Error on Geographic Routing Performance

In this section, we develop a location error model and investigate the impact of location errors on the performance of geographic routing. As mentioned in the introduction, errors in the location information affect the forwarding scheme in the greedy mode, which could cause unnecessary transitions into the recovery mode of the algorithm in order to find an alternative route to the destination. The goal of the greedy mode is to *succeed* in transmitting packets to a neighbor with *forward progress* (i.e., the neighbor is closer to the destination). Thus, it is important to account for both *transmission failures* and *backward progress* to analyze how the location error affects the performance of geographic routing.

2.1 Error Modeling

Location errors occur during the process of estimating the location (via GPS or other techniques) [5,6,10,21]. In GPS, the performance of an estimated location depends on the geometry of satellites, location sensing techniques, radio environments, etc. [5,23,24]. For example, when GPS uses a single frequency, the *root mean square* measurement error is typically 6 m [6]. On the other hand, the typical measurement error is about 3 m for a dual-frequency receiver [6]. The geometry of satellites and location sensing techniques are known parameters to the GPS receiver. Other factors are also estimated and adjusted by the GPS receiver.

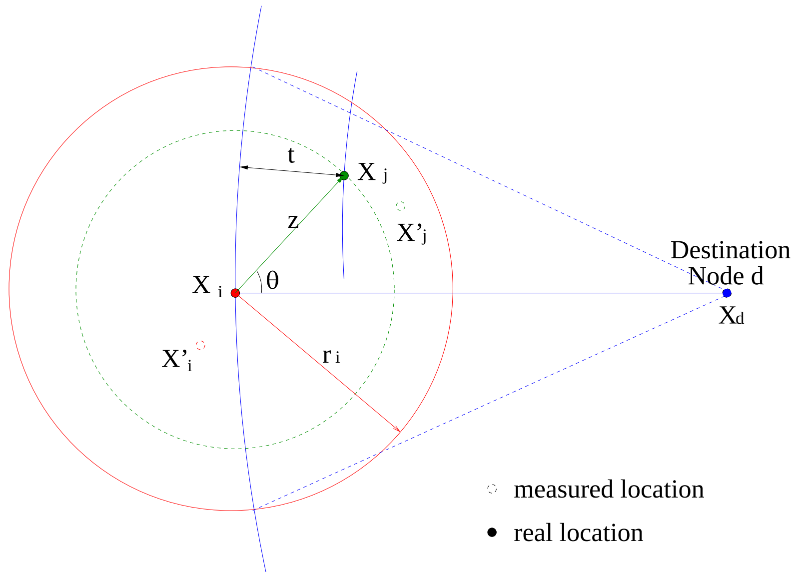

Fig. 2. Location error modeling when node i with the transmission range r_i has a packet to send the destination node d.

In this paper we make the following assumptions. All nodes are equipped with GPS [2] to measure their own locations. These locations are proactively broadcasted. The location errors at different nodes are independent. The location error at each node is modeled by a Gaussian distribution [3] with zero mean and finite standard deviation. The zero-mean assumption implies that, for an given environment, the average of location errors over all nodes is equal to zero, i.e., it follows from the strong law of large numbers [25] that $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} W_k$ is equal to zero almost surely, where W_i represents a measurement error of node i. *However, note that for a given environment, the time average of location errors at a single node could be non-zero.* Therefore, a scheme that uses simple averaging of samples in time at each node will not be able to overcome the errors.

Let X_i be the real position of node i and let X_i' be its measured position. Then X_i can be expressed as $X_i = X_i' + W_i$, where W_i is a Gaussian random vector with zero mean and standard deviation σ_i.

For convenience, we assign node i to be at the origin and destination node d to be on the x-axis, as in Figure 2. Let node j be a neighbor of node i, and let z be the real distance between the two nodes. Since X_i and X_j are independent and Gaussian, the probability density function $f_j(z, \theta)$ that node j is located at (z, θ) is

[2] Our analysis and algorithm are applicable to other location estimation technologies, however in this paper we focus on networks where the nodes are equipped with GPS.

[3] In Section 4, we will also study cases when the errors are not Gaussian and their impact on the routing performance.

$$f_j(z,\theta) = \frac{z}{2\pi\sigma_{ij}^2} \exp\left(-\frac{z^2+\eta_{ij}^2}{2\sigma_{ij}^2}\right) \exp\left(\frac{z\eta_{ij}}{\sigma_{ij}^2}\cos\theta\right),$$

where σ_{ij} is the standard deviation of $X_i - X_j$, θ is an angle of X_j with respect to x-axis, and $\eta_{ij} = \|X_i' - X_j'\|$ as in Figure 2. Hence the probability density function $f(z)$ that the distance between two nodes is z is

$$\begin{aligned} f_j(z) &= \int_0^{2\pi} f_j(z,\theta)d\theta \\ &= \frac{z}{\sigma_{ij}^2} \exp\left(-\frac{z^2+\eta_{ij}^2}{2\sigma_{ij}^2}\right) I_0\left(\frac{z\eta_{ij}}{\sigma_{ij}^2}\right) \text{ for } z \geq 0, \end{aligned}$$

where $I_0(x)$ is the modified Bessel function of the first kind and zero order, which is defined by $I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} \exp(x\cos\theta)\, d\theta$.

2.2 Transmission Failure Probability

A packet transmission failure occurs when a chosen node is out of the transmission range of a sender, as shown in Figure 3. Most work assumes that the transmission range of each node is perfectly circular and identical so that a neighbor is within the transmission range of a node if and only if the node is located within the transmission range of the neighbor. In practice, nodes have imperfect circular transmission patterns [26] and the transmission ranges deviate from the ideal case [20]. Moreover, a network may be composed of heterogeneous nodes that have different transmission ranges. In these cases, even though a node is located in the transmission range of a neighbor, the neighbor may be out of the transmission range of the node. Such a link is an asymmetric communication link. In the case of asymmetric communication links, transmission failures can happen in the presence of location errors even though each node precisely knows its own transmission range and pattern. In a mobile environment, displacements of nodes induced by mobility cause transmission failures that displacement prediction may reduce [20,27]. However, this is still vulnerable to the transmission failure due to location errors in measurement even though the prediction is perfect and the communication links are symmetric. In the case when the transmission range is controllable, the adjusted transmission range of a sender can also fail to transmit a packet to its neighbor that is still within the maximum transmission range of the sender. Here, we study failures caused only by inaccurate location information.

Assume that node i has a packet to transmit and node j is chosen as the next node. We assume that node i calculates the distance between nodes i and j as η_{ij} such that $\eta_{ij} = \|X_i' - X_j'\|$, and sets its transmission range to r_i, which may

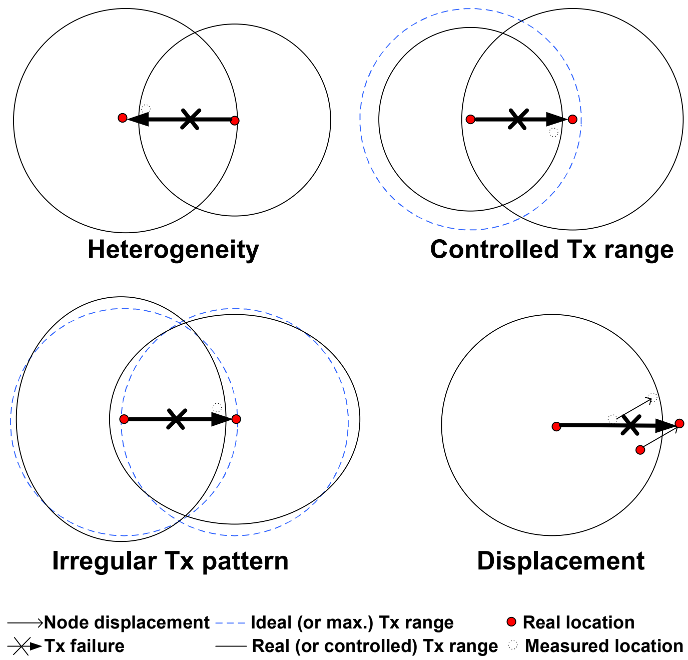

Fig. 3. Examples for transmission failure

be fixed or controllable, in order to forward data to node j. The probability that a packet transmission from node i to node j fails is

$$
\begin{aligned}
&\Pr\{\text{transmission failure at node } j\} \\
&= \Pr\{Z > r_i\} \\
&= \int_{r_i}^{\infty} f(z)dz \\
&= \int_{r_i}^{\infty} \frac{z}{\sigma_{ij}^2} \exp\left(-\frac{z^2+\eta_{ij}^2}{2\sigma_{ij}^2}\right) I_0\left(\frac{z\eta_{ij}}{\sigma_{ij}^2}\right) dz \\
&= \int_{\frac{r_i}{\sigma_{ij}}}^{\infty} r \exp\left(-\frac{1}{2}\left(r^2+\frac{\eta_{ij}^2}{\sigma_{ij}^2}\right)\right) I_0\left(r\frac{\eta_{ij}}{\sigma_{ij}}\right) dr \\
&= Q_1\left(\frac{\eta_{ij}}{\sigma_{ij}}, \frac{r_i}{\sigma_{ij}}\right), \qquad (1)
\end{aligned}
$$

where $Z = \|X_i - X_j\|$, σ_{ij} is the standard deviation of $X_i - X_j$, and $Q_1(a, b)$ is a Marcum's Q function with $m = 1$ defined as in [28].

It follows from Marcum's Q function with $m = 1$ that *the transmission failure probability increases when the standard deviation of location errors σ_{ij} increases.* When σ_{ij} is fixed and the chosen node is closer to the edge of the transmission range, the transmission failure probability increases. In other words, *given an error environment, a longer transmission range reduces the* *transmission failure probability.*

2.3 Backward Progress Probability

Backward progress occurs when a chosen node j is located farther from a destination than the sending node i. Note that there may exist a route to the destination even though there is no neighbor in the forward region. This case is typically called a “local minimum” in the geographic routing literature and cannot be solved by using only the greedy mode, so recovery mode is needed. Assume that node i has a packet to transmit and node j is chosen as the next node. For simplicity, in this subsection, we assume that the destination location X_d in the packet has no error [4].

The probability that the chosen node j, such that $\|X_i' - X_d\| \geq \|X_j' - X_d\|$, is located *behind* (is further away from the destination than node i) the sender i is

$$\begin{aligned}
&\Pr\{\text{backward progress at node } j\}\\
&=\Pr\{(z,\theta)|\|X_i - X_d\| \leq \|X_j - X_d\|\}\\
&=\int_{\{(z,\theta)|\|X_i-X_d\|\leq\|X_j-X_d\|\}} f_j(z,\theta)dzd\theta\\
&=\int_{\{(z,\theta)|\|X_i-X_d\|\leq\|X_j-X_d\|\}} \frac{z}{2\pi\sigma_{ij}^2} \times \exp\left(-\frac{z^2+\eta_{ij}^2}{2\sigma_{ij}^2}\right)\exp\left(\frac{z\eta_{ij}}{\sigma_{ij}^2}\cos\theta\right)dzd\theta,
\end{aligned}$$

where $\eta_{ij} = \|X_i' - X_j'\|$ and σ_{ij} is the standard deviation of $X_i - X_j$.

In general, the integral above does not reduce to a closed form and must be numerically evaluated. However, if r_i is much smaller than the distance between node i and the destination, θ is close to 0, we can approximate equation (2) as follows:

$$\Pr\{\text{backward progress at node } j\} \simeq Q\left(\frac{\eta_{ij}}{\sigma_{ij}}\right), \qquad (2)$$

where $\eta_{ij} = \|X_i' - X_j'\|$ and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{x^2}{2}\right)dx$.

[4] The case where the destination location is in error can be similarly treated, although the equations become more notationally complex. For the simulations, we assume that the location information of all nodes, including the destination, has errors.

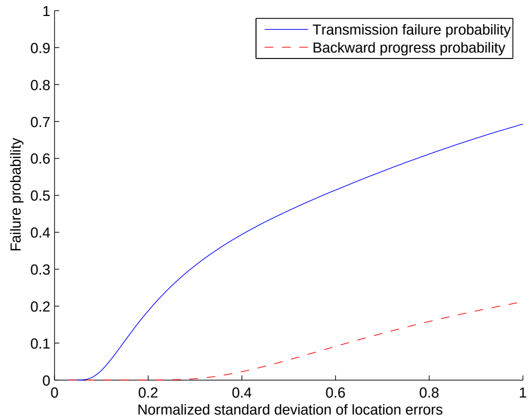

Fig. 4. Failure probabilities versus the standard deviation of location errors when $\eta_{ij} = 0.8r_i$. The standard deviation of location errors is normalized by the transmission range r_i.

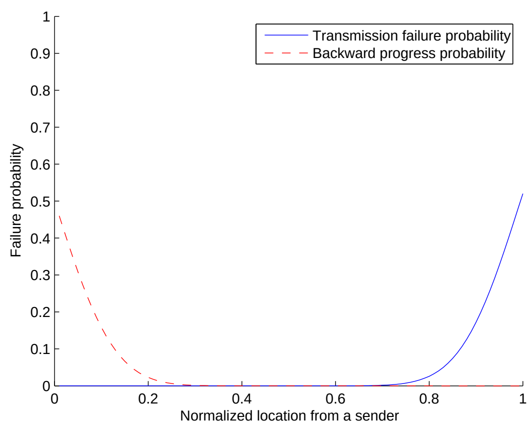

Fig. 5. Failure probabilities versus the location of node j when the standard deviation of location errors is $0.1r_i$. The location of the chosen node is normalized by the transmission range r_i.

It follows from the Q function that *the backward progress probability increases when the standard deviation of location errors* σ_{ij} *increases.* When σ_{ij} is fixed and the chosen node is closer to the sender, the backward progress probability increases.

Figures 4 and 5 show these two probabilities. In Figure 4, we fix the location of node j and show the relationship between the failure probabilities and the standard deviation of the location error. In Figure 5, we fix the location error and show the relationship between the failure probabilities and the distance between node i and node j. Given the location error, when the chosen node is closer to the sender or the edge of the transmission range, the failure probability increases.

All forwarding schemes in geographic routing suffer from the above mentioned failures. GRS (or MFR) selects the closest neighbor to a destination (or the farthest neighbor from the source in the direction of the destination), so the node is more likely close to an edge of a transmission range than any other neighbors. They are susceptible to a transmission failure. NFP chooses a neighbor which is closest to the sender. This scheme is susceptible to backward progress. Compass routing does not consider the distance between the sender and the intermediate node, but cares for only the angle with respect to the line between the sender and the destination. Hence, compass routing is vulnerable to both factors described above. When the number of nodes increases in a given area, a chosen node is closer to the sender or the edge of the transmission range. Therefore, denser nodes can potentially worsen the performance (as will also be shown via numerical studies in Section 4).

3 Geographic routing scheme with location errors

In this section we propose a new geographic routing scheme that can mitigate the impact of location errors. Since MFR is similar to GRS in most cases [4], we focus on improving GRS. For ease of illustration, from here on we assume that the transmission range is fixed. However, it should be readily apparent that the methodology can be extended to the case when the transmission range is controllable. Since each node measures its location and estimates its own error characteristic, we attach an error information field in a message for geographic routing and announce the statistical characteristics of the location error to neighbors with location information.

3.1 Objective Function

Let nodes i and j be located at X_i and X_j, respectively. We assume that X_i' and X_j' are the measured locations of nodes i and j, respectively. As before, these are expressed as $X_i = X_i' + W_i$ and $X_j = X_j' + W_j$, where W_i and W_j are Gaussian random vectors with zero means and standard deviations σ_i and σ_j, respectively. Then the real position of node j with respect to node i is a Gaussian random vector with mean $X_j' - X_i'$ and standard deviation $\sigma_{ij} = \sqrt{\sigma_i^2 + \sigma_j^2}$. Hence, the probability that node j is located within u_j from $X_j' - X_i'$ is $\Pr\{\|X - (X_i' - X_j')\| \leq u_j\} = 1 - \exp\left(-\frac{u_j^2}{2\sigma_{ij}^2}\right)$, where X is the real position of node j with respect to node i.

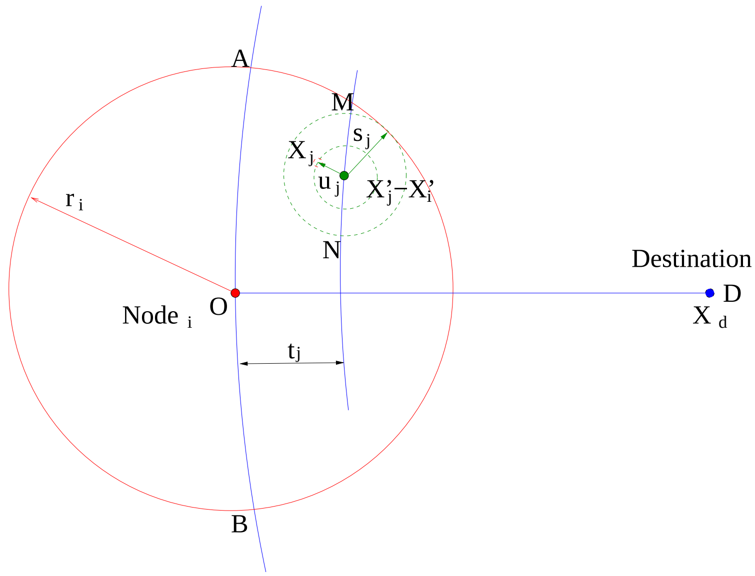

Fig. 6. The expected progress of node j with respect to node i that has the transmission range r_i and is located at O.

Fix a sender i and the destination d. We define *measured progress* to node j to be $t_j = \|X_d\| - \|X_d - (X_j' - X_i')\|$ and the *measured margin* from the boundary to be $s_j = r_i - \|X_i' - X_j'\|$, where r_i is the transmission range of node i and X_d is the destination position with respect to node i. Then we can express GRS in this (location) error-free environment as follows. If $\|X_i' - X_d\| \leq r_i$, choose node d. If $\|X_i' - X_d\| > r_i$, choose node k such that

$$\begin{aligned} k &= \arg\max_{j \in N_i}\ t_j \\ &\text{subject to } \|X_i' - X_j'\|\ \leq r_i\ \forall j \in N_i, \end{aligned} \tag{3}$$

where N_i is the set of neighbors of node i.

When we have location errors, instead of using the measured progress t_j in (3), we propose using a different metric to determine which neighbor to forward the packets.

We define *true progress* to node j to be $\tau_j = \|X_d\| - \|X_d - (X_j - X_i')\|$ when node j is actually located at position X_j. Then, $\tau_j(X_j)$ is a random variable with probability density function $f_j(X_j)$. Since the probability density function, $f_j(X_j)$, that node j is located at X_j is circularly symmetric with respect to the point $X_j' - X_i'$, we consider area A_j such that $A_j = \{X \in R^2 | \|X - (X_j' - X_i')\| \leq u_j\}$ and the expected progress of node j over A_j as follows.

$$E\{\tau_j(X_j) 1_{A_j}\} = \int_{A_j} \tau_j(X_j) f_j(X_j) dX. \tag{4}$$

If $u_j = \infty$, (4) becomes the expected progress over the entire domain of X. However, if $u_j > s_j$, node j may be out of the transmission range of node i. If $u_j > t_j$, node j may be located behind node i. In order to find a neighbor to be able to successfully transmit to and result in forward progress, we let $u_j = \min\{s_j, t_j\}$.

For simplicity, we use an approximation of (4). For realistically sized wireless networks, in most cases $\|X_d\|$ is much greater than r_i. Then, arc $\widehat{AB}$ and arc $\widehat{MN}$ in Figure 6 are nearly straight. Since $f_j(z)$ is circularly symmetric, we can simplify (4) as follows:

$$E\{\tau_j(X_j)1_{A_j}\} \simeq t_j \int_{A_j} f_j(X_j)dX_j,$$

where $A_j = \{X \in R^2 | \|X - (X_j' - X_i')\| \leq u_j\}$ for $u = \min\{s_j, t_j\}$.

Hence, we define E_j as the *revenue* of node j as follows.

$$E_j \overset{\triangle}{=} t_j F_j(u_j), \tag{5}$$

where

$$\begin{aligned} F_j(u_j) &= Pr\{\|X - (X_j' - X_i')\| \leq u_j\} \\ &= 1 - \exp\left(-\frac{u_j^2}{2\sigma_{ij}^2}\right) \end{aligned}$$

for $u_j = \min\{s_j, t_j\}$.

Based on the calculated revenue of each node, node i selects the next node which has a Maximum Expectation within transmission Range r_i (MER). Our MER algorithm for forwarding packets is given as follows. If $\|X_i' - X_d\| \leq r_i - \delta_i$, where $0 \leq \delta_i < r_i$, choose node d. If $\|X_i' - X_d\| > r_i - \delta_i$, choose node k such that

$$\begin{aligned} &k = \arg\max_{j \in N} E_j \\ &\text{subject to } \|X_i' - X_j'\| \ \leq r_i \ \forall j \in N_i, \end{aligned}$$

where N_i is the set of node i's neighbor nodes stored in the node i's DB. Here, δ_i is a function of the location errors and could be a tuning parameter for specific implementations. δ_i can be simply the standard deviation of location errors or the distance from the maximizer of the objective function to the transmission

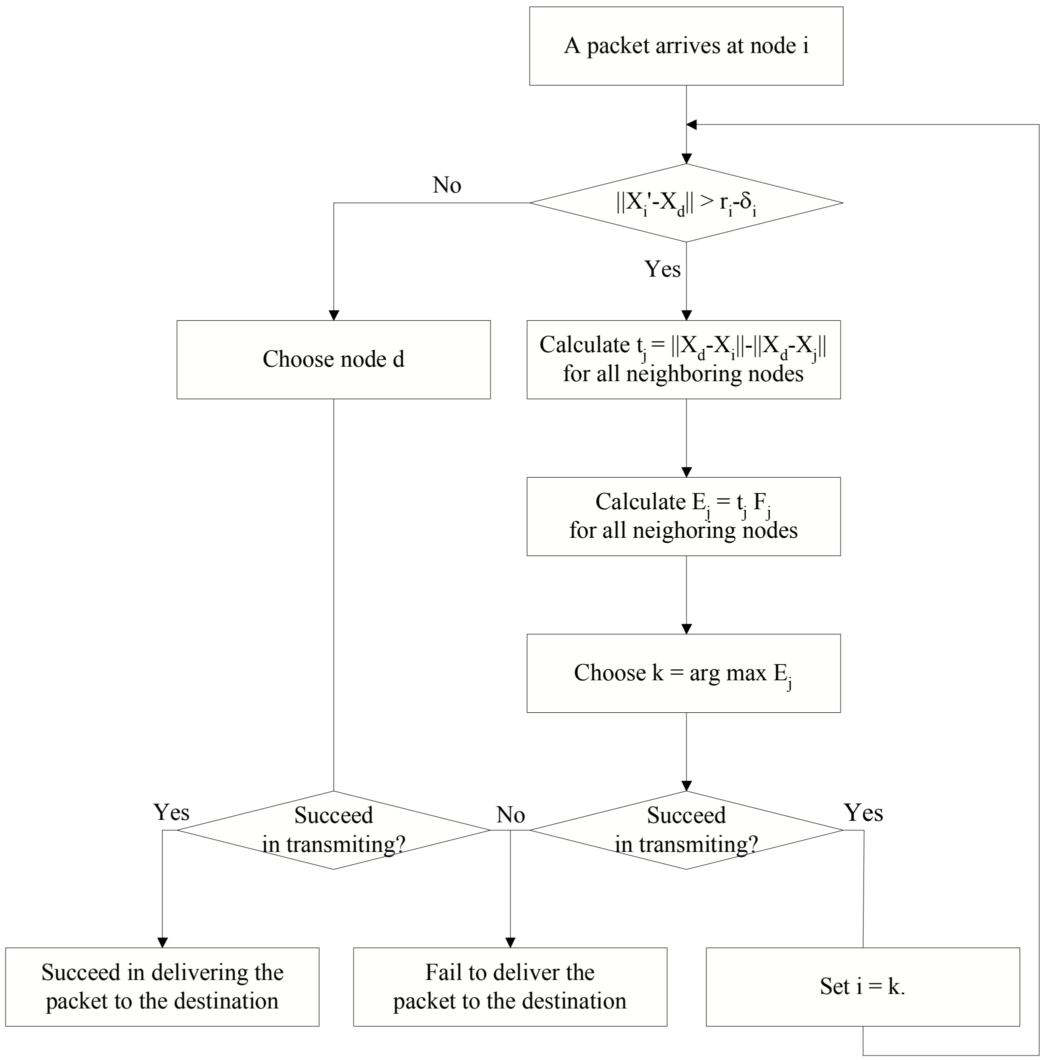

Fig. 7. Block diagram for MER

range edge. In [20], the authors proposed a scheme to improve the forwarding performance as follows: if the destination node exists in the neighbor list, the sender forwards the packet to the destination without any effort. However, since in the location error environment the destination may be located out of the transmission range, it is imperative to consider the parameter δ_i to increase the transmission success rate. In Figure 7, we summarize our algorithm. For the simulations in Section 4, we also add functionalities to the protocol to detect loops and local minima.

3.2 Properties of the Algorithm

We now analyze the MER algorithm. For simplicity, we assume that a selected node is located on the line between the sender and the destination since the node on the line ($\overline{OD}$ in Figure 6) is the most likely chosen among neighbors that have the same measured progress. Let the transmission range of the sender r_i be 1. We let σ denote the standard deviation for the overall location error between the sender and the intermediate node. Then, (5) becomes

$$\begin{aligned} E(t) &= tF(u) \\ &= t\left(1 - \exp\left(-\frac{u^2}{2\sigma^2}\right)\right), \end{aligned} \tag{6}$$

where $t \in (0, 1)$ is the position of the selected node and $u = \min\{t, 1-t\}$.

We define an *effective search range* to be an area from the source to the most likely position chosen by the algorithm and $t_{\max}$ to be a maximizer of $E(t)$ such that $E(t_{\max}) \geq E(t)$ for $t \in (0, 1)$.

Property 1 *Let $\sigma \ll 1$, then MER is identical to GRS within an effective search range for a next node. The range is reduced by δ such that $E'(1-\delta) = 0$, where $E'(t)$ is the derivative of $E(t)$ and the maximizer of $E(t)$, $t_{\max}$, is a decreasing function of σ.*

Proof: Since the exponential term in (6), $\exp\left(-\frac{u^2}{2\sigma^2}\right)$, goes to zero faster than σ^2 when σ goes to zero, for $\sigma \ll 1$, we have

$$E(t) = t + o(\sigma^2). \tag{7}$$

Note that (7) is equivalent to the objective function of GRS (3). Since $E^{'}(t) = tF(t)$ is always an increasing function of $t \in (0, 0.5)$, we need to consider $t \in (0.5, 1)$ to check the maximum of $E(t)$. The derivative of the objective function $E^{'}(t)$ for $t \in (0.5, 1)$ is

$$\begin{aligned} E^{'}(t) &= F(1-t) - tf(1-t) \\ &< 1 - \frac{1}{2}f(1-t). \end{aligned}$$

Since $\sigma \ll 1$ from the assumption above and $f(t)$ has a maximum $\frac{1}{\sigma}\exp\left(-\frac{1}{2}\right)$ at $t = \sigma$, $E^{'}(t) \ll -1$. The values of $E(t)$ precipitates from a maximum value to zero, and the effective search range reduces to $t_{\max}$ such that $E^{'}(t_{\max}) = 0$. In other words, the effective search range is reduced by δ such that $\delta = 1 - t_{\max}$. Since $\sigma^2 > 0$ and

$$\begin{aligned} E^{'}(t) &= F(1-t) - tf(1-t) \\ &= F(1-t) + (1-t)f(1-t) - f(1-t) \end{aligned}$$

$$= \frac{(1-t_{\max})^2}{2\sigma^2} - \frac{(1-t_{\max})^4}{8\sigma^4} + \frac{(1-t_{\max})^2}{\sigma^2} - \frac{(1-t_{\max})^4}{2\sigma^4}$$
$$+(1-t_{\max}) - \frac{(1-t_{\max})^3}{2\sigma^2} + o\left(\frac{(1-t_{\max})^4}{\sigma^4}\right),$$

we have

$$\sigma^2 = \frac{(1-t_{max})\left(\sqrt{(4-t_{max})^2 + 10(1-t_{max}))}\right)}{4} \times \frac{(1-t_{max})(4-t_{max})}{4} + o((1-t_{max})^3), \tag{8}$$

where $t_{\max}$ is the maximizer of (6). From (8) above, σ is a monotonically decreasing function of $t_{\max}$ for $t_{\max} \in (0.5, 1)$. Hence $t_{\max}$ is a monotonic decreasing function of σ. ■

Property 2 *There exists a threshold σ_{th} such that the decrement of σ results in an increment of $t_{\max}$ for $\sigma \geq \sigma_{th}$, but $t_{\max}$ does not depends on σ for $\sigma \leq \sigma_{th}$. The threshold σ_{th} is numerically 0.315.*

Proof: $tF(t)$ is an increasing function for $t \in [0, 1]$. $tF(1-t)$ has a maximizer $t_{\max} \in [0, 1]$ and a minimum value 0 at $t = 0$ and 1. $tF(1-t)$ increases for $t \in [0, t_{\max}]$ and decreases for $t \in [t_{\max}, 1]$ monotonically. It follows from (6) that if $t_{\max} \in [0, 0.5]$, $E(t)$ has a maximum at t = 0.5 and if $t_{\max} \in [0.5, 1)$, $E(t)$ has a maximum at t = $t_{\max}$. σ_{th} is σ such that $E'(\frac{1}{2}) = 0$. The numerical value of σ_{th} is 0.315. ■

Figures 8, 9 and 10 depict how the objective function of the proposed algorithm (MER) works. For these figures, we assume that the selected node is located on $\overline{OD}$ in Figure 6. Property 1 shows that MER works identically to GRS except in the outskirts of the transmission range, as shown in Figure 8. However, MER may require more hops to route packets from source to destination compared with the case of perfect location environment, since the position of the most likely chosen node, $t_{\max}$, decreases due to location error, as in Figures 8 and 10.

Property 2 gives us some insights on geographic routing in an environment with location errors. If the standard deviation of the location error is greater than some threshold, i.e. $\sigma \geq \sigma_{th}$, the maximizer of (6), $t_{\max}$, decreases when σ increases. If $\sigma \leq \sigma_{th}$, $t_{\max}$ does not depend on σ, as in Figure 10 and only the maximum revenue decreases as in Figure 9. $t_{\max}$ is fixed at 0.5 after $\sigma = 0.315$. In this region, increasing σ does not affect the selected node, but decreases the revenue. This means that for a standard deviation of error larger than σ_{th}, taking error estimates into account does not help in improving the performance

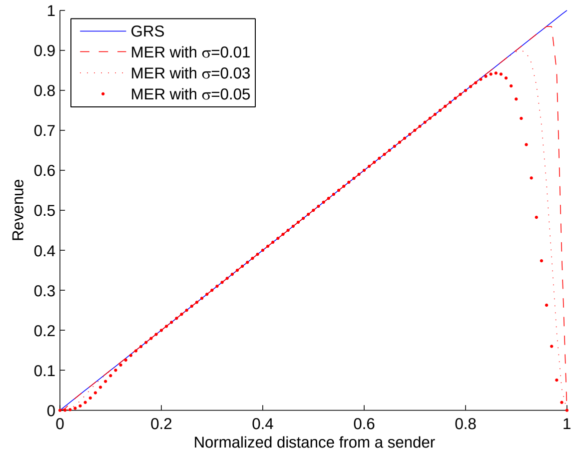

Fig. 8. The revenue versus the distance from the sender when the transmission range is 1.

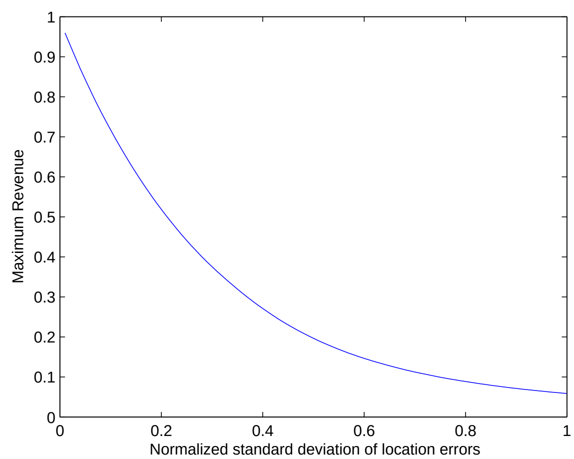

Fig. 9. The maximum revenue versus the standard deviation normalized by the transmission range.

of geographic routing. The algorithm is simply reduced to selecting the node that is closest to the middle point of the forward region. Hence, σ_{th} becomes the critical point that determines the utility of the algorithm. For example, when each node is equipped with a GPS receiver that has a standard deviation of 3 m of the location error, Bluetooth [29] which has a transmission range of 10 m cannot use geographic routing. However, a geographic routing scheme that incorporates error information (such as MER) is applicable in the case of IEEE 802.11 with a nominal transmission range 250 m.

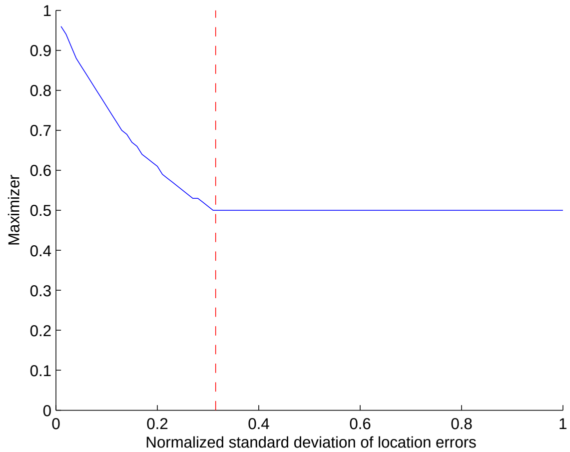

Fig. 10. The maximizer versus the standard deviation when the transmission range is 1.

4 Simulation Results

In this section, we use numerical simulations to verify the performance of the proposed algorithm, MER, in a mobile environment as in [11,18,20]. We use the following random way point (RWP) mobility model. Each node chooses a destination in a given area and moves at a constant speed, which is uniformly chosen between 0 and 50 m/s. The node stays for a pause time, which is uniformly distributed between 0 seconds and 30 seconds. Each node broadcasts its own location periodically and proactively. To avoid collisions, the interval of these broadcasts is uniformly chosen from 1 second to 3 seconds, as in [18]. We also set the neighbor timeout interval at 9 seconds. Since location information errors in the mobile environment can be categorized into two classes, displacement due to mobility and location errors in measurement, we use two cases of simulations in the mobile environment in order to study the impact of location errors on geographic routing. First, we assume that location information errors exist only in measurement. The displacements of the nodes are assumed to be perfectly estimated in the mobile environment. In the case when there is no location error due to measurement, the authors in [20] study the performance of geographic routing due to errors in displacement using extensive simulations with several mobility models. In our simulation environment, we compare the performance of MER versus GRS when the parameters of the location error model are known in Subsection 4.1. We further demonstrate the robustness of the algorithm to changing parameters in the error model in Subsection 4.2. Finally, we study the performance of MER in a noisy environment due to inaccurate measurement and mobility in Subsection 4.3. In order to reduce the impact of mobility on the routing performance, we adopt a displacement estimation method used in [20,27]. Through our simulations we assume that there is no time delay to route data from a source to destination.

In our simulations we compare two different schemes: GRS and MER. We use the performance of GRS with perfect location information as an upper bound on the routing performance of all schemes in the presence of location errors.

To compare the fundamental performance in Section 4.1, we use two measures: delivery success rate and the number of attempts. In the other sections we compare delivery success rates. For the delivery success rate, we do not use retransmissions or algorithms to find alternative routes when packet forwarding failures occur. Packet delivery is said to succeed only if the packet is delivered to the destination by the schemes. For the number of attempts, we employ an algorithm to find another node when packet forwarding fails. When the chosen node is not available, a sender immediately finds the next available node without retransmitting to the same node. We define this as a reattempt. When there is no available node in the forward area, the routine stops finding an alternative node. In the case when there is no reattempt, the total number of attempts is equivalent to the number of hops. The reattempts require extra energy as well as time delay so that the throughput will be degraded. Since the time delay per reattempt is protocol-specific (such as the number of retransmission before declaring the chosen node to be unavailable), hence we focus only on the number of reattempts. We count the number of attempts and reattempts until a hundred packets are successfully delivered to their own destinations.

In MER, we use the distance from the maximizer of the objective function to the transmission range edge as a tuning parameter δ_i. In order to study how system parameters affect the performance of geographic routing in the presence of location errors, for our numerical results, we focus on the case when the wireless environment is homogeneous across all nodes. In practice, the wireless environment at each node in the network may be different. For instance, in the case of GPS, each node could receive a different number of satellite signals due to obstacles. Our algorithm described in Section 3 can handle such a heterogeneous environment, and we show that our algorithm works well in the heterogeneous environment by simulations (Section 4.1).

We run 20 simulations with different random seeds for each scenario and average the results.

4.1 The Performance of MER when the Distribution of the Location Error is known

We investigate three scenarios, as illustrated in Table 1. In the first scenario, we deploy 100 nodes in an area of 1000×1000 m^2. The standard deviation of the location error for each node is 10 m. We vary the transmission range

Table 1
Scenarios for simulations: A, N, R and σ represent a deployed area, the number of deployed nodes, the transmission range of nodes and the standard deviation of location errors, respectively

Scenario	A (m^2)	N	R (m)	σ (m)
1	1000×1000	100	25 $\sim$ 500	10 (2% $\sim$ 40%)
2	1000×1000	100	250	3 $\sim$ 50 (1.2% $\sim$ 20%)
3	1000×1000	25 $\sim$ 500	250	5 (2%)

from 25 m to 500 m. Figure 11 shows that the transmission success rate of MER is close to that of GRS with perfect location information. However, the performance of GRS degrades severely in the presence of location errors. Table 2 provides a comparison between MER and GRS in terms of the total number of attempts required for 100 successful transmissions. Table 2 shows that MER with location errors needs additional attempts (or hops) to reach its destination when compared with GRS with perfect location information. The reason is that the location errors cause MER to reduce its effective search range when forwarding packets to its neighbors, as shown by Property 1. Hence, MER requires additional attempts to deliver packets from source to destination. When we compare the total number of attempts of MER and GRS in the presence of location errors, they are similar so that the reattempt routine seems to dilute the benefits of MER. However, GRS requires many more reattempts than our algorithm. The reattempts may induce time delay in the system. When the transmission ranges increase, the number of attempts of GRS with location errors decreases since the failure probability decreases, as in (1).

In the second scenario, the transmission range is fixed at 250 m, which is the nominal transmission range of IEEE 802.11, and we vary the standard deviation of the location error from 3 m (1.2 %) to 50 m (20 %). As expected, Figure 12 shows that MER performs much better than GRS when there are location errors. The performance of GRS starts to degrade when the standard deviation is above 3 m (1.2 %). However, the performance of MER does not decrease significantly. As the error increases, the effective search range in MER is further reduced. This reduction in the effective search range decreases the number of neighbor nodes to be selected and degrades the performance. Similarly to Scenario 1, MER with location errors requires additional attempts to route packets when compared to the case with perfect information, as shown in Table 3. When we compare GRS with MER, GRS needs more reattempts than our algorithm in the erroneous environment. Since MER and GRS tend to choose the closest node to the transmission range, they are less susceptible to backward progress. As analyzed in Section 2, backward progress rarely happens, however, backward progress increases with larger location errors.

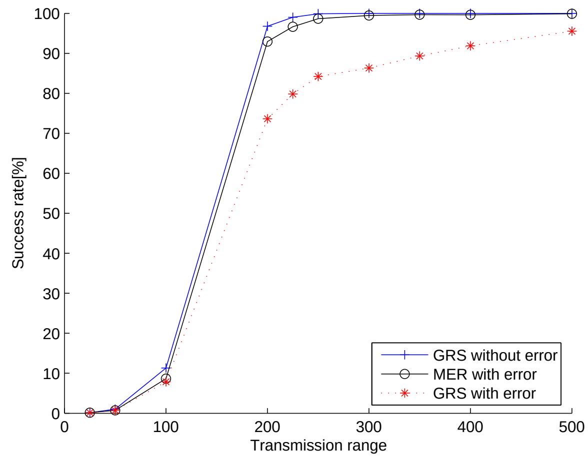

Fig. 11. The performance comparison of the forwarding schemes in Scenario 1.

In the third scenario, we vary the number of deployed nodes from 25 to 500. We fix the transmission range at 250 m and the standard deviation of the location error at 5 m. Figure 13 shows that the performance MER is not affected by the number of nodes while the performance of GRS is. The larger density reduces the distance between two adjacent nodes. This reduction in distance means that the selected node is closer to the edge of the transmission range. Hence, transmission failure is more likely to happen in GRS. Similarly to Scenario 1, MER with location errors needs additional attempts (Table 4) compared to the case without location errors, however the success rate does not decrease when the node density of the network is increased. Note that in Table 4, unlike previous scenarios, the ratio of the additional attempts for MER does not change significantly as the number of nodes increases, since the maximizer of the objective function depends only on the ratio of the location error to the transmission range. In the case of GRS with errors, the total number of attempts decreases as the number of nodes, N, increases between 100 and 200. However, after that region, the total number of attempts increases due to the increment of the failure probability as the number of nodes increases.

Depending on RF environments, the location errors in the same system may be different. We use the same settings as Scenario 3 except that now there exits a shadowing area ($250 \times 250\ m^2$) in which nodes have twice the location errors of the other areas. Figure 14 shows that our algorithm still works well in such a heterogeneous environment.

4.2 Robustness to Estimation Error

In Section 2, we modeled the location error by a Gaussian error distribution. In practice, the error may not follow a Gaussian distribution and/or the parame-

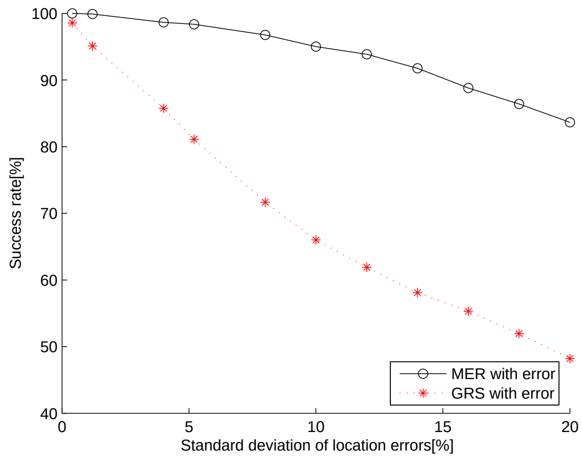

Fig. 12. The performance comparison of the forwarding schemes in Scenario 2.

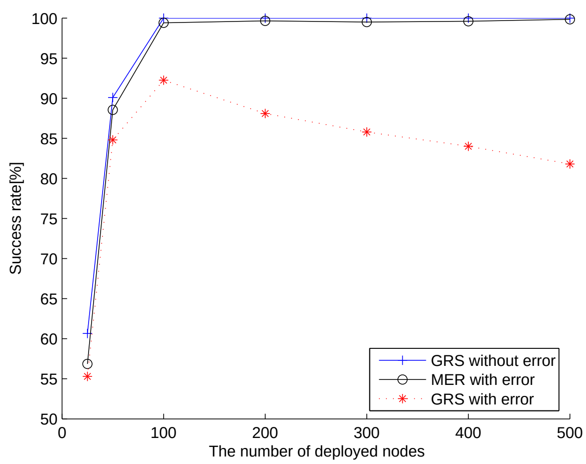

Fig. 13. The performance comparison of the forwarding schemes in Scenario 3.

ters of the model may be incorrect. In this subsection, we study the robustness of MER with respect to these two kinds of modeling errors: the distribution function and parameter error.

In Figure 15, we simulate three different location error models: uniformly distributed error, exponentially distributed error, and Gaussian error. However, the MER algorithm always assumes a Gaussian model. The transmission range of each node is 250 m and the standard deviation of the location error is 5 m. The simulation results show that MER is quite robust to difference in error distribution functions.

In Figure 16, the underlying error model is also Gaussian. However the parameter used by MER is different from the true parameter of the underlying model. In the simulation the transmission range of each node is 250 m and the standard deviation of the location error is assumed to be 5 m. However,

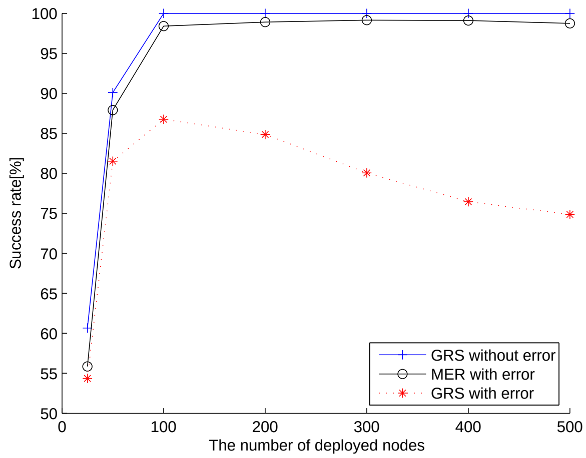

Fig. 14. The performance comparison of the forwarding schemes in Scenario 3 with a heterogeneous RF environment.

Table 2
The comparison of the number of attempts in Scenario 1: (a) the average number of total attempts to deliver packets (b) the average number of reattempts to find alternative nodes (c) the average number of times backward progress is made (d) $\frac{\text{(a) of MER with error}}{\text{(a) of GRS with no error}}(\%)$

$r_i(m)$	GRS						MER			
	No error			Error			Error			
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(d)
300	238.15	0.0	0.0	260.00	17.85	0.0	255.70	1.20	0.0	107.37
350	208.50	0.0	0.0	225.65	14.75	0.0	219.05	0.60	0.0	105.06
400	184.95	0.0	0.0	195.95	9.40	0.0	193.45	0.50	0.0	104.60
500	155.45	0.0	0.0	162.05	5.70	0.0	159.85	0.10	0.0	102.83

the actual standard deviation of the location error is varied from 0 m from 10 m. Figure 16 illustrates that MER is robust to the estimation error and outperforms GRS.

4.3 Noisy Mobile Environment

In this subsection, we compare the performance of MER and GRS in the presence of location errors due to inaccurate measurement and mobility. When each node chooses the next node, we use the predicted positions of neighbors at the transmission time similar to [20,27] in order to improve the routing performance. The authors in [20,27] predict the neighbor positions at the transmission time by using two positions reported at two recent times as fol-

Table 3
The comparison of the number of attempts in Scenario 2: (a) the average number of total attempts to deliver packets (b) the average number of reattempts to find alternative nodes (c) the average number of times backward progress is made (d) $\frac{\text{(a) of MER with error}}{\text{(a) of GRS with no error}}(\%)$

$\sigma(m)$	GRS						MER			
	No error			Error			Error			
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(d)
3	289.75	0.0	0.0	295.95	5.30	0.00	295.45	0.10	0.00	101.97
5	289.75	0.0	0.0	300.35	8.55	0.00	300.10	0.65	0.00	103.57
7	289.75	0.0	0.0	303.05	11.15	0.05	304.35	1.20	0.05	105.04
10	289.75	0.0	0.0	310.60	17.05	0.10	310.35	1.35	0.10	107.11

Table 4
The comparison of the number of attempts in Scenario 3: (a) the average number of total attempts to deliver packets (b) the average number of reattempts to find alternative nodes (c) the average number of times backward progress is made (d) $\frac{\text{(a) of MER with error}}{\text{(a) of GRS with no error}}(\%)$

N	GRS						MER			
	No error			Error			Error			
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(d)
100	289.75	0.0	0.0	300.35	8.55	0.0	300.10	0.65	0.0	103.57
200	275.95	0.0	0.0	291.55	13.75	0.0	285.90	0.45	0.0	103.61
400	271.05	0.0	0.0	293.15	19.75	0.0	281.65	0.45	0.0	103.91
500	268.95	0.0	0.0	294.60	23.45	0.0	279.85	0.25	0.0	104.05

lows.

$$X^{(0)} = X^{(1)} + \frac{X^{(1)} - X^{(2)}}{t^{(1)} - t^{(2)}} \times (t^{(0)} - t^{(1)}), \tag{9}$$

where $X^{(0)}$ is the predicted position at the current time $t^{(0)}$, $X^{(1)}$ is the reported location at the first recent time $t^{(1)}$, and $X^{(2)}$ is the reported location at the second recent time $t^{(2)}$. The method improves the routing performance by reducing the mobility error when the reported location information is assumed to be perfect. However, this method will accumulate measurement and prediction errors when reported location information is noisy. Hence, we use the instant velocity, which is available to GPS equipped nodes, since the velocity measured by GPS is considered to be very accurate [6]. The instant velocity is

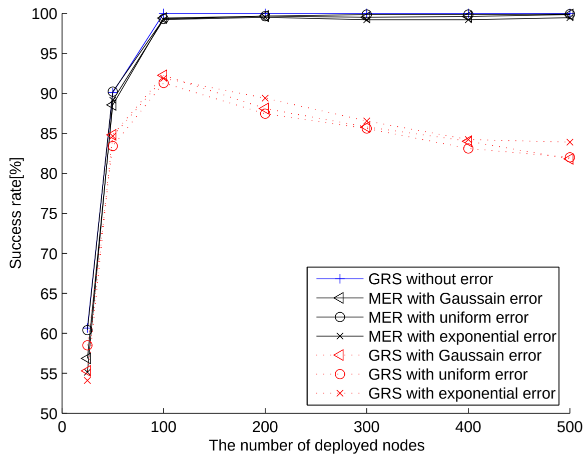

Fig. 15. The performance of forwarding schemes versus the number of nodes with different error distributions and the fixed transmission range.

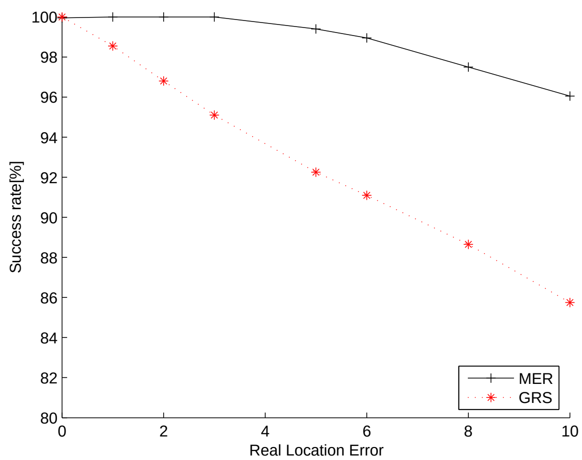

Fig. 16. The performance of forwarding schemes versus the parameter error in estimation with the fixed number of nodes and the fixed transmission range.

announced with location information. Each node predicts neighbors' positions when forwarding a packet as follows.

$$X^{(0)} = X^{(1)} + v^{(1)} \times (t^{(0)} - t^{(1)}), \tag{10}$$

where $X^{(0)}$ is the predicted position at current time $t^{(0)}$, $X^{(1)}$ is the reported location at the first recent time $t^{(1)}$, and $v^{(1)}$ is the reported velocity at $t^{(1)}$.

Figure 17 shows the simulation results when the location error at each node has a Gaussian distribution with standard deviation 5 m and the transmission range is fixed at 250 m. GRS with the displacement prediction performs better than GRS without the displacement prediction, as shown in [20]. MER is slightly affected by mobility but outperforms GRS in all cases. As shown in

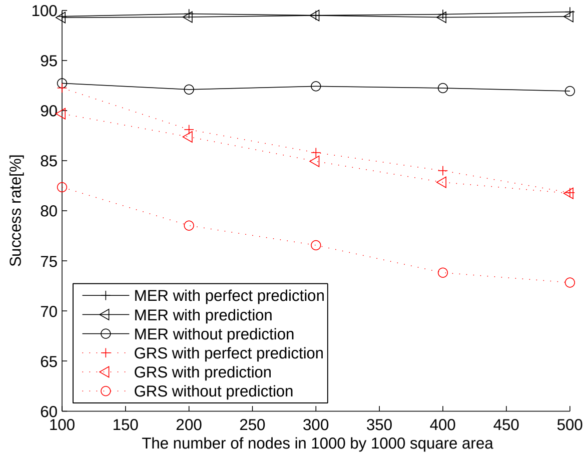

Fig. 17. The performance of forwarding schemes versus the number of nodes with Gaussian location errors and the RWP mobility model.

[11], the performance degradation from mobility can be further improved by reducing time discrepancy. However, the measurement error of $X^{(1)}$ in (10) does not decrease, as the time discrepancy reduces. The reduction of the time discrepancy also does not affect the location error of a sender. Hence, any effort to reduce the time discrepancy from mobility cannot help mitigate the impact of measurement errors on geographic routing.

4.4 Discussion

In order to solve the fundamental problem of geographic routing with location errors, in this paper we do not use a protocol specific solution to help mitigate errors. However, such a protocol-specific solution could in certain cases help combat location errors [30]. For example, in the case when nodes are static and have fixed transmission ranges, the transmission failures caused by asymmetric communication links can be reduced by a three-way handshake protocol when nodes join the network. However, the three-way communication does not alleviate backward progress. Even in a static wireless network, such a protocol cannot avoid transmission failures when the transmission range is controllable in the presence of location errors. Further, the solution is not suitable in a dense mobile wireless network, where frequent topology changes may take place. Moreover, in contrast to our proactive announcement (one-way communication) of location information, the three-way communication results in $2n(n-1)$ overhead messages per time interval in a transmission range, where n represents the number of nodes within a transmission range. The excessive overhead messages may in fact worsen network performance such as throughput. For the above mentioned reasons, in this paper, we do not focus on protocol-specific solutions to alleviate location errors. However, such approaches can be potentially used in conjunction with our method on a case by case basis.

Another feature of our approach is to provide an understanding of the intrinsic performance achievable using geographic routing with location errors. This serves to provide design guidelines for implementers to choose equipment that provides an appropriate level of accuracy.

5 Conclusions

In this paper, we consider the impact of location errors on geographic routing in multi-hop wireless networks. We have shown that location errors can significantly impact the performance of geographic routing. The degradation in the routing performance depends on the transmission range of the sender, error characteristics of the sender and its neighbors, and the deployed density of nodes. We have proposed a new algorithm called MER in order to mitigate the effect of noisy location information by explicitly considering the error probability when making routing decisions. In doing so we find that our algorithm performs quite well, in many cases, even approximating the performance of geographic routing without location errors. We have also used simulations to show that MER is robust to different location error models and errors in model parameters.

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