Optimal Anycast Technique for Delay-Sensitive Energy-Constrained Asynchronous Sensor Networks

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Abstract

In wireless sensor networks, asynchronous sleep-wake scheduling protocols can be used to significantly reduce energy consumption without incurring the communication overhead for clock synchronization needed for synchronous sleep-wake scheduling protocols. However, these savings could come at a significant cost in delay performance. Recently, researchers have attempted to exploit the inherent broadcast nature of the wireless medium to reduce this delay with virtually no additional energy cost. These schemes are called "anycasting," where each sensor node forwards the packet to the first node that wakes up among a set of candidate next-hop nodes. In this paper, we develop a delay-optimal anycasting scheme under periodic sleep-wake patterns. Our solution is computationally simple and fully distributed. Further, we show that periodic sleep-wake patterns result in the smallest delay among all wake-up patterns under given energy constraints. Simulation results illustrate the benefit of our proposed schemes over the state-of-the art.

Index Terms

Anycast, Sleep-wake scheduling, Sensor network, Energy-efficiency, Delay, Periodic wake-up process

I. Introduction

The most efficient method to save energy in wireless sensor networks (WSNs) is to put nodes to sleep when there is no need to relay or transmit packets. Such mechanisms are called *sleep-wake scheduling* and have been used to dramatically reduce energy consumption in energy-constrained WSNs. However, it is well known that sleep-wake scheduling can significantly increase the packet-delivery delay because, at each hop, an event-reporting packet has to wait for its next-hop node to wake up. Such additional delays can be detrimental to delay-sensitive applications, such as Tsunami/fire detection, environmental monitoring, security surveillance, etc. In this paper, we study how to improve this tradeoff between energy-savings and delay, by using a technique called "*anycasting*" (to be described later) that exploits the broadcast nature of the wireless medium.

In the literature, many synchronous sleep-wake scheduling protocols have been proposed [1]–[5]. In these protocols, sensor nodes periodically exchange synchronization messages with neighboring nodes. However, this message exchange inevitably incurs additional communication overhead, and consumes a considerable amount of energy. In this paper, we focus on asynchronous sleep-wake scheduling, where nodes do not synchronize their clocks with other nodes and thus wake up independently [6]–[8]. Asynchronous sleep-wake scheduling is simpler to implement, and it does not consume energy required for synchronizing sleep-wake schedules across the network. However, because nodes do not know the wake-up schedules of other nodes, they have to estimate the wake-up schedule, which can result in additional delays that could detrimental to delay-sensitive applications.

Recently, *anycast packet-forwarding schemes* have been shown to substantially reduce the one-hop delay under asynchronous sleep-wake scheduling [9]–[18]. Note that in traditional packet-forwarding schemes, nodes forward packets to their designated next-hop nodes. In contrast, in anycast-based forwarding schemes, nodes maintain multiple candidates of next-hop nodes and forward packets to the *first* candidate node that wakes up. Hence, an anycast forwarding scheme can substantially reduce the one-hop delay over traditional schemes, especially when nodes are densely deployed, as is the case for many WSN applications. (See the example in Section I and Fig. 1 of [18] that illustrates the advantage of anycasting over traditional schemes.) However, the reduction in the one-hop delay may not necessarily lead to a reduction in the expected end-to-end delay experienced by a packet because the first candidate node that wakes up may not have a small expected end-to-end delay to the sink. Hence, the anycast forwarding policy (with which nodes decide whether or not to forward a packet to an awake node) needs to be carefully designed.

Exiting solutions that exploit path diversity attempt to address this issue by dealing with some local metrics. The anycast protocols in [9]–[11] let each node use the geographical distance from each neighboring node to the sink node to prioritize the forwarding decision to its neighboring nodes. The work in [12], [13] proposes anycast packet-forwarding protocols that work on top of a separate routing protocol in the network layer. The anycast protocols in [14]–[17] use the hop-count information (i.e., the number of hops for each node to reach the sink) such that at each hop the forwarding decision is chosen to reduce the hop count to the sink as soon as possible. However, these aforementioned approaches are heuristic in nature and do not directly minimize the expected end-to-end delay.

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In our prior work [18], we developed a distributed anycast forwarding policy that simultaneously minimizes the expected end-to-end delays from all nodes to the sink, when the wake-up rates of the nodes are given. (The wake-up rate represents the frequency with which a node wakes up.) However, the delay-optimal anycast policy in [18] was derived based on the assumption that nodes wake up according to a Poisson process (i.e., the wake-up intervals of a node are i.i.d. exponential random variables). Hence, the following questions remain unanswered: (1) If we can control the wake-up patterns (subject to given wake-up rates) in addition to the anycast forwarding policy, is there a wake-up pattern that results in optimal delay performance? and (2) If such a pattern exists, which forwarding policy is delay-optimal for the wake-up pattern? These questions make the problem more complex than the one considered [18] because we can no longer exploit the memoryless property of a Poisson Process.

In this paper, we extend the results in [18] to address these questions. For given wake-up rates of nodes (in other words, given energy budget at each node), we obtain the anycast forwarding policy and the wake-up pattern that can minimize the expected end-to-end delays from all nodes to the sink. Specifically, we show that using asynchronous periodic wake-up patterns along with an optimal forwarding policy can minimize the expected end-to-end delay over all asynchronous wake-up patterns. Further, we provide an efficient distributed algorithm that can implement the delay-optimal anycast forwarding policy for the periodic wake-up pattern.

The rest of this paper is organized as follows. In Section II, we describe our system model and formulate the delay-minimization problem that we intend to solve. In Section III, we study the delay-optimal anycast forwarding policy when nodes wake up periodically. In Section IV, we show that given an average wake-up rate, the periodic wake-up pattern is the best in terms of delay performance. In Section VI, we provide simulation results that illustrate the superior performance of our proposed solution.

II. System Model

We consider an event-driven WSN with N sensor nodes. Let $\mathcal{N}$ be the set of all nodes. We assume that event information is reported to a single sink node s, although the analysis can also be readily extended to the scenario with multiple sink nodes. Each node i has a set $\mathcal{N}_i$ of neighboring nodes to which node i is able to directly transmit packets. We assume that the network is connected, i.e., there is a finite-length path from every node to the sink.

The lifetime of an event-driven WSN under asynchronous sleep-wake scheduling consists of two phases: *the configuration phase* and *the operation phase*. When sensor nodes are deployed, the configuration phase begins, during which the nodes determine their packet-forwarding and sleep-wake scheduling policies. It is also during this phase that the optimization on these policies (which we will study in this paper) is carried out. Once the optimal policies are determined, the operation phase begins, during which the nodes apply the policies determined in the configuration phase to perform their main functions: detecting events and reporting the event information. Specifically, during this phase, sensor nodes wake up occasionally and sleep most of the time *when there is no need to relay or transmit a packet*. The sleep-wake scheduling policy that is developed in the configuration phase determines when nodes sleep and wake up. We assume asynchronous sleep-wake scheduling, with which nodes sleep and wake up independently of other nodes according to their own sleep-wake scheduling policies. If an event occurs during the operation phase, the source node generates the event-reporting packet and immediately turns on its transceiver from the sleeping mode to find a relaying node as soon as possible. If a neighboring node wakes up from the sleeping mode and hears a packet-relaying request (in the form of a beacon signal) from the source node, the neighboring node first checks whether it is an eligible forwarder of the source node based on the packet-forwarding policy that is determined during the configuration phase. If it is, it receives the packet and then finds a new relaying node without turning back to the sleeping mode in order to reduce the delay. Following the same procedure, subsequent nodes relay the packet to the sink. If the nodes successfully forward these packets, they return to sleep and follow their sleep-wake scheduling policy to determine the next-time when they wake up. Note that since the forwarding policy is determined before an event occurs, and nodes are not synchronized, we assume that nodes cannot dynamically adjust their forwarding policy based on what happened in the network before the packet is received.

A. Basic Forwarding and Sleep-Wake Scheduling Protocols

We first introduce the basic packet-forwarding and sleep-wake scheduling protocols that are used in the operation phase.

Packet-Forwarding Protocol: When a node i has a packet to deliver to the sink, it must wait for its neighboring nodes to wake up. Under asynchronous sleep-wake scheduling, we simply assume that the clocks at different nodes are not synchronized. Hence, the sending node i does not know exactly when its neighboring nodes will wake up (although it may have some statistical information of their wake-up patterns and wake-up rates). Fig. 1 describes the protocol with which sending node i transmits its packet to one of its neighboring nodes. As soon as node i is ready to transmit the packet, it sends a beacon signal (Beacon 1 in Fig. 1) of duration t_B, and ID signal of duration t_C, and then listens for acknowledgements (CTS: Clear-To-Send) for duration t_A. The sending node repeats this sequence until it hears an acknowledgement. The ID signal contains the identity of the sending node and the sequence number of the last beacon signal. When a node wakes up and senses the h-th beacon signal, it will stay awake to decode the following ID signal, in which case we say that the node receives the h-th ID signal. (If a node wakes up in the middle of the ID signal, it must stay awake to decode the next ID signal.) Then, such a node has two

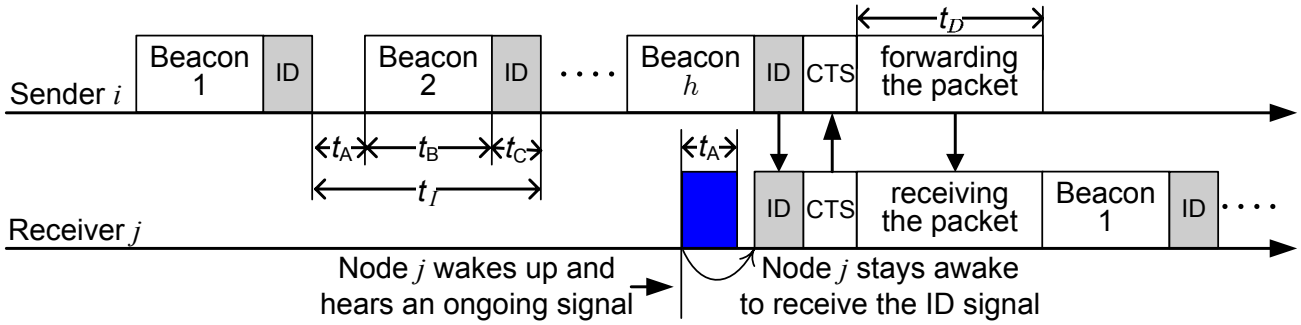


Fig. 1. System model

choices. **Choice 1:** If the node chooses to receive the packet, it responds with a CTS message containing its identity during the acknowledgement period t_A that immediately follows the ID signal. Once the sending node hears the CTS, it forwards the packet to the awake node during the data transmission period t_D. **Choice 2:** If the awake node decides not to receive the packet, it goes back to sleep. For simplicity of notation, let $t_I = t_B + t_C + t_A$, which denotes the duration of each beacon-ID signaling iteration (See Fig. 1).

Remark: In the above basic protocol, we have ignored the possibility of collisions, which can be due to either multiple awake nodes or multiple sending nodes. In Section V, we describe an extended packet-forwarding protocol that addresses these collision scenarios using random or deterministic back-offs. However, in a low duty-cycle WSN where nodes seldom wake up (i.e., the duration t_I is much smaller than a wake-up interval), chances are small that multiple neighboring nodes wake up at the same beacon signal. Due to this reason, we use the basic protocol for our analysis and study the effect of collisions using simulation in Section VI.

Sleep-Wake Scheduling Protocol: In order to save energy, each node wakes up infrequently and goes back to sleep *if there is no need to receive or transmit a packet.* Note that if the duration for which the node stays awake is shorter than t_A, the node may stay awake only within an acknowledgement period t_A and miss on-going beacon-ID signals. In order to avoid such a case, we assume that nodes must stay awake for at least t_A. Further, since a longer awake duration results in higher energy consumption, we set the awake duration to be exactly equal to t_A. The next time to wake up is determined by the sleep-wake scheduling policy of the node.

B. Sleep-Wake Scheduling and Anycast Forwarding Policies

In this subsection, we define the sleep-wake scheduling and anycast forwarding policies that are computed during the configuration phase and applied during the operation phase. These policies affect the end-to-end delay experienced by a packet, and the energy consumption of the network.

Sleep-Wake Scheduling Policy: The sleep-wake scheduling policy that is chosen by each node determines when the node wakes up.[1] We define the counting process $\kappa_i(t)$ as the number of times node i wakes up in the time interval $[0, t]$. We call this process *the wake-up process of node* i. This sleep-wake scheduling is fully characterized by the following two factors: (1) how frequently the node has to wake up and (2) the distribution of the wake-up interval. The former and the latter are called *the wake-up rate* and *the wake-up pattern*, respectively. In this paper, we assume that nodes independently control these two variables to determine the sleep-wake scheduling policy.

Wake-up Rate: We assume that every node chooses its wake-up process among counting processes for which the following time-average limit exists:

$$\lim_{t\to\infty} \frac{\kappa_i(t)}{t} = r_i \text{ almost surely.} \tag{1}$$

We then define *the wake-up rate* r_i of node i as the time average of the times that node i wakes up. *Since a higher wake-up rate results in faster energy consumption, the wake-up rate directly impacts the lifetime of a node.* Let $\vec{r} = (r_1, r_2, \cdots, r_N)$ be the global wake-up rate (or simply the wake rate).

Wake-up Pattern: Consider the time-scaled wake-up process $\kappa_i(t/r_i)$ that has a wake-up rate of 1. We then define the wake-up pattern w_i of node i as the control variable that fully characterizes this scaled wake-up process of node i. For example, if the scaled process $\kappa_i(t/r_i)$ is given by $\lfloor t \rfloor$, node i will wake up every $1/r_i$ time unit. We call this wake-up pattern as *a periodic wake-up pattern* and express it as $w_i = w_{\text{per}}$. If the scaled process is given by a Poisson process with mean 1, the wake-up intervals will be *i.i.d* exponential random variables with mean $1/r_i$. We call this wake-up pattern *a Poisson wake-up pattern* and express it as $w_i = w_{\text{Poisson}}$. Node i can also choose a wake-up pattern such that the wake-up intervals are correlated, e.g.,

[1]Recall that sleep-wake scheduling runs only if there is no need to transmit or receive a packet. Otherwise, nodes need to stay awake to transmit the packets to the sink as soon as possible.

node i can alternate with the wake-up intervals of length $\frac{1}{2r_i}$ and $\frac{3}{2r_i}$. Let $\vec{w} = (w_1, w_2, \cdots, w_N)$ denote the global wake-up pattern (or simply the wake-up pattern).

Remark: While the wake-up rate determines the expected wake-up interval in the operational phase, the wake-up pattern determines the distribution of the interval. Hence, the wake-up rate and the wake-up pattern of a node are the independent control variables.

Recall that we focus on asynchronous sleep-wake scheduling where nodes do not synchronize their clocks with their neighboring nodes. In practice, due to variations of the clock frequency, the clocks at different nodes will drift from each other. Note that after an event occurs, a sending node i needs to predict when a neighboring node j will wake up. Due the aforementioned clock-drift, even for periodic wake-up patterns, there will be a random time-offset in the wake-up process of the node j as it is observed by the sending node i. When events occur rarely and no clock synchronization is performed between events, it can be difficult for node i to predict this offset *immediately after an event occurs*. Hence, in this paper, we assume that due to a random time-offset, the wake-up process of a node is of stationary increments, as observed by other nodes. In other words, even though a node might have an opportunity to synchronize with its neighbor when the previous event occurred, by the time the next event occurs the wake-up process is assumed to have "completely forgotten the past."

We define the residual time $R_j(t)$ as the interval from time t to the next-wake-up time of node j, i.e., $R_j(t) \triangleq \inf\{s-t | s > t, \kappa_j(s) - \kappa_j(t) = 1\}$. Since the wake-up process has stationary increments, we can assume that the residual time $\{R_j(t)\}_{t\geq 0}$ is a stationary and ergodic process. Hence, we can drop the variable t and use the random variable R_j to denote the residual time. Let $F_{R_j}(y)$ be the cumulative distribution function (CDF) of R_j, i.e., $F_{R_j}(y) = \Pr(R_j \leq y)$. For a neighboring node j to wake up and hear the h-th beacon signal that node i has sent, the residual time R_j must be within $((h-1)t_I, h\cdot t_I]$. Define the awake probability $p_{j,h}$ as the conditional probability that node j wakes up at the h-th beacon signal, conditioned on not having woken up at earlier beacon signals, i.e., $p_{j,h} = \Pr\{R_j \in ((h-1)t_I, ht_I] \mid R_j \notin [0, (h-1)t_I)\}$. Using the cdf $F_{R_j}(y)$, we can express the awake probability as

$$p_{j,h} = \frac{F_{R_j}(ht_I) - F_{R_j}((h-1)t_I)}{1 - F_{R_j}((h-1)t_I)}. \tag{2}$$

For example, if node j wakes up periodically, the residual time R_j is disturbed on $[0, 1/r_j]$. Let $F^*_{R_j}(y)$ and $p^*_{j,h}$ denote the CDF of the residual time and the awake probability, respectively, under the periodic wake-up pattern. Then, the CDF $F^*_{R_j}(y)$ is given by $F^*_{R_j}(y) \triangleq r_j y 1_{\{0\leq y\leq \frac{1}{r_j}\}} + 1_{\{y>\frac{1}{r_j}\}}$. From (2), the awake probability for the periodic wake-up pattern is given by

$$p^*_{j,h} = \begin{cases} \dfrac{t_I}{1/r_j - (h-1)t_I} & \text{if } h < \lfloor \frac{1/r_j}{t_I} \rfloor, \quad (3) \\ 1 & \text{otherwise.} \quad (4) \end{cases}$$

Anycast Forwarding Policy: For given wake-up rates and given wake-up patterns of neighboring nodes, a node determines its anycast forwarding policy in the configuration phase. To define the anycast forwarding policy, suppose that a sending node i has sent the h-th beacon-ID signal, and a set $X \subset \mathcal{N}_i$ of the neighboring nodes wakes up and receives the ID signal. We let $f_{i,h}(X)$ denote the corresponding decision of the sending node i, which is to be specified next. We let $f_{i,h}(X) = j$ if the sending node i decides to transmit the packet to node $j \in X$, and we let $f_{i,h}(X) = i$ if the sending node i decides to send out the $(h+1)$-st beacon-ID signal, i.e., the packet remains at node i. This forwarding decision may seem inconsistent with the packet-forwarding protocol described in Subsection II-A, in which the sending node is restricted to transmit the packet whenever it receives a CTS. However, we only use this more general setting to find the optimal forwarding decisions and then show that such optimal decisions can be implemented by our packet-forwarding protocol that lets the sending node always transmit the packet whenever it receives a CTS. Let $f_i = \{f_{i,1}, f_{i,2}, \cdots\}$ denote the anycast forwarding policy of node i (or simply the anycast policy of node i), where $f_{i,h}$ is the forwarding decision of node i at beacon signal h. We further denote by $f = \{f_1, f_2, \cdots, f_N\}$ the global anycast forwarding policy (or simply the anycast policy), where the indices $1, 2, \cdots, N$ denote nodes.

C. Performance Metrics and Optimization

In this section, we define the notion of the end-to-end delay. We then formulate the problem of minimizing the end-to-end delay by jointly controlling the anycast forwarding policy and the sleep-wake scheduling policy.[2]

Expected end-to-end delay: During the operation phase, we define the end-to-end delay as the delay from the time when a source node detects an event and generates the event-reporting packet (or packets) to the time the *first* packet is received at the sink. For applications that use a single packet to carry the event information, the above definition captures the actual delay for reporting the event information. For applications that use multiple packets, if the nodes that relayed the first packet stay awake for a while, the delay to relay subsequent packets will be much smaller than that experienced by the first packet. (For instance, these subsequent packets may be sent a few nodes behind the first packet, and hence they can reach the sink

[2]As mentioned earlier, our goal during the configuration phase is to design the system to minimize the delay of interest during the operation phase.

soon after the first packet reaches the sink.) Hence, the actual event-reporting delay can still be approximated by the delay experienced by the first packet.

The sleep-wake scheduling policy $(\vec{r}, \vec{w})$ and anycast forwarding policy f fully determine the stochastic process with which the first packet traverses the network from the source node to the sink. Hence, we use $D_i(\vec{r}, \vec{w}, f)$ to denote the expected end-to-end delay from node i to the sink under the joint policy $(\vec{r}, \vec{w}, f)$. For simplicity, from now on, we simply call the expected end-to-end delay from node i to the sink as "the delay from node i."

Delay-Minimization Problem: The objective of this paper is to find the optimal joint policy $(\vec{w}, f)$ that solves the following delay-minimization problem for given wake-up rate $\vec{r}$:

$$\min_{\vec{w}, f} D_i(\vec{r}, \vec{w}, f). \tag{5}$$

Note that $\vec{r}$ controls the duty cycle of the sensor network, which in turn controls the energy expenditure. Hence, the problem can also be viewed as minimizing the delays for a given energy budget. In Sections III and IV, we develop an algorithm that solves this problem for all nodes i, i.e., our solution can simultaneously minimize the delays from all nodes.

III. Delay-Optimal Anycast Policy for a given Sleep-Wake Scheduling Policy

As a preliminary step to solving the delay-minimization problem, in this section we first fix a sleep-wake scheduling policy $(\vec{r}, \vec{w})$ and study delay-optimal anycast policies for the fixed sleep-wake scheduling policy. This problem can be formulated as a stochastic shortest path (SSP) problem, where the state corresponds to the node that is holding the packet, and the cost corresponds to the delay for each packet to reach the sink.[3] In Section III-A, we will derive a solution to this problem, by using the value-iteration algorithm. A key part of the value-iteration algorithm is, assuming that node i knows the end-to-end delay from its neighboring nodes to the destination, how node i should update its own forwarding policy. This corresponds to a sub-problem, in which the sending node needs to decide whether to forward the packet to an awake node, or to send the next beacon signal and wait for another node to wake up. This problem can again be formulated as an infinite-horizon dynamic programming problem where the state corresponds the set of awake nodes after each beacon signal. We derive the solution to this sub-problem in Section III-B. However, the optimal policy in Sections III-A and III-B can be difficult to compute in practice due to the infinite horizon. In Section III-C, we proposed a more practical truncated version of the forwarding policy, and show that the optimal truncated policy will converge to the original optimal policy as a parameter approaches infinity. Finally, in Section III-D, we study the important properties of periodic wakeup patterns and show that the truncated policy becomes exactly optimal under the periodic wake-up pattern.

A. Value-Iteration Algorithm

In this subsection, we develop the value-iteration algorithm. Given a sleep-wake scheduling policy $(\vec{r}, \vec{w})$, the delay-minimization problem can be formulated as a stochastic shortest path (SSP) problem [19, Chapter 2], where the sensor node that has a packet corresponds to the "state", and the delay corresponds to the "cost" that we intend to minimize. The sink s corresponds to the terminal state, where no further cost (delay) will be incurred. Let $i_0, i_1, i_2, \cdots, i_L = s$ be the sequence of nodes that relay the packet from the source node i_0 to the sink s in L steps. Note that under anycasting, this sequence is random because each node has a set of candidate next-hop nodes and does not know which of them will wake up first to receive the packet. Let $D_{\text{hop},i}(\vec{r}_i, \vec{w}_i, f_i)$ be the expected one-hop delay at node i under the forwarding policy f_i, where the wake-up rates and patterns of neighboring nodes are given by $\vec{r}_i \triangleq (r_j, j \in \mathcal{N}_i)$ and $\vec{w}_i \triangleq (w_j, j \in \mathcal{N}_i)$. We note that the wake-up rates and patterns of the other nodes not in $\mathcal{N}_i$ do not affect the one-hop delay of node i. Then, the end-to-end delay $D_i(\vec{r}, \vec{w}, f)$ from each node i_0 to the sink can be expressed as

$$D_i(\vec{r}, \vec{w}, f) = E\Big\{ \sum_{l=0}^{L} D_{\text{hop},i_l}(\vec{r}_{i_l}, \vec{w}_{i_l}, f_{i_l}) \Big\}, \tag{6}$$

where the expectation is taken with respect to the random sequence $i_1, i_2, \cdots, i_L$. (Note that the source node i_0 is deterministic.) Given the sleep-wake scheduling policy $(\vec{r}, \vec{w})$, let $D_i^*(\vec{r}, \vec{w}) \triangleq \min_f D_i(\vec{r}, \vec{w}, f)$ be the minimum expected delay from node i. Then, according to the Bellman equation [19, Section 2.2], for all nodes i, the minimum delay $D_i^*(\vec{r}, \vec{w})$ of node i must satisfy

$$D_i^*(\vec{r}, \vec{w}) = \min_{f_i} \Big(D_{\text{hop},i}(\vec{r}_i, \vec{w}_i, f_i) + \sum_{j \in \mathcal{N}_i} q_{i,j}(\vec{r}_i, \vec{w}_i, f_i) D_j^*(\vec{r}, \vec{w}) \Big), \tag{7}$$

[3] In dynamic programming, a policy is proper if there is a positive probability that any initial state can transit to the terminal state within some finite decision stages. In our SSP model, as long as the network is connected, any policy (including our delay-optimal policy) that can transport a packet in every node to the sink within some finite number of hops is a proper policy.

TABLE I
TABLE OF NOTATIONS

Notation	Definition
D_j	Expected delay from neighboring node j to the sink
X_h	Set of awake neighboring nodes right after beacon signal h
x_h	Node j that has the smallest delay value D_j among the nodes in X_h
$P^{(h)}_{x,x'}$	Conditional probability that $x_h = x'$ conditioned on that $x_{h-1} = x$ and node i sends beacon signal h
$p_{j,h}$	Conditional probability that node j wakes up at stage h conditioned on not having woken up at earlier stages
$d^{(h)}(x_h)$	Expected delay after node i sends beacon signal h conditioned on that the current state is x_h, and the optimal forwarding policy will be used afterward
$d^{(h)}_{\text{wait}}(x_h)$	Expected delay after node i sends beacon signal h conditioned on that the current state is x_h, and node i sends beacon signal $h+1$ and uses the optimal forwarding policy afterward

where $q_{i,j}(\vec{r}_i, \vec{w}_i, f_i)$ is the probability that node j is chosen as the next-hop node of node i under the forwarding policy f_i and the sleep-wake scheduling policy $(\vec{r}_i, \vec{w}_i)$. The minimization is taken with respect to all feasible forwarding policies of node i. Further, using the following value-iteration algorithm [19, Section 1.3], we can find the delay-optimal forwarding policy that achieves $D^*_i(\vec{r}, \vec{w})$ for all nodes i:

Value Iteration Algorithm: At the initial iteration $k = 0$, all nodes i set their initial delay values $D^{(0)}_i$ to ∞, and the sink s sets its delay value $D^{(0)}_s$ to zero. At each iteration $k = 1, 2, \cdots$, every node i collects the delay values $D^{(k-1)}_j$ from its neighboring nodes j and then updates its delay value $D^{(k)}_i$ by solving

$$D^{(k)}_i = \min_{f_i} \Big(D_{\text{hop},i}(\vec{r}_i, \vec{w}_i, f_i) + \sum_{j \in \mathcal{N}_i} q_{i,j}(\vec{r}_i, \vec{w}_i, f_i) D^{(k-1)}_j \Big). \tag{8}$$

Let $f^{(k)}_i$ be the forwarding policy of node i that minimizes (8). Then, according to [19, Proposition 2.2.2], the delay value $D^{(k)}_i$ of each node i converges to the minimum delay $D^*_i(\vec{r}, \vec{w})$, i.e., $\lim_{k\to\infty} D^{(k)}_i = D^*_i(\vec{r}, \vec{w})$, and the corresponding forwarding policy $f^{(k)} = \{f^{(k)}_1, f^{(k)}_2, \cdots\}$ also converges to the delay-optimal forwarding policy, i.e., $\lim_{k\to\infty} f^{(k)} \in \arg\min_f D_i(\vec{r}, \vec{w}, f)$ for all nodes i.

The key step in this value iteration algorithm is how every node i solves the sub-problem in (8) at each iteration k. Note that the probability $q_{i,j}(\vec{r}_i, \vec{w}_i, f_i)$ may be difficult to derive in closed form. Instead, we reformulate (8) as another dynamic programming problem: we find forwarding decisions $f_{i,1}, f_{i,2}, \cdots$ of node i for each beacon signal $1, 2, \cdots$ that minimize the expected delay from node i to the sink when the delays from neighboring nodes j to the sink are given by $D^{(k-1)}_j$, and the sleep-wake scheduling policies of all nodes are given by $(\vec{r}, \vec{w})$. The solution to this second dynamic program is presented next as the LOCAL-OPT algorithm, which does not need to compute the probability $q_{i,j}(\vec{r}_i, \vec{w}_i, f_i)$. Recall that the value iteration algorithm minimizes the expected end-to-end delay when each state corresponds to the node that has the packet and the state transition occurs when the packet is transmitted from one node to another. In contrast, the LOCAL-OPT algorithm aims to minimize the expected end-to-end delay from a given node to the sink *when the delays from its neighboring nodes to the sink are fixed*. State transitions occur after each beacon signal, and the state corresponds to the available (awake) neighboring node that are ready to receive after each beacon signal.

B. LOCAL-OPT Algorithm

In the next two subsections, we provide a thorough analysis of the LOCAL-OPT algorithm. For readers who may be more interested in the implementation, we also provide pseudo-code algorithms that compute our solution in a fully distributed manner

To solve the above sub-problem, we focus on a node i that has a packet. For ease of exposition, let the expected delays from neighboring nodes j be denoted by $D_j = D^{(k-1)}_j$ $(j \in \mathcal{N}_i)$, which is equal to $D^{(k-1)}_j$ for iteration k in the value-iteration algorithm. Without loss of generality, we assume that the node i has neighboring nodes $1, 2, \cdots, N_i$ $(N_i = |\mathcal{N}_i|)$, and their expected delays are sorted in increasing order, i.e., $D_1 \le D_2 \le \cdots \le D_{N_i}$. To avoid confusion, we further assume that the index i of the sending node is larger than $N_i + 1$. In Table I, we summarize the definition of the notations that will be used in this section.

After the sending node i sends out the h-th beacon signal, it has to choose either to **transmit** the packet to one of the awake nodes, or to **wait** for the other node to wake up by sending the next beacon signal. We call this moment the decision stage h (or simply stage h) and denote the set of the awake nodes at this moment by X_h. By definition, $f_{i,h}(X_h) = j$ $(j \in X_h)$ implies that node i decides to transmit to node j, and $f_{i,h}(X_h) = i$ implies that node i decides to wait and send the $(h+1)$-st beacon signal. (Recall that for non-Poisson wake-up patterns that do not have the memoryless property, the forwarding decision must be controlled differently at each beacon signal in order to minimize the delay.) Since stage 0 is the moment when node i is about to send the first beacon signal, we set $X_0 = \emptyset$ and $f_{i,0}(X_0) = i$. This state transition terminates whenever the sending

node transmits the packet to an awake node j. When this happens, the packet will be relayed by the node j and eventually arrive at the sink after D_j time (the delay from node j). We denote this terminal state by state 0.

Since the number of possible states at each stage increases exponentially with the number N_i of neighboring nodes (2^{N_i} states at each stage), it is more convenient to deal with a simpler transition model as follows. Note that if node i decides to transmit the packet to one of the awake nodes in X_h, clearly it should choose the node j with the smallest delay D_j among all the awake nodes in order to minimize the delay from the next-hop node to the sink. Hence, at each stage h, node i only needs to remember the awake node with the smallest delay. In other words, if a delay-optimal policy is applied, only the awake node with the smallest delay affects the state transition dynamics. We denote this node by $x_h = \arg\min_{j \in X_h} D_j$. (Ties are broken arbitrarily.) If no nodes are awake ($X_h = \emptyset$), we denote the corresponding state x_h by $x_h = N_i + 1$. (For example, since $X_0 = \emptyset$, the initial state is always given by $x_0 = N_i + 1$.) From now on, we can use a simpler state transition model $x_0, x_1, x_2, \cdots$ to solve the sub-problem (8) without any loss of optimality. Due to the same principle, we abuse notation slightly, and use $f_{i,h}(x_h)$ to denote the decision of node i at state x_h as follows: $f_{i,h}(x_h) = x_h$ if the sending node i decides to transmit the packet to node x_h, and $f_{i,h}(x_h) = i$ if the node i decides to wait. We further use the following assumption to simplify the dynamics for the state transitions $x_0, x_1, \cdots$. (However, this is not a required assumption, as we will soon see.)

Assumption 1: If an awake node is not chosen as the next-hop node, we assume that the node stays awake to remain eligible to be chosen as the next-hop node at following stages. Under this assumption, the state transition must satisfy $x_0 \geq x_1 \geq \cdots$.

Remark: Assumption 1 not only simplifies the analysis, but it also clearly leads to smaller delay, compared to the case where an awake node can return to sleep when it is not immediately chosen as the next-hop node. However, one could argue that keeping nodes awake consumes more energy. In Section III-D, we will show that the optimal anycast forwarding policy achieves the minimum delay without Assumption 1, and thus the awake nodes in fact do not need to stay awake. But for now, we use the assumption to simplify the analysis.

We next consider the state transition probability. Let $P^{(h)}_{x,x'}$ be the state transition probability from state $x_{h-1} = x$ to state $x_h = x'$, given that node i decides to wait at stage $h-1$, i.e., $P^{(h)}_{x,x'} \triangleq \Pr\big(x_h = x' | x_{h-1} = x \text{ and } f_{i,h-1}(x) = i\big)$. Using the awake probability $p_{j,h}$, we can express the state transition probability as

$$P^{(h)}_{x,x'} = \begin{cases} p_{x',h} \prod_{j=1}^{x'-1} (1 - p_{j,h}) & \text{if } x' < x, \\ \prod_{j=1}^{x'-1} (1 - p_{j,h}) & \text{if } x' = x, \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

The state transition probability conditioned on $f_{i,h-1}(x) = x$ is trivial because, if the sending node decides to transmit the packet to node x, the next state must be 0. Note that if the wake-up pattern of node 1 is such that node 1 must wake up before beacon signal h, the probability $P^{(h)}_{x,x'}$ is not well defined for $x > 1$ because the conditional event $x_{h-1} = x$ cannot happen. Hence, we say that state $x_h = x$ is *admissible* if $\Pr(x_h = x | f_{i,h'}(x_{h'}) = i, \forall h' < h) > 0$, and we define the state transition probability only for admissible states. We also define $x_{h,\max}$ as the upperbound of the admissible state at stage h, i.e., $x_h \leq x_{h,\max}$.

In our dynamic programming problem, the cost to be minimized is delay. Let $g(x_h, f_{i,h}(x_h))$ be the one-step delay between stages h and $h+1$ when decision $f_{i,h}$ is used at state x_h. If the sending node i sends out the next beacon signal ($f_{i,h}(x_h) = i$), the delay incurred by this decision is the beacon-ID duration t_I. If node i transmits the packet, the packet will be transmitted to the next-hop node x_h for the packet transmission period t_D and will arrive at the sink D_{x_h} time later. Hence, the delay incurred by this decision is $t_D + D_{x_h}$. Once the packet reaches the sink, no more delay will be incurred. Hence, the one-step delay can be expressed as

$$g(x_h, f_{i,h}(x_h)) = \begin{cases} t_I & \text{if } f_{i,h}(x_h) = i, \quad (10) \\ t_D + D_{x_h} & \text{if } f_{i,h}(x_h) = x_h. \quad (11) \end{cases}$$

for $x_h \neq 0$ and $g(x_h, f_{i,h}(x_h)) = 0$ for $x_h = 0$. Using the above state transition probability and the one-step delay, we can represent the sub-problem (8) as the following infinite-horizon dynamic program (DP) problem [19, Chapter 1]: given the delays D_j from the neighboring nodes j, we want to find the anycast forwarding policy f_i of node i that minimizes the overall cost (delay) function

$$d_{f_i} = \lim_{\bar{h} \to \infty} E\left\{ \sum_{h'=0}^{\bar{h}-1} g(x_{h'}, f_{i,h'}(x_{h'})) \right\} \tag{12}$$

where $x_0, x_1, x_2, \cdots$ are the states visited, and the expectation is taken with respect to these states. Then, $\min_{f_i} d_{f_i}$ and $\arg\min_{f_i} d_{f_i}$ corresponds to $D_i^{(k)}$ and $f_i^{(k)}$ of the value-iteration algorithm in (8), respectively.

To solve this DP problem, we define $d^{(h)}(x_h)$ as the expected delay from state $x_h \geq 1$ at stage h, given that the optimal forwarding policy is applied afterward, i.e.,

$$d^{(h)}(x_h) \triangleq \min_{f_{i,h}, f_{i,h+1}, \cdots} \left(\lim_{\bar{h} \to \infty} E\left\{ \sum_{h'=h}^{\bar{h}-1} g(x_{h'}, f_{i,h'}(x_{h'})) \right\} \right)$$

where $x_{h+1}, x_{h+2}, \cdots$ are the states to be visited after stage h, and the expectation is taken with respect to these states. By definition, it immediately follows that $d^{(0)}(N_i+1) = \min_{f_i} d_{f_i}$. The delay function $d^{(h)}(x_h)$ can be interpreted as the minimum expected delay from state x_h. Suppose that the sending node i at state x_h decides to **transmit** the packet to node x_h ($f_{i,h}(x_h) = x_h$). By (11), the minimum expected delay conditioned on this decision is $t_D + D_{x_h}$. If node i decides to **wait** ($f_{i,h}(x_h) = i$), the minimum expected delay $d^{(h)}_{\text{wait}}(x_h)$ conditioned on this decision is given by

$$d^{(h)}_{\text{wait}}(x_h) = t_I + \sum_{x_{h+1}=1}^{x_h} P^{(h+1)}_{x_h,x_{h+1}} d^{(h+1)}(x_{h+1}), \tag{13}$$

assuming that the optimal decisions $f^*_{i,h+1}, f^*_{i,h+2}, \cdots$ are applied afterward. Clearly, if just transmitting the packet to the awake node x_h incurs a smaller delay (i.e., $t_D + D_{x_h}$) than the delay $d^{(h)}_{\text{wait}}(x_h)$ that is incurred by the waiting decision, the node should forward the packet to node x_h in order to reduce the delay, and *vice versa*. Hence, the optimal forwarding decision at stage h can be expressed as

$$f^*_{i,h}(x_h) = \begin{cases} x_h & \text{if } t_D + D_{x_h} < d^{(h)}_{\text{wait}}(x_h), \\ i & \text{otherwise.} \end{cases} \tag{14}$$

(Readers can refer to [19, Equation (1.3) on Page 5] for further information.) Further, the minimum expected delay $d^{(h)}(x_h)$ under the optimal decision at stage h is given by

$$d^{(h)}(x_h) = \min(d^{(h)}_{\text{wait}}(x_h), t_D + D_{x_h}). \tag{15}$$

Although (14) and (15) are not well defined for $x_h = N_i + 1$, by setting $D_{N_i+1} = \infty$, we can still use (14) and (15) even when $x_h = N_i + 1$. In this case, $d^{(h)}(N_i+1)$ is always equal to $d^{(h)}_{\text{wait}}(N_i+1)$. (In other words, if no nodes are awake, the only choice left is to send the next beacon-ID signal.)

Clearly, whenever node 1 has woken up, the optimal decision is to forward the packet to node 1. Hence, the optimal forwarding decision must satisfy

$$f^*_{i,h}(1) = 1 \text{ and } d^{(h)}(1) = t_D + D_1 \text{ for all } h. \tag{16}$$

Furthermore, since the packet will be forwarded to a neighboring node j eventually, taking $t_D + D_j$ expected time, it must hold that

$$d^{(h)}(x_h) \geq t_D + D_1 \text{ for } x_h > 0. \tag{17}$$

Both equations (16) and (17) provide an important property about $d^{(h)}(x_h)$, which is independent on h. We will use this property to prove Propositions 1 and 9.

We have shown that for an arbitrary sleep-wake process the optimal forwarding decision $f^*_{i,h}$ and the delay value $d^{(h)}$ must satisfy the necessary conditions in (14) and (15), respectively. If there is a reference stage $\bar{h}$ such that the minimum delay $d^{(\bar{h})}(x_{\bar{h}})$ is known for all admissible states $x_{\bar{h}}$, we can then use (13) and (15) as a backward iteration from stage $\bar{h}$ to stage 0, and can find the optimal forwarding decisions. However, such a reference stage may not exist in general. In practice, we can artificially impose a reference stage $\bar{h}$ and use a truncated policy after $\bar{h}$. In the next section, we will study the performance of such a truncated packet-forwarding policy as $\bar{h} \to \infty$.

C. A Truncated Forwarding Policy

We use $\hat{f}_{i,h}$ to denote a packet-forwarding policy that uses truncated decisions after a given stage $\bar{h}$. In the rest of the paper, we refer to it as *the truncated policy*. Specifically, if the sending node has not chosen its next-hop node until stage $\bar{h}$, it then waits only for node 1 (the node with the smallest delay) to wake up and then forwards the packet to node 1. Let H be the number of beacon signals that the sending node has to send until node 1 wakes up. Then, if node 1 has not woken up for the first $\bar{h}$ beacon signals, i.e, $x_{\bar{h}} > 1$, the sending node has to send $H - \bar{h}$ more beacon signals until node 1 wakes up. Similar to $d^{(h)}(x_h)$, we define $\hat{d}^{(h)}(x_h)$ as the expected delay from state x_h at stage h under the truncated policy. Then, the expected delay $\hat{d}^{(\bar{h})}(x_{\bar{h}})$ at stage $\bar{h}$ is given by

$$\hat{d}^{(\bar{h})}(x_{\bar{h}}) = \begin{cases} t_D + D_1 & \text{if } x_{\bar{h}} = 1, \\ E[H - \bar{h} | H > \bar{h}] \cdot t_I + t_D + D_1 & \text{if } x_{\bar{h}} > 1. \end{cases} \tag{18}$$

(Recall that D_1 is the delay value of node i from the previous iteration of the value-iteration algorithm.) Since we now know the value of $\hat{d}^{(\bar{h})}(x_{\bar{h}})$ for all admissible states $x_{\bar{h}}$ at stage $\bar{h}$, we can compute the optimal forwarding decision at stages $h < \bar{h}$ for the truncated policy. Similarly to (13) and (15), we compute $\hat{d}^{(h)}_{\text{wait}}(x_h)$ (the minimum expected delay conditioned on the WAIT decision) and $\hat{d}^{(h)}(x_h)$ for $h = \bar{h}-1, \bar{h}-2, \cdots, 1, 0$, using

$$\hat{d}^{(h)}_{\text{wait}}(x_h) = t_I + \sum_{x_{h+1}=1}^{x_h} P^{(h+1)}_{x_h,x_{h+1}} \hat{d}^{(h+1)}(x_{h+1}), \tag{19}$$

and

$$\hat{d}^{(h)}(x_h) = \min(\hat{d}^{(h)}_{\text{wait}}(x_h), t_D + D_{x_h}). \tag{20}$$

Once we obtain these values, the optimal truncated policy can be expressed as follows:

$$\hat{f}_{i,h}(x_h) = \begin{cases} 1 & \text{if } x_h = 1, \\ x_h & \text{if } x_h > 1, h < \bar{h}, \text{and} \\ & D_{x_h} < \hat{d}^{(h)}_{\text{wait}}(x_h) - t_D, \\ i & \text{otherwise.} \end{cases} \tag{21}$$

Since the delay under the truncated policy cannot be smaller than that under the optimal policy, we have

$$d^{(h)}(x_h) \leq \hat{d}^{(h)}(x_h), \tag{22}$$

for all h and admissible states x_h. Note that $\hat{d}^{(0)}(N_i + 1)$ corresponds to the expected delay of the sending node under the truncated policy, and $d^{(0)}(N_i + 1)$ corresponds to that under the optimal forwarding policy. In the following proposition, we show that the delay gap between the optimal and the truncated forwarding policies will approach to zero as $\bar{h} \to \infty$.

Proposition 1: The truncated forwarding policy $\hat{f}_i$ has the following properties:
(a) $\hat{d}^{(0)}(N_i + 1) - d^{(0)}(N_i + 1) \leq \Pr(H > \bar{h}) E[H - \bar{h} | H > \bar{h}] \cdot t_I$,
(b) $\hat{d}^{(0)}(N_i + 1) - d^{(0)}(N_i + 1) \to 0$ as $\bar{h} \to \infty$.
The proof is provided in Appendix A. Proposition 1 implies that (a) *the truncated forwarding policy is asymptotically optimal,* and (b) the rate of convergence depends on the decay rate of the tail probability $\Pr(H > \bar{h})$. If nodes, for instance, wake up according to the Poisson wake up pattern, $E[H - \bar{h} | H > \bar{h}]$ will be given by a constant because of the memoryless property, and the probability $\Pr(H > \bar{h})$ will decay exponentially. Hence, the delay gap between the truncated policy and the optimal policy will decrease exponentially.

Although we can compute the optimal truncated policy $\hat{f}_i$, it is still difficult to implement such a policy because of the following reasons. First, the policy requires the sender to know the list (X_h or x_h) of awake nodes at each stage h. It can be difficult for the sender to acquire this information during a short period t_A between two beacon-ID signals because of collisions. Second, the optimal policy is based on Assumption 1, which requires that an awake node stay awake even if it is not immediately chosen as the next-hop node. However, if the node is not chosen as the next-hop node in the end, the additional energy that it has spent to remain awake is then wasted. The following proposition contains an important result to address the above implementation issues.

Proposition 2: For $h = 0, 1, \cdots, \bar{h} - 1$ and all admissible states $x_h = x', x''$ such that $1 < x' \leq x''$, we have

$$\hat{d}^{(h)}_{\text{wait}}(x'') - \hat{d}^{(h)}_{\text{wait}}(x') \leq D_{x''} - D_{x'}. \tag{23}$$

Proof: We prove this result by induction. For $h = \bar{h} - 1$, by applying (18) to (19), we can verify that $\hat{d}^{(h)}_{\text{wait}}(x'')$ is equal to $\hat{d}^{(h)}_{\text{wait}}(x')$ for all states $x'' \geq x' > 1$. Hence, (23) holds for $h = \bar{h} - 1$.

We now assume that (23) holds for stage $h + 1 \leq \bar{h} - 1$. From (20), we also have

$$\hat{d}^{(h+1)}(x'') - \hat{d}^{(h+1)}(x') \leq D_{x''} - D_{x'}. \tag{24}$$

for $0 < x' \leq x''$. Using (9) and (13), we have

$$\begin{aligned} &\hat{d}^{(h)}_{\text{wait}}(x'') - \hat{d}^{(h)}_{\text{wait}}(x') \\ &= \sum_{x_{h+1}=x'}^{x''} P^{(h+1)}_{x'',x_{h+1}} d^{(h+1)}(x_{h+1}) - P^{(h+1)}_{x',x'} d^{(h+1)}(x'). \end{aligned}$$

Note that $\sum_{x_{h+1}=x'}^{x''} P^{(h+1)}_{x'',x_{h+1}} = P^{(h+1)}_{x',x'}$ from (9). Hence, the R.H.S. of the above can be expresses as

$$\begin{aligned} &\sum_{x_{h+1}=x'}^{x''} P^{(h+1)}_{x'',x_{h+1}} (d^{(h+1)}(x_{h+1}) - d^{(h+1)}(x')) \\ &\leq \sum_{x_{h+1}=x'}^{x''} P^{(h+1)}_{x'',x_{h+1}} (D_{x_{h+1}} - D_{x'}) \leq D_{x''} - D_{x'}, \end{aligned}$$

where in the last step we have used (24). Hence, (23) also holds for stage h. By induction, the result follows. ■

Using Proposition 2, the sending node i can implement the optimal truncated policy as follows during the operation phase:
Implementation of the optimal truncated policy: In the configuration phase, every neighboring node i computes the set $h^{(i)}_j$ of beacon signals for $j \in \mathcal{N}_i$ such that

$$h^{(i)}_j \triangleq \{h < \bar{h} \mid j \leq x_{h,\max}, D_j < \hat{d}^{(h)}_{\text{wait}}(j) - t_D\} \tag{25}$$

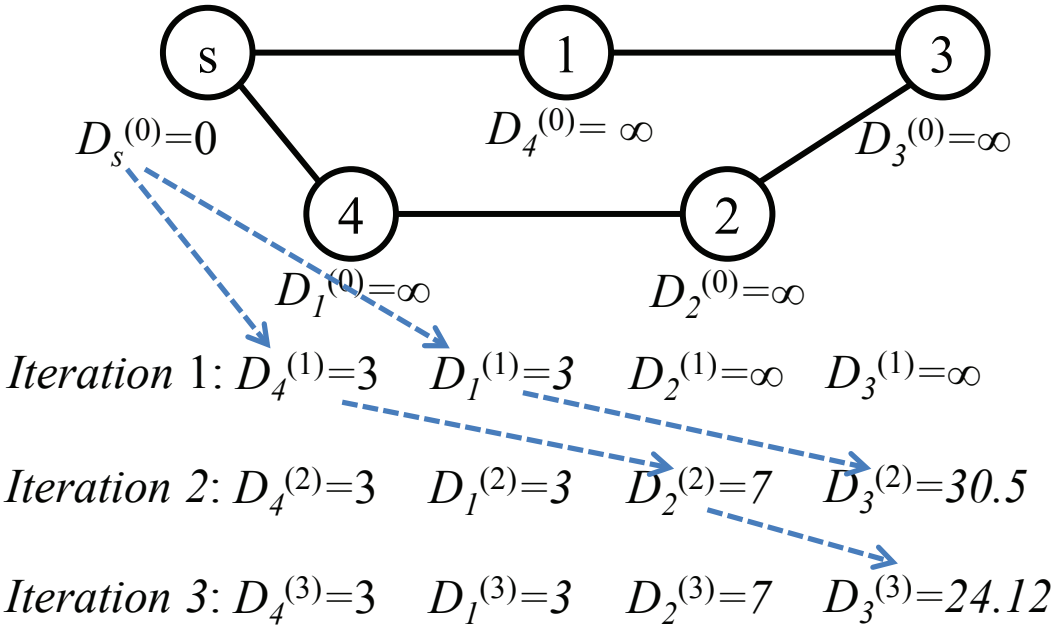


Fig. 2. Example of the value-iteration algorithm: the dotted line shows that the change in the delay value at one node affects the delay value of its neighboring nodes at the next iteration.

and then informs $h_j^{(i)}$ to each neighboring node j. In the operation phase, if node j wakes up and hears beacon signal h from node i, it sends a CTS if and only if $h \in h_j^{(i)}$. If $h \notin h_j^{(i)}$, node j returns to sleep and wakes up at the next beacon signal in $h_j^{(i)}$. Among the neighboring nodes that have sent a CTS, the sending node i forwards the packet to the node j with the smallest delay value D_j.

We now show that the above method implements the optimal truncated policy. If node j wakes up and hears beacon signal h, according to Assumption 1 the current state x_h must be at least state j, i.e., $x_h \leq j$. We now consider three cases. **Case (A):** If $j > x_{h,\max}$, there must exist another awake node that has a smaller delay value than D_j. Hence, node j has no chance to be a next-hop node, and thus it does not need to respond. **Case (B-1):** If $j \leq x_{h,\max}$ and $D_j < \hat{d}_{\text{wait}}^{(h)}(j) - t_D$, it immediately follows from Proposition 2 that $D_{x_h} < \hat{d}_{\text{wait}}^{(h)}(x_h) - t_D$. Hence, from (21), the decision must be $\hat{f}_{i,h}(x_h) = x_h$, and node x_h will receive the packet. (If $x_h = 1$, in which case Proposition 2 does not apply, we still have $\hat{f}_{i,h}(x_h) = x_h$ from (21).) In the above implementation, since both x_h and j will respond, the correct decision is reached. **Case (B-2):** If $j \leq x_{h,\max}$ and $D_j \geq d_{\text{wait}}^{(h)}(j) - t_D$, node j cannot be the next-hop node according to the truncated policy in (21). Hence, node j does not need to respond, and it can wait until the next beacon signal h' such that $D_{h'} < d_{\text{wait}}^{(h')}(j) - t_D$. *From all the cases **(A)**, **(B-1)**, and **(B-2)**, we can conclude that the above method exactly implements the optimal truncated policy, and does not require for the sending node to know the current state x_h. However, we still need Assumption 1 because node j in case **(B-2)** has to wake up at a later beacon signal.* In the next subsection, we will show that when all neighboring nodes wake up periodically, Assumption 1 is not even necessary for the implementation.

Simple Example: Using a simple network in Fig. 2, we show how nodes compute the optimal forwarding policy in a distributed manner during the configuration phase. Let $t_I = 1$ and $t_D = 2$ for all nodes. We assume that all nodes 1, 2, 3, and 4 wake up periodically, and their wake-up intervals are given by $1/r_1 = 1/r_2 = 1/r_3 = 50t_I$ and $1/r_4 = 3t_I$. Initially, all nodes set their delay values to infinity, except the sink node sets it to zero. At the first iteration of the value-iteration algorithm, only nodes 1 and 4 updates their delay values because they have a sink as a neighboring node that has a finite delay value. Since the sink always stay awake, the expected delays from nodes 1 and 4 are $t_I + t_D = 3$. At the second iteration, based on the updated delay values of node 1 and 4 at the previous iteration, nodes 2 and 3 now have a chance to update their delay values using the LOCAL-OPT algorithm. Running the LOCAL-OPT algorithm in this case is straightforward because both nodes have only one neighboring node with a finite delay value. For example, since the wake-up interval of nodes 1 is 50 beacon-signals long, the expected number of beacon signals that are sent until node 1 wakes up is 25.5. Hence, the delay value of node 3 is $25.5 + t_D + D_1 = 30.5$. Following the same procedure, the delay value of node 2 is given by 7. Note that nodes 1 and 4 do not need to change their delay values at the second iteration because none of their neighboring nodes have changed their delay values at the first iteration.

We next see how node 3 runs the LOCAL-OPT algorithm at the third iteration. By definition, the awake probabilities of nodes 1 and 2 in (3) are given by $p_{i,h} = \frac{1}{51-h}$ for $i = 1, 2$ and $h = 1, 2, \cdots, 50$. Further, the state transition probabilities in

TABLE II
EXAMPLE OF THE LOCAL-OPT ALGORITHM

h	50	49	48	47	46	45	44	43	42	$\cdots$
$d^{(h)}(1)$	5	5	5	5	5	5	5	5	5	$\cdots$
$d^{(h)}_{\text{wait}}(2)$		6	6.5	7	7.5	8	8.5	9	9.5	$\cdots$
$t_D + D_2$		9	9	9	9	9	9	9	9	$\cdots$
$d^{(h)}(2)$		6	6.5	7	7.5	8	8.5	9	9	$\cdots$
$d^{(h)}_{\text{wait}}(3)$		6	6.5	7	7.5	8	8.5	9	9.5	$\cdots$
$d^{(h)}(3)$		6	6.5	7	7.5	8	8.5	9	9.5	$\cdots$

(9) are given by $P^{(h)}_{2,1} = P^{(h)}_{3,1} = \frac{1}{51-h}$, $P^{(h)}_{2,2} = \frac{50-h}{51-h}$, $P^{(h)}_{3,2} = \frac{50-h}{(51-h)^2}$, and $P^{(h)}_{3,3} = \frac{(50-h)^2}{(51-h)^2}$ for $h = 1, 2, \cdots, 50$. By (16), $d^{(h)}(1)$ is given by $t_D + D_1 = 5$ for all beacon signals h (as reported in Table II). Since node 1 (that has the smallest delay) must wake up within 50 beacon signals, we start computing $d^{(h)}_{\text{wait}}(x_h)$ and $d^{(h)}(x_h)$ from $h = 49$. For $h = 49$, we first compute $d^{(49)}_{\text{wait}}(2) = t_I + d^{(50)}(1) = 6$ using (13). Hence, by (15), we have $d^{(49)}(2) = 6$. We can also compute $d^{(49)}_{\text{wait}}(3) = 6$ simply by using (13). Recall that since state 3 implies that there is no awake node in the neighborhood, node 3 at this state has to wait, i.e., $d^{(h)}_{\text{wait}}(3) = d^{(h)}(3)$ for $h = 1, 2, \cdots, 50$. For $h = 48$, we have

$$d^{(48)}_{\text{wait}}(2) = t_I + \frac{1}{2}d^{(49)}(1) + \frac{1}{2}d^{(49)}(2) = 6.5,$$

from (13) and thus $d^{(48)}(2) = 6.5$. We have repeated the same computation for $h = 48, 47, \cdots, 1$ and have provided the result in Table II. We have also computed $d^{(0)}(3) = 24.12$, which is the updated delay value of node 3 at iteration 3 of the value-iteration algorithm in Fig 2. Note that from beacon signal 42, $d^{(h)}_{\text{wait}}(2)$ becomes larger than $t_D + D_2 = 9$. Hence, if node 2 wakes up before beacon signal 42, the sending node 3 can forward the packet to node 2. Otherwise, if node 2 wakes up after beacon signal 42, node 3 will wait for node 1 to wake up. By (14), the set $h^{(3)}_2$ of the beacon signals from node 3 that node 2 can respond to is $\{1, 2, \cdots, 42\}$, while $h^{(3)}_1 = \{1, 2, \cdots, 50\}$.

We summarize the LOCAL-OPT algorithm in pseudo code that every node i runs during the configuration phase. We also

LOCAL-OPT Algorithm

Receive $(D^{(k-1)}_j, j \in \mathcal{N}_i)$
Sort $(D^{(k-1)}_j, j \in \mathcal{N}_i)$ in an increasing order
Let $D_1, D_2, \cdots, D_{N_i}$ be the sorted delay and $m(1), m(2), \cdots, m(N_i)$ be the corresponding node indices.
Set $\bar{h}$
for $j = 1$ to $N_i + 1$ **do**
  Set $\hat{d}^{(\bar{h})}(j)$ using (18)
  $h^{(i)}_j \leftarrow \emptyset$
end for
for $h = \bar{h} - 1$ to 0 **do**
  for $j = 1$ to $x_{h,\max}$ **do**
    Compute $\hat{d}^{(h)}_{\text{wait}}(j)$ using (19)
    if $D_j < d^{(h)}_{\text{wait}}(j) - t_D$ **then**
      $h^{(i)}_{m(j)} \leftarrow h^{(i)}_{m(j)} \cup \{h\}$
    end if
    $\hat{d}^{(h)}(j) \leftarrow \min(\hat{d}^{(h)}_{\text{wait}}(j), t_D + D_{x_h})$
  end for
end for
$h^{(i)}_{m(1)} \leftarrow h^{(i)}_{m(1)} \cup \{\bar{h}, \bar{h} + 1, \cdots\}$
return $\hat{d}^{(0)}(N_i + 1)$, $(h^{(i)}_j, j \in \mathcal{N}_i)$

summarize the value-iteration algorithm in pseudo code as follows:

Value-Iteration Algorithm

1: $D_i^{(0)} \leftarrow \infty$
2: **for** $k = 1$ to $k_{\max}$ **do**
3: Collect $D_j^{(k-1)}$ from neighboring nodes j
4: $(D_i^{(k)}, (h_j^{(i)}, j \in \mathcal{N}_i)) \leftarrow$ **LOCAL-OPT**$((D_j^{(k-1)}, j \in \mathcal{N}_i))$
5: **end for**
6: **return** $D_i^{(k)}$,$(h_j^{(i)}, j \in \mathcal{N}_i)$

During the operation phase that follows the configuration phase, each node j uses the implementation for the optimal truncated policy.

Sleep-wake Scheduling Protocol

1: **loop**
2: Set up the next time t_{wake} that node j has to wake up according the sleep-wake scheduling policy (r_j, w_j).
3: Wake up at time t_{wake}.
4: **if** Hear beacon signal h from a neighboring node i **then**
5: **if** $h \in h_j^{(i)}$ **then**
6: Respond a CTS signal to the sending node i
7: Break the loop and follow the packet relay protocol
8: **else if** There exists $h' > h$ such that $h' \in h_j^{(i)}$, **then**
9: $t_{\text{wake}} \leftarrow t_{\text{wake}} + t_I \cdot (h' - h)$
10: Go to Line 3
11: **end if**
12: **end if**
13: **end loop**

The value-iteration algorithm is a synchronous algorithm that requires all nodes to execute the value-iteration (8) in locked steps. Depending on the application setting, the following asynchronous version of the value-iteration algorithm may be more useful: each node chooses either to solve (8) or to skip it (i.e., $D_i^{(k)} = D_i^{(k-1)}$) independently, of other nodes. Then, the following proposition states the convergence of the asynchronous value-iteration algorithm.

***Proposition** 3:* If each node i updates its delay value $D_i^{(k)}$ using (8) infinitely often, then the delay values and the forwarding policies of all nodes converge to the optimal, i.e., $\lim_{k\to\infty} D_i^{(k)} = D_i^*(\vec{r}, \vec{w}_{\text{per}})$, and $\lim_{k\to\infty} f^{(k)} \in \arg\min_f D_i(\vec{r}, \vec{w}_{\text{per}}, f)$ for all nodes i

Proof: The proof follows from the standard result of Proposition 1.3.5 in [19]. ■

D. Optimal Anycast Policy for Periodic Wake-Up Processes

So far, we have developed the value-iteration algorithm and a truncated version of the local-opt algorithm, which are asymptotically optimal for a general sleep-wake scheduling policy. In this subsection, we show that for periodic wake-up patterns these algorithms are exactly optimal for appropriately chosen parameters $\bar{h}$ and $k_{\max}$. In the next section, we will then study why the periodic wake-up pattern is delay-optimal over all the other wake-up patterns.

Assume that all nodes wake up periodically ($\vec{w} = \vec{w}_{\text{per}}$). Then, each neighboring node j must wake up every $1/r_j$ time. Since an awaken node keeps awake according to Assumption 1, node j must be awake after stage $\lfloor \frac{1/r_j}{t_I} \rfloor$. If we set $\bar{h}$ to the beacon signal $\lfloor \frac{1/r_1}{t_I} \rfloor$, state $x_{\bar{h}} = 1$ is the only admissible state at stage $\bar{h}$. Then, under the periodic wake-up pattern, the result of Proposition 1 becomes stronger as follows:

***Proposition** 4:* If all neighboring nodes wake up periodically, and $\bar{h}$ is set to $\lfloor \frac{1/r_1}{t_I} \rfloor$, the truncated forwarding policy $\hat{f}_i$ is optimal, i.e.,

$$\hat{d}^{(0)}(N_i + 1) = d^{(0)}(N_i + 1).$$

Proof: Since $x_{\bar{h}} = 1$ is the only admissible state, it holds that $\hat{d}^{(\bar{h})}(x_{\bar{h}}) = d^{(\bar{h})}(x_{\bar{h}})$ for admissible states $x_{\bar{h}}$. Then, from (13) and (19), we also have $\hat{d}_{\text{wait}}^{(\bar{h}-1)}(x_{\bar{h}-1}) = d_{\text{wait}}^{(\bar{h}-1)}(x_{\bar{h}-1})$ for all admissible states $x_{\bar{h}-1}$. ($P_{x',x''}^{(h)} = 0$ for inadmissible state $x_h = x''$.) From (15) and (20), it follows that $\hat{d}^{(\bar{h}-1)}(x_{\bar{h}-1}) = d^{(\bar{h}-1)}(x_{\bar{h}} - 1)$ for all admissible states $x_{\bar{h}-1}$. By induction, we can conclude that $\hat{d}^{(0)}(N_i + 1) = d^{(0)}(N_i + 1)$. ■

Proposition 4 implies that the truncated forwarding policy becomes exactly optimal under the periodic wake-up pattern. Hence, when the wake-up pattern of neighboring nodes are periodic, i.e., $\vec{w}_i = \vec{w}_{\text{per}}$, we can completely solve the subproblem in (8).

The periodic wake-up pattern not only makes the truncated policy optimal, but also simplifies the implementation by the following proposition.

***Proposition** 5:* If all neighboring nodes wake up periodically, and $\bar{h}$ is set to $\lfloor \frac{1/r_1}{t_I} \rfloor$, the conditional delay $\hat{d}^{(h)}_{\text{wait}}(x_h)$ is non-increasing, i.e.,

$$\hat{d}^{(h-1)}_{\text{wait}}(x_{h-1}) \geq \hat{d}^{(h)}_{\text{wait}}(x_h), \tag{26}$$

for $h = 1, 2, \cdots, \bar{h} - 1$, and all admissible states x_h.

The detailed proof is provided in Appendix B. The result of Proposition 5 can be interpreted as follows: as more stages pass by, the neighboring nodes are more likely to wake up, and the conditional delay $d^{(h)}_{\text{wait}}$ then decreases. This property can further simplify the implementation of our solution. Recall that in the original truncated policy, if node j wakes up at beacon signal h and satisfies the condition $D_j + t_D \geq \hat{d}^{(h)}_{\text{wait}}(j)$, it has to sleep and wake up again at the next beacon signal when the condition is satisfied. However, under the periodic wake-up pattern, such a node j will never satisfy the condition in the following beacon signals because $\hat{d}^{(h)}_{\text{wait}}(j)$ is non-increasing. *Hence, instead of maintaining the set of $h^{(i)}_j$ of all beacon signals that it has to respond with a CTS, each neighboring node j only needs to maintain the last beacon signal that it has to respond.* Furthermore, this property provides an opportunity to reduce the complexity of the LOCAL-OPT algorithm. In [20], we provide the simplified LOCAL-OPT algorithm for the periodic wake-up pattern, whose complexity is reduced $\mathcal{O}(\hat{h}N_i^2)$ to $\mathcal{O}(\hat{h}N_i)$.

We now study the convergence properties of the value-iteration algorithm under the periodic wake-up pattern. Define $\bar{h}^{(k)}_i = \min_{j \in \mathcal{N}_i} \{ \lfloor \frac{1/r_j}{t_I} \rfloor | j \in \arg\min D^{(k-1)}_j \}$ as the maximum number of beacon signals until the neighboring node j with the smallest delay value $D^{(k-1)}_j$ wakes up. Then, the next proposition states the convergence of the value-iteration algorithm:

***Proposition** 6:* If all nodes i wake up periodically and set $\bar{h} = \bar{h}^{(k)}_i$ at each iteration k of the value-iteration algorithm, the algorithm converges to the optimal solution within N iterations, i.e., $D^{(N)}_i = D_i(\vec{r}, \vec{w}_{\text{per}}, f^{(N)}) = D^*_i(\vec{r}, \vec{w}_{\text{per}})$.

The proof is provided in Appendix D. From Proposition 6, every node needs to run the LOCAL-OPT algorithm for only N iterations, and the last forwarding policy $f^{(N)}$ is delay-optimal when all nodes wake up periodically. Hence, under the periodic wake-up pattern, the overall complexity experienced by each node i is alleviated from $O(k_{\max}\bar{h}N_i^2)$ to $O(N\bar{h}N_i)$. We remind the reader again that this computation overhead only occurs at the configuration phase.

IV. Optimal Wake-up Pattern

In the previous section, we have developed an asymptotically optimal anycast forwarding policy for a general sleep-wake policy $(\vec{r}, \vec{w})$. In this section, we fix the wake-up pattern to the periodic ($\vec{w} = \vec{w}_{\text{per}}$), i.e., all nodes wake up periodically, and study the special properties of the periodic wake-up pattern. We will show that, among all wake-up patterns, the periodic wake-up pattern and the corresponding optimal forwarding policy attain the smallest delay. Hence, they are the solution to the delay-minimization problem (5) that we originally intend to solve.

A. Fundamental Properties of Wake-up Patterns

We begin by studying the fundamental properties of the wake-up patterns. As mentioned in Subsection II-B, we have assumed that the residual time $\{R_j(t)\}_{t \geq 0}$ is a stationary and ergodic process. Hence, the CDF of the residual time must satisfy

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T 1_{\{R_j(t) \leq y\}} dt = F_{R_j}(y) \text{ almost surely}, \tag{27}$$

where $1_{\{\cdot\}}$ is an indicator function. Recall that $F^*_{R_j}(y)$ denotes the CDF of the residual time under the periodic wake-up pattern. The following proposition then shows the essential properties of the *cdf* of the residual time.

***Proposition** 7:* For any stationary and ergodic wake-up process with rate r_j, the *cdf* $F_{R_j}(y)$ of the residual time R_j satisfies the following properties:

(a) $F_{R_j}(y) \leq F^*_{R_j}(y)$,

(b) $\frac{dF_{R_j}(y)}{dy} \leq \frac{dF^*_{R_j}(y)}{dy}$ for $0 \leq y \leq \frac{1}{r_j}$.

Proof: We first show Property (a). We first estimate

$$\int_0^T 1_{\{R_j(t) < y\}} dt. \tag{28}$$

Let $t_1, t_2, \cdots$ be the sequence of times the node wakes up (as shown in Fig. 3.) To satisfy $R_j(t) < y$, time $t \in [0, T]$ must be in the shaded area, *i.e.*,

$$t \in \cup_{k=1}^{\infty} [t_k - y, t_k]. \tag{29}$$

Hence, we can express (28) as follows:

$$\int_0^T 1_{\{R_j(t)<y\}}dt \le \sum_{k=1}^{\infty}\int_0^T 1_{\{t\in[t_k-y,t_k]\}}dt \tag{30}$$

$$\le \sum_{k=1}^{\infty} y 1_{\{t_k\in[0,T+y]\}} = y\cdot\kappa_j(T+y), \tag{31}$$

where equalities in (30) and (31) are attained when the point "*" of each shaded area in Fig. 3 does not lie within other shaded areas. From (27), we have $F_{R_j}(y) \le y\lim_{T\to\infty}\frac{\kappa_j(T+y)}{T}$. Using (1), we have $F_{R_j}(y) \le yr_j$ almost surely. This establishes Property (a).

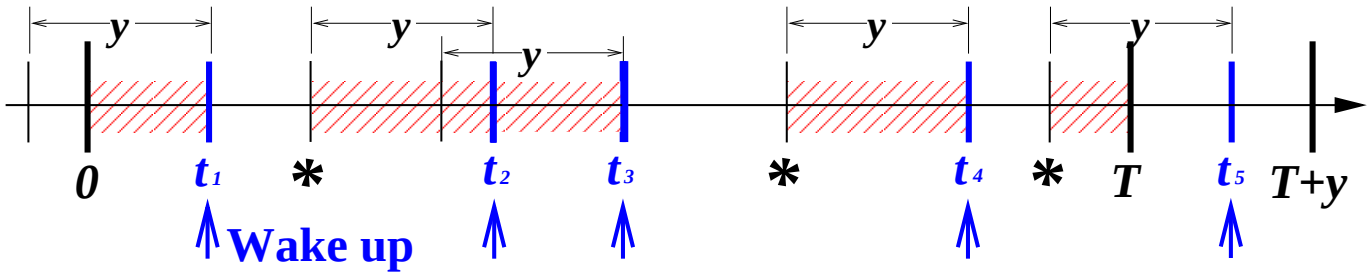


Fig. 3. Example of the sequence of times a node wakes up

We next show Property (b). To show this, we need to compute $F_{R_j}(y_2) - F_{R_j}(y_1)$ for $0 \le y_1 < y_2 \le \frac{1}{r_j}$. As we did to show Property (a), we first estimate $\int_0^T 1_{\{R_j(t)\in[y_1,y_2]\}}dt$. We follow the same logic used for showing Property (a). We replace $R_j(t) < y$ and $[t_k - y, t_k]$ with $R_j(t) \in [y_1, y_2]$ and $[t_k - y_2, t_k - y_1]$, respectively, in (28)-(30). Then, the right-hand side of (31) is replaced with $(y_2 - y_1)\cdot\kappa_j(T + y_2)$. From (27), we have $F_{R_j}(y_2) - F_{R_j}(y_1) \le (y_2 - y_1)\lim_{T\to\infty}\frac{\kappa_j(T+y_2)}{T}$. Using (1) again, we have $F_{R_j}(y_2) - F_{R_j}(y_1) \le r_j(y_2 - y_1)$, which corresponds to Property (b). ■

Proposition 7 shows that for all $0 \le y \le 1/r_j$, the *cdf* $F_{R_j}(y)$ and the derivative $\frac{dF_{R_j}(y)}{dy}$ are maximized when the wake-up pattern is periodic.[4]

Recall that in Section II-B the awake probability $p_{j,h}$ is defined as the conditional awake probability that the given node j wakes up and receives the h-th beacon-ID signal, conditioned on that it has not woken up at earlier beacon-ID signals. Then, from Proposition 7, we obtain the following two important properties about the awake probability.

***Proposition** 8:* For $h = 1, \cdots, \lfloor\frac{1/r_j}{t_I}\rfloor$, we have
(a) $p^*_{j,h-1} < p^*_{j,h}$, and **(b)** $p^*_{j,h} \ge p_{j,h}$.

Proof: (a) For $h = \lfloor\frac{1/r_j}{t_I}\rfloor$, Property (a) trivially holds by (4). For $h < \lfloor\frac{1/r_j}{t_I}\rfloor$, the numerator in (2) for $F^*_{R_j}(y)$ is a constant of $r_i \cdot t_I$, and the denominator decreases with h. Hence, Property (a) still holds.

(b) By Proposition 7(a), the denominator is minimized under the periodic wake-up pattern. Further, by Proposition 7(b), the numerator is maximized under the periodic wake-up pattern. Hence, we directly obtain Property (b). ■

Property (a) implies that under the periodic wake-up pattern, the awake probability $p^*_{j,h}$ increases with respect to the number h of the beacon-ID signals sent. Property (b) implies that the conditional awake probability is maximized when the neighboring node wakes up periodically.

B. Optimality of Periodic Wake-up Patterns

Using the properties of the periodic wake-up patterns, we show that the periodic wake-up patterns result in the smallest delay from all nodes. To show this, we first revisit the subproblem (8) that we have solved in Section III-B and in Section III-D.

Consider two scenarios:

(Scenario 1) Each neighboring node j wakes up periodically every $1/r_j$ time. The optimal forwarding policy f^*_i that we obtained in Section III-D is applied. For this scenario, we use the same notations that are used for the optimal forwarding policy, e.g, $d^{(h)}_{\text{wait}}(x_h)$, $d^{(h)}(x_h)$, $P^{(h)}_{x_{h-1},x_h}$, $h_{j,\max}$, and $x_{h,\max}$. Recall that the packet at the sending node is forwarded no later than stage $\bar{h} = h_{1,\max}$.

(Scenario 2) The wake-up process of each neighboring node j is arbitrary, but the wake-up rate is still given by r_j. We denote by $\tilde{f}_i$ the optimal forwarding policy for the given wake-up processes of the neighboring nodes. To differentiate from Scenario

[4]Proposition 7 is closely related to the standard results for renewal processes that periodic renewal processes have the smallest mean residual time [21, Chapter 5.2]. These standard results require the wake-up intervals to be independent, while Proposition 7 does not require such an assumption. Since we were unable to find a result in the literature that covered the non-independent case, we have provided the full proof here.

1, we put a tilde ($\sim$) on all notations in this scenario, *e.g.*, $\tilde{d}^{(h)}_{\text{wait}}(x_h)$, $\tilde{d}^{(h)}(x_h)$, $\tilde{P}^{(h)}_{x_{h-1},x_h}$, etc. Similarly, node j must have woken up no later than stage $\tilde{h}_{j,\max}$, and let $\tilde{x}_{h,\max}$ be the node with the smallest delay among the nodes that must be awake at stage h. By simply setting $\tilde{h}_{j,\max}=\infty$ and $\tilde{x}_{h,\max}=N_i+1$, we can still use these notations for the wake-up processes under which there is no such a finite limit point. For instance, if all neighboring nodes j follow the Poisson wake-up pattern, then the residual times until they wake up are independent exponential random variables, and we thus have $\tilde{h}_{j,\max}=\infty$ for $j\in\mathcal{N}_i$ and $\tilde{x}_{h,\max}=N_i+1$ for all $h\geq 0$. Since the awake probability is maximized when nodes wake up periodically, it follows that $h_{j,\max}\leq\tilde{h}_{j,\max}$ and $x_{h,\max}\leq\tilde{x}_{h,\max}$. Further, the optimal policy $\bar{f}_i$ must satisfy the necessary conditions (15) and (14).

We now compare the delays from both scenarios.

***Proposition** 9:* $d^{(h)}(x_h)\leq\tilde{d}^{(h)}(x_h)$ for $h=0,1,\cdots,\bar{h}$ and $x_h\leq x_{h,\max}$,

Proof: We prove this by induction. By (16), we must have $d^{(\bar{h})}(1)=\tilde{d}^{(\bar{h})}(1)=t_I+t_D+D_1$. At stage $\bar{h}$, node 1 must be awake under the periodic wake-up process (i.e., $x_{\bar{h},\max}=1$). Hence, Proposition 9 holds for $h=\bar{h}$.

Assume that $d^{(h)}(x_h)\leq\tilde{d}^{(h)}(x_h)$ holds for $h=h'+1,h'+2,\cdots,\bar{h}$ and $x_h\leq x_{h,\max}$. We then show that this also holds for $h=h'$. From (13), we have the following inequality:

$$\begin{aligned}
&\tilde{d}^{(h')}_{\text{wait}}(x_{h'})-t_I=\textstyle\sum_{x_{h'+1}=1}^{x_{h'}}\tilde{P}^{(h'+1)}_{x_{h'},x_{h'+1}}\tilde{d}^{(h'+1)}(x_{h'+1})\\
&\geq\textstyle\sum_{x_{h'+1}=1}^{x_{h'}}\tilde{P}^{(h'+1)}_{x_{h'},x_{h'+1}}d^{(h'+1)}(x_{h'+1}) &(32)\\
&\geq\textstyle\sum_{x_{h'+1}=1}^{x_{h'}}P^{(h'+1)}_{x_{h'},x_{h'+1}}d^{(h'+1)}(x_{h'+1}) &(33)
\end{aligned}$$

To obtain (32), we have used the induction hypothesis. The inequality in (33) can be understood as follows: according to Proposition 8(b), neighboring nodes are more likely to wake up under the periodic wake-up patterns, and thus the delay is also minimized under the periodic wake-up pattern. To obtain (33), we have used Lemma 1 in Appendix C, where $L=x_{h'}$, $\alpha^{(1)}_j=p'_{j,h'+1}$ (equivalently, $\beta^{(1)}_j=\tilde{P}^{(h'+1)}_{x_{h'},j}$), $\alpha^{(2)}_j=p_{j,h'+1}$ (equivalently, $\beta^{(2)}_j=P^{(h'+1)}_{x_{h'},j}$), and $\theta_j=d^{(h'+1)}(j)$. Since $\theta_1\leq\cdots\leq\theta_{\bar{h}}$ by Proposition 23, and $\alpha^{(1)}_j\leq\alpha^{(2)}_j$ by Proposition 8, the conditions for the lemma hold.

Since (33) is equal to $d^{(h')}_{\text{wait}}(x_{h'})-t_I$, we have $d^{(h')}_{\text{wait}}(x_{h'})\leq\tilde{d}^{(h')}_{\text{wait}}(x_{h'})$. Then, from (15), we have

$$\begin{aligned}
\tilde{d}^{(h')}(x_{h'})&=\min(\tilde{d}^{(h')}_{\text{wait}}(x_{h'}),t_D+D_{x_{h'}})\\
&\geq\min(d^{(h')}_{\text{wait}}(x_{h'}),t_D+D_{x_{h'}})=d^{(h')}(x_{h'}).
\end{aligned}$$

Hence, Proposition 9 holds for $h=h'$. By induction, this also holds for $h=0,1,\cdots,\bar{h}$. ■

From Proposition 9, we can infer that $d^{(0)}(N_i+1)\leq\tilde{d}^{(0)}(N_i+1)$, which implies $D^{(k)}_i\leq\tilde{D}^{(k)}_i$ in the value iteration algorithm. Hence, when the delays from the neighboring nodes are given, the delay from the sending node i is minimized when the neighboring nodes wake up periodically and the corresponding optimal forwarding policy is applied.

We next apply this result to the Stochastic Shortest Path (SSP) problem in (6). Assume that each node i can control the wake-up patterns $\vec{w}_i$ of its neighboring nodes j, as well as its forwarding policy f_i. Then, to minimize (6) with respect to $(\vec{w},f)$, every node i should carry out the following value-iteration algorithm, which is a generalized version of (8): for $k=1,2,\cdots$,

$$D^{(k)}_i=\min_{\vec{w}_i,f_i}(D_{\text{hop},i}(\vec{r},\vec{w}_i,f_i)+\sum_{j\in\mathcal{N}_i}q_{i,j}(\vec{r},\vec{w}_i,f_i)D^{(k-1)}_j).$$

In this equation, the expected one-hop delay $D_{\text{hop},i}(\vec{r},\vec{w}_i,f_i)$ and the probability $q_{i,j}(\vec{r},\vec{w}_i,f_i)$ that node i forwards the packet to node j depend only on $\vec{w}_i$ (instead of $\vec{w}$). This is because the wake-up patterns of nodes other than the neighboring nodes do not affect the one-hop delay and the transition probability from node i. From Proposition 9, $D^{(k)}_i$ is maximized when $\vec{w}_i$ is given by $\vec{w}_{\text{per}}$ and the corresponding optimal forwarding policy is chosen. Hence, the following proposition holds:

***Proposition** 10:* $\min_f D_i(\vec{r},\vec{w}_{\text{per}},f)=\min_{\vec{w},f}D_i(\vec{r},\vec{w},f)$ for all nodes i.

Let $f^*(\vec{r})$ be the optimal forwarding policy for a given sleep-wake scheduling policy $(\vec{r},\vec{w}_{\text{per}})$. From Proposition 6, $f^*(\vec{r})$ is equal to $f^{(N)}$ in the value-iteration algorithm. Then, Proposition 10 implies that $(\vec{w}_{\text{per}},f^*(\vec{r}))$ is the solution to the delay-minimization problem (5).

V. Collision Resolution Protocols

In this section, we extend our basic-packet forwarding and sleep-wake scheduling protocols to account for the cases when collisions occur. (These protocols originate from our previous paper [18].) We classify collisions into *collision by multiple receivers* and *collision by multiple senders*. We provide here the main insight of how to resolve these collisions. The detailed protocol is provided in Appendix E.

Collision by multiple receivers: When multiple eligible next-hop nodes wake up and hear the same beacon-ID signal, these nodes will send CTS simultaneously, which collide at the sender's end. In this case, the sender only know the existence of

multiple awake nodes from the colliding CTSs, but does not know which node can deliver a packet to the sink most quickly. To address this problem, the nodes use the following deterministic back-off. For each sending node i, its eligible next-hop nodes j are assigned their priority $b_{i,j}$ such that the node j with the k-th smallest delay is assigned $b_{i,j} = k$. Then, once the collision occur, the sending node i does not acknowledge the CTS. If each awake eligible node j does not receive an ACK_cts (acknowledgement of the CTS) from the sender, it waits for $b_{ij}t_A$ time and resend the CTS. Then, among the awake nodes, the node with the smallest delay will send the CTS first and receive the packet from the sender. During the time, the other nodes will hear the CTS from the first priority node or packet transmission from the sender. Then, the other nodes go back to sleep.

Collision by multiple senders: If two or more events occur concurrently in different locations, their first event-reporting packets will be delivered to the sink. If some of these packets converge in a place, their packet transmission or beacon-ID signaling will interfere with the others' transmission power. In this case, the nodes involved in this collision use a random back-off technique: when a packet transmission or the beacon-ID signaling fails due to the collision, the involved senders suspend transmission for a random amount of time, and then resume their transmission if the medium is clear. In the case when the colliding senders are hidden terminals, i.e., they are not in the transmission ranges of each other, the senders cannot recognize the collision, but a newly waken node in the common transmission range will hear the colliding beacon-ID signals or packet transmission. In this case, the awake node broadcasts a strong noise signal for a sufficiently long time to inform the hidden terminals the collision. Then, this noise signal will be detected in the acknowledgement period following the packet transmission or the beacon-ID signal and then the sending node can run the random back-off mechanism described above.

VI. Simulation Results

In this section, we provide simulation results to evaluate the delay performance of the proposed solution. In this paper, our analytical results have captured the delay gain from both delay-optimal anycast policy and delay-optimal wake-up pattern, which is the periodic wake-up pattern. Since extensive simulations in [18] have already shown substantial gains of the optimal anycast policy for the Poisson wake-up pattern over other anycast policies, here we only focus on evaluating the gain from the periodic wake-up pattern over the Poisson wake-up pattern. To simulate more realistic scenarios, we randomly deploy 690 nodes over a 1 km-by-1km area with obstructions as shown in Fig 4(c). We set the transmission range to 70 m and the duration t_I and t_D to 6 ms and 30 ms, respectively. We simulated our proposed solutions using Matlab. For the case when multiple packets converge to the same area, we have used our extended packet forwarding protocol in our technical report [20]. We have intentionally generated 50 events at each node and took the average on the measured delay. We have also intentionally placed the events in neighboring nodes to observe how the collision will affect the overall delay performance.

We will compare the delay performance of the following algorithms:

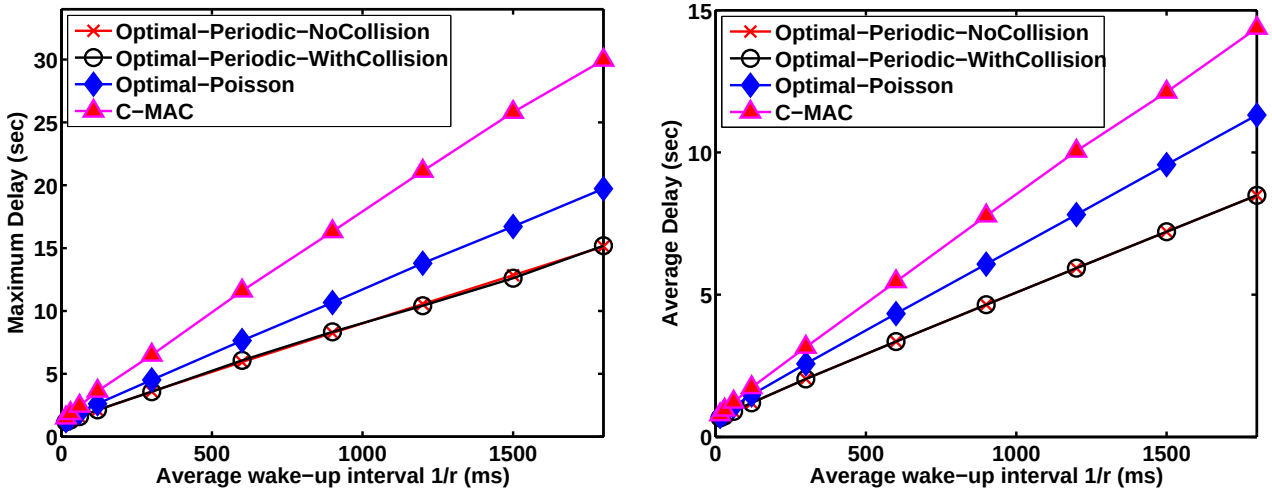
Optimal-Periodic-NoCollision: This corresponds to the optimal anycast forwarding policy with periodic wake-up patterns, and the effect of collision is ignored. We obtain the expected delay simply from the output of the value iteration algorithm in (8).

Optimal-Periodic-WithCollision: This corresponds to the optimal anycast policy with periodic wake-up patterns. We simulate the policy with the collision resolution component in Appendix E.

Optimal-Poisson: This corresponds to the optimal anycast forwarding policy in [18] with Poisson wake-up patterns. We also simulate the policy with the same collision resolution component in Optimal-Periodic-WithCollision. (Refer to [18] to see the performance advantage of **Optimal-Poisson** over existing solutions (including CMAC) over different simulation environments.)

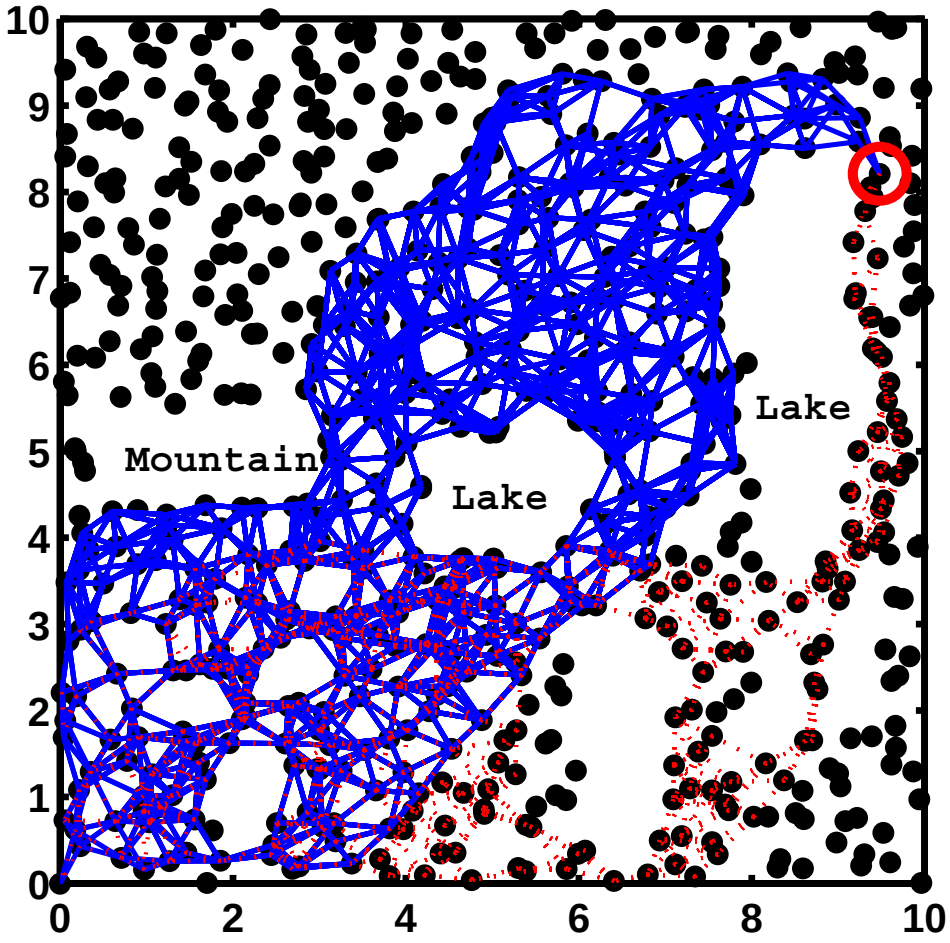
CMAC (Convergent MAC): This corresponds to the heuristic algorithm with Poisson wake-up pattern that was proposed in [11]. CMAC uses geographical information to choose the packet forwarding policy. Let D and R be the random variables that denote the one-hop delay and process in reducing the Euclidean distance to the sink when a packet is forwarded to the next-hop node. Then, under CMAC, each node i selects the set of eligible next-hop nodes that can minimize the expected normalized-latency $E[D/R]$. Since the performance advantage of CMAC over other existing anycast-based heuristics has been extensively studied in [11] and [18], we only compare the performance of our optimal algorithm to that of CMAC.

In Fig. 4(a) and Fig. 4(b), we compare the maximum and the average expected end-to-end delays, respectively, over all nodes as we vary wake-up rates r. We observe that 'Optimal-Periodic-NoCollision' and 'Optimal-Periodic-WithCollision' significantly reduce the end-to-end delay compared with the other algorithms. This is consistent with our result that the periodic wake-up pattern is delay-optimal. We also observe the significant performance gap between 'CMAC' and 'Optimal-Periodic-WithCollision.' To explain this performance gap, we show in Fig. 4(c) the possible routing paths under both algorithms. Under CMAC, packets tend to be forwarded to the nodes with higher progress. However, overall the packets may take longer paths to go around the obstructions. In contrast, under 'Optimal-Periodic-Withcollision,' the next-hop nodes are chosen by delay. Hence, it is possible for a packet to be first forwarded to nodes with negative progress, if doing so reduces the delay beyond the next-hop node. For example, in Fig. 4(c), 'Optimal-Periodic-WithCollision' results in paths that are shorter than those under 'CMAC.' From Fig. 4(c), we can infer that if there is no strong correlation between distance and delay (e.g. where there are obstructions), the heuristic anycast solutions such as CMAC can perform poorly. (In [18], we have also showed that the



(a) Delay Comparison

(b) Average Delay



(c) Node Deployment and Possible Paths

Fig. 4. (a) Maximum and (b) Average delays under different wake-up rate r and (b) Node deployment and the possible routing paths for 300 ms average wake-up interval under Optimal-Periodic-NoCollision (blue solid lines) and CMAC (red dotted lines). The paths under Optimal-Periodic-NoCollision pass through the network diagonally while the paths under CMAC circumvent the lake.

performance gap between the CMAC and the optimal anycast policy becomes significant when the wake-up rates of nodes are heterogenous.) Finally, we can observe from Fig. 4(a) that the performance gap between 'Optimal-Periodic-NoCollision' and 'Optimal-Periodic-WithCollision' is negligible over average wake-up intervals (from 30 ms to 1800 ms). The reason behind this result is that the beacon-ID duration 6 ms is much smaller than the wake-up interval, so that there is little chance that multiple nodes wake up and respond simultaneously. Hence, as long as collisions are resolved properly, they will not significantly

impact the performance of our proposed solution at reasonable wake-up rates.

VII. Conclusion

In this paper, we have studied the optimal anycast forwarding and sleep-wake scheduling policies that minimize the end-to-end delay. We have shown that among all wake-up patterns with the same wake-up rate, the periodic wake-up pattern maximizes the probability that a neighboring node wakes up at each beacon signal. Using this result, we have developed the optimal anycast forwarding algorithms for periodic wake-up patterns and have shown that the algorithms guarantee the minimum end-to-end delay of all nodes for given wake-up rates (which correspond to given energy budgets). Through simulation results, we have illustrated the benefits of using asynchronous periodic sleep-wake scheduling.

Appendix A
Proof of Proposition 1

Proof: We first show by induction that

$$\hat{d}^{(h)}(x_h) - d^{(h)}(x_h) \leq \Pr(H > \bar{h}|x_h) E[H - \bar{h}|H > \bar{h}] \cdot t_I. \tag{34}$$

holds for $h \leq \bar{h}$ and all admissible states $x_h > 0$. At stage $\bar{h}$, if $x_{\bar{h}} = 1$, we have $\hat{d}^{(\bar{h})}(1) - d^{(\bar{h})}(1) = 0$ from (16) and (18), and thus (34) holds. If $x_{\bar{h}} > 1$, from (17), it holds that $d^{(\bar{h})}(x_{\bar{h}}) \geq t_I + D_1$. Hence, using (18) and (22), we have

$$\hat{d}^{(\bar{h})}(x_{\bar{h}}) - d^{(\bar{h})}(x_{\bar{h}}) \leq E[H - \bar{h}|H > \bar{h}] \cdot t_I.$$

Since $x_{\bar{h}} > 1$, i.e., node 1 has not woken up until stage $\bar{h}$, we have $\Pr(H > \bar{h}|x_{\bar{h}}) = 1$. Hence, (34) holds for $h = \bar{h}$.

We now assume that (34) holds for stage $h+1$. Since $\hat{d}^{(h+1)}_{\text{wait}}(1) = d^{(h+1)}_{\text{wait}}(1)$, using (13) and (19), we have

$$\begin{aligned}
&\hat{d}^{(h)}_{\text{wait}}(x_h) - d^{(h)}_{\text{wait}}(x_h) \\
&\leq \sum_{x_{h+1}=2}^{x_h} P^{(h+1)}_{x_h, x_{h+1}} \Pr(H > \bar{h}|x_{h+1}) E[H - \bar{h}|H > \bar{h}] \cdot t_I \\
&= \Pr(H > \bar{h}|x_h) E[H - \bar{h}|H > \bar{h}] \cdot t_I.
\end{aligned} \tag{35}$$

From (15) and (20), we have $\hat{d}^{(h)}(x_h) - d^{(h)}(x_h) \leq \hat{d}^{(h)}_{\text{wait}}(x_h) - d^{(h)}_{\text{wait}}(x_h)$. Hence, from (35), Inequality (34) holds for h. Then, by induction, (34) holds for all $h = 0, 1, \cdots, \bar{h}$.

Since $x_0 = N_i + 1$ with probability 1, it holds that $\Pr(H > \bar{h}|x_0 = N_i + 1) = \Pr(H > \bar{h})$. Hence, for $h = 0$, we have

$$\begin{aligned}
&\hat{d}^{(0)}(N_i + 1) - d^{(0)}(N_i + 1) \\
&\leq \Pr(H > \bar{h}) E[H - \bar{h}|H > \bar{h}] \cdot t_I &(36)\\
&= E[(H - \bar{h}) 1_{\{H > \bar{h}\}}] \cdot t_I, &(37)
\end{aligned}$$

where $1_{\{\cdot\}}$ is an indicator function. From (36), Property (a) follows. Since $E[H] < \infty$, (37) must converge to 0 as $\bar{h}$ increases. Hence, Property (b) follows. ■

Appendix B
Proof of Proposition 5

Proof: We prove by induction that $\hat{d}^{(h-1)}_{\text{wait}}(x) \geq \hat{d}^{(h)}_{\text{wait}}(x)$ holds for $h = \bar{h} - 1, \cdots, 1, 0$ and admissible states $x_h = x$. At stage $\bar{h} - 1$, from (19), we have $\hat{d}^{(\bar{h}-1)}_{\text{wait}}(x) = t_I + \hat{d}^{(\bar{h})}(1)$ because state 1 is the only admissible state at stage $\bar{h}$. By (20), we have $\hat{d}^{(\bar{h}-1)}_{\text{wait}}(x) = t_I + t_D + D_1$. At stage $\bar{h} - 2$, we must have $\hat{d}^{(\bar{h}-2)}_{\text{wait}}(x) \geq t_I + t_D + D_1$ since $\hat{d}^{(\bar{h}-1)}(x_{\bar{h}-1}) \geq t_D + D_1$. Thus, it holds that $\hat{d}^{(\bar{h}-2)}_{\text{wait}}(x) \leq \hat{d}^{(\bar{h}-1)}_{\text{wait}}(x)$ for admissible states $x_{\bar{h}-1} = x$.

Assume that $d^{(h-1)}_{\text{wait}}(x) \geq d^{(h)}_{\text{wait}}(x)$ holds for $h = h' + 1, h' + 2, \cdots, \bar{h}$ and admissible states $x_h = x$. We then show that this also holds for $h = h'$. To this end, we need the following lemma:

Lemma 1: Suppose $\alpha^{(1)}_j$, $\alpha^{(2)}_j$, $\beta^{(1)}_j$, $\beta^{(2)}_j$, and θ_j for $j = 1, \cdots, L$ such that $0 \leq \alpha^{(1)}_j \leq \alpha^{(2)}_j \leq 1$, $\alpha^{(m)}_L = 1$, $\beta^{(m)}_j = \prod_{k=1}^{j-1}(1 - \alpha^{(m)}_k)\alpha^{(m)}_j$ for $m = 1, 2$, and $\theta_1 \leq \theta_2 \leq \cdots \leq \theta_L$. Then, the following inequality holds:

$$\sum_{j=1}^{L} \beta^{(1)}_j \theta_j \geq \sum_{j=1}^{L} \beta^{(2)}_j \theta_j. \tag{38}$$

The detailed proof is provided in Appendix C. Lemma 1 has the following interpretation. Assume that there are two users $m = 1, 2$ and each user m picks up at least one θ_j's from $\{\theta_1, \theta_2, \cdots, \theta_L\}$ independently of the other user. $\alpha^{(m)}_j$ is the

probability that user m will pick θ_j, independently of whether it picks other θ_k's ($k \neq j$). Since $\alpha_L^{(m)} = 1$, at least θ_L must be picked up by each user. If $\theta_1 \leq \theta_2 \leq \cdots \leq \theta_L$, then for the user with a larger value of $\alpha_j^{(m)}$, the expected value of the *smallest* θ_j picked will be lower.

Using Lemma 1, we can show the following inequality: for all $x \leq x_{h',\max}$

$$\begin{aligned} d_{\text{wait}}^{(h'-1)}(x) =& t_I + \sum_{x'=1}^{x} P_{x,x'}^{(h')} d^{(h')}(x') \\ \geq& t_I + \sum_{x'=1}^{x} P_{x,x'}^{(h'+1)} d^{(h')}(x'). \end{aligned} \tag{39}$$

Let L in Lemma 1 be x. For $m = 1, 2$, let $\alpha_j^{(m)} = p_{j,h'-1+m}$ if $1 \leq j < L$, and let $\alpha_L^{(m)} = 1$. Since $\beta_j^{(m)} = \prod_{h'=1}^{j-1}(1 - \alpha_{h'}^{(m)})\alpha_j^{(m)}$, $\beta_j^{(m)}$ is given by $P_{x,j}^{(h'-1+m)}$ from (9). Note that under the periodic wake-up process, the awake probability $p_{j,h}$ in (3) increases with h, which means $0 \leq \alpha_j^{(1)} \leq \alpha_j^{(2)} \leq 1$. Let $\theta_j = d^{(h')}(j)$. By Proposition 23, we have $d^{(h')}(1) \leq d^{(h')}(2) \leq \cdots \leq d^{(h')}(x_{h',\max})$, which satisfies the condition $\theta_1 \leq \theta_2 \leq \cdots \leq \theta_L$. Since all conditions for $\alpha_j^{(m)}$, $\beta_j^{(m)}$, and θ_j ($j = 1, 2, \cdots, L$ and $m = 1, 2$) are satisfied, we obtain the inequality in (39) from Lemma 1.

Combining the induction hypothesis and (15), we can obtain $d^{(h')}(x') \geq d^{(h'+1)}(x')$ for $1 \leq x' \leq x_{h'+1,\max}$. Since $P_{x,x'}^{(h'+1)} = 0$ for $x' > x_{h'+1,\max}$, we can obtain the following inequality from (39): for $x \leq x_{h',\max}$,

$$\begin{aligned} & t_I + \sum_{x'=1}^{x} P_{x,x'}^{(h'+1)} d^{(h')}(x') \\ & \geq t_I + \sum_{x'=1}^{x} P_{x,x'}^{(h'+1)} d^{(h'+1)}(x') \\ & = d_{\text{wait}}^{(h')}(x). \end{aligned} \tag{40}$$

Combining (39) and (40), we have $d_{\text{wait}}^{(h'-1)}(x) \geq d_{\text{wait}}^{(h')}(x)$ for $1 \leq x \leq x_{h',\max}$. By induction, Proposition 5 follows. ■

APPENDIX C
PROOF OF LEMMA 1

Proof: We prove this lemma by induction. First, the lemma holds for $L = 1$ because $\alpha_1^{(1)} = \alpha_1^{(2)} = 1$ and

$$\beta_1^{(1)}\theta_1 = \alpha_1^{(1)}\theta_1 = \alpha_1^{(2)}\theta_1 = \beta_1^{(2)}\theta_1.$$

We now assume that (38) holds for $L = 1, 2, \cdots, K-1$ and suppose $L = K$. Let $\tilde{\alpha}_j^{(m)} \triangleq \alpha_{j+1}^{(m)}$, $\tilde{\theta}_j \triangleq \theta_{j+1}$, and

$$\tilde{\beta}_j^{(m)} \triangleq \frac{\beta_{j+1}^{(m)}}{1 - \alpha_1^{(m)}} = \prod_{k=1}^{j-1}(1 - \tilde{\alpha}_k^{(m)})\tilde{\alpha}_j^{(m)}$$

for $m = 1, 2$ and $j = 1, 2, \cdots, K-1$. Then, by induction hypothesis, we have

$$\sum_{j=1}^{K-1} \tilde{\beta}_j^{(1)}\tilde{\theta}_j \geq \sum_{j=1}^{K-1} \tilde{\beta}_j^{(2)}\tilde{\theta}_j. \tag{41}$$

Using the above, we can obtain the following inequality:

$$\begin{aligned} \sum_{j=1}^{K} \beta_j^{(1)}\theta_j &= \alpha_1^{(1)}\theta_1 + \sum_{j=2}^{K} \beta_j^{(1)}\theta_j \\ &= \alpha_1^{(1)}\theta_1 + (1 - \alpha_1^{(1)}) \sum_{j=1}^{K-1} \tilde{\beta}_j^{(1)}\tilde{\theta}_j \\ &\geq \alpha_1^{(1)}\theta_1 + (1 - \alpha_1^{(1)}) \sum_{j=1}^{K-1} \tilde{\beta}_j^{(2)}\tilde{\theta}_j \qquad (42) \\ &= \alpha_1^{(1)} \left(\theta_1 - \sum_{j=1}^{K-1} \tilde{\beta}_j^{(2)}\tilde{\theta}_j \right) + \sum_{j=1}^{K-1} \tilde{\beta}_j^{(2)}\tilde{\theta}_j. \qquad (43) \end{aligned}$$

To obtain (42), we have used (41). Since $\sum_{j=1}^{K-1} \tilde{\beta}_j^{(2)} \tilde{\theta}_j$ is a weighted average of $\theta_2, \theta_3, \cdots, \theta_K$, and all these values are no smaller than θ_1, the term $(\theta_1 - \sum_{j=1}^{K-1} \tilde{\beta}_j^{(2)} \tilde{\theta}_j)$ is non-positive. Since $\alpha_1^{(1)} \leq \alpha_1^{(2)}$, we can rewrite (43) as

$$\begin{aligned}\sum_{j=1}^{K} \beta_j^{(1)} \theta_j &\geq \alpha_1^{(2)} \left(\theta_1 - \sum_{j=1}^{K-1} \tilde{\beta}_j^{(2)} \tilde{\theta}_j \right) + \sum_{j=1}^{K-1} \tilde{\beta}_j^{(2)} \tilde{\theta}_j \\ &= \alpha_1^{(2)} \theta_1 + (1 - \alpha_1^{(2)}) \sum_{j=1}^{K-1} \tilde{\beta}_j^{(2)} \tilde{\theta}_j. \\ &= \sum_{j=1}^{K} \beta_j^{(2)} \theta_j. \end{aligned} \tag{44}$$

Hence, (38) holds for $L = K$. By induction, the result of the lemma follows. ■

APPENDIX D
PROOF OF PROPOSITION 6

Proof: To show the convergence within N iterations, we first show that there exists an acyclic optimal solution, which minimizes the delays from all nodes simultaneously for given sleep-wake scheduling policy $(\vec{r}, \vec{w}_{\text{per}})$, and does not incur any cyclic routing paths. Let f denote an optimal solution. Then, this optimal policy must satisfy the Bellman equation in (7). Hence, for each node i, f_i must minimize the R.H.S. of the sub-problem (8), when the delays of other nodes are given by $D_j^{(k-1)} = D_j^*(\vec{r}, \vec{w}_{\text{per}})$. $D_i^*(\vec{r}, \vec{w}_{\text{per}})$ must be the corresponding delay value $D_i^{(k)}$ in (8). In the sub-problem, to be an eligible next-hop node under the optimal policy f, the neighboring nodes j must satisfy $D_j^*(\vec{r}, \vec{w}) + t_D \leq \hat{d}_{\text{wait}}^{(h)}(j)$ for some h. By repeatedly applying Proposition 5, we have $\hat{d}_{\text{wait}}^{(0)}(N_i + 1) \geq \hat{d}_{\text{wait}}^{(h)}(j)$. Hence, all eligible next-hop nodes j of node i must satisfy $\hat{d}_{\text{wait}}^{(0)}(N_i + 1) \geq D_j^*(\vec{r}, \vec{w}) + t_D$. Since $\hat{d}_{\text{wait}}^{(0)}(N_i + 1)$ corresponds to $D_i^*(\vec{r}, \vec{w})$, we have $D_j^*(\vec{r}, \vec{w}) < D_i^*(\vec{r}, \vec{w})$. This implies that under policy f, a packet at a node i will only be forwarded to a node j, whose delay value $D_j^*(\vec{r}, \vec{w})$ is smaller than $D_i^*(\vec{r}, \vec{w})$. Hence, the solution does not incur any cyclic path.

We have shown the existence of an acyclic solution. Then, based on the proof in [19, Page 107], $D_i^{(h)}$ converges to $D_i^*(\vec{r}, \vec{w})$ for each i within N iterations, and $f^{(N)}$ becomes the corresponding optimal forwarding policy. ■

APPENDIX E
COLLISION-RESOLVING PROTOCOL

In this section, we explain each step of the collision resolution protocols at the receiver's end (Fig. 5) and at the sender's end (Fig. 6). Each step in the sender's protocol is labeled as "S#" in Fig. 6, while that in the receiver's protocol is labeled as "R#" Fig. 5. We first identify the steps that correspond to the basic protocol in Section II. When a node i has a packet to relay, it repeatedly does the following: sends the beacon-ID signal and listens to the medium (S1→S2↔S3). If an eligible next-hop node j wakes up and hears the beacon-ID signal (R1→R2→R3→R4→R5), then it responds with a CTS (R6). If this CTS is received by the sender without a collision, the sending node i will acknowledge the CTS (S4→S5→S6 and R6→R7→R8), and transmit the packet to node j (S7→S8 and R9→R10→R11) until the packet is successfully forwarded. Then, the sender will stay awake for a while to relay the subsequent packets (S9), and the receiving node then becomes a transmitting node and follows a similar procedure to find its next-hop node (R12→S1). The steps from S1 to S9 and from R1 to R12 correspond to the basic packet-forwarding and sleep-wake scheduling protocols that we have introduced in Section II.

Along with these basic steps, both sender and receiver need additional mechanism to accommodate the possibility of collisions. If the awake node hears a signal, but the signal is not an ID signal (R4→R13), there are two possibilities. Case 1: An event-reporting packet is being forwarded in the neighborhood or Case 2: multiple signals are colliding. If Case 1 occurs, the awake node should not interfere with the packet transmission. If Case 2 occurs, the node should inform multiple sending nodes of the collision and let them run random back-off. The receiving node can distinguish these cases by hearing the medium for a period of time $t_D + 2t_A$, which is longer than the duration of packet transmission. In Case 1, the awake node can hear an ACK_packet (if the receiving node is in the neighborhood) or nothing (if the receiving node is outside of the transmission range) right after the packet transmission. In this case, the awake node can return to sleep (R16) because there is no need for relaying a packet. In Case 2, the awake node broadcasts a noise signal for a period of time $3t_I$ (the duration of three beacon signaling iterations) to inform the senders of the collision (R15). Note that if the ACK_packet collides with other signals (e.g. beacon signals), the awake node will recognize the current situation as Case 2, and thus will send the noise signal. However, since the awake node have waited for an ACK for $t_D + 2t_A$, the noise signal does not overlap the ACK_packet and thus will not disturb the packet transmission.

At the sender's end, if the sender hears a signal right after a beacon-ID signal, but is unable to decode, there are two

possibilities. Case 1: multiple neighboring nodes have sent CTSs simultaneously, or Case 2: other signals are colliding. The sender can distinguish these cases using the duration of the signal (S11). If it is the duplicated CTSs, the duration will be t_A, and then silence will follow. If it is a mixture of other types of signals, the duration will be longer than t_A. In Case 1, the sender i will not send an ACK to the awake nodes. Thus, each awake node runs a deterministic back-off and then retransmit the CTS (R18→R17). If any awake node j hears an CTS or a packet transmission during back-off, this means that a node with a smaller back-off time has sent the CTS. Hence, the node j returns to sleep (R18→R15). During this time, the sender waits for the first CTS (S12) and forwards the packet to the node with the smallest back-off (and also smallest delay). In Case 2 where there is another sending node in the neighborhood, the sender suspends sending beacon-ID signals for a random amount of time (S10).

The actual packet transmission can be corrupted by the signals from other nodes. In this case, a pair of the sending node and the receiving node retries the packet transmission $K_{\max}$ times (S14 and R20). If all trials are unsuccessful, this implies that there are other nodes that keep sending signals and interrupting the transmission. In this case, the sender gives up the packet transmission and waits for a random amount of time (R14→R10). During the same time, the receiver broadcasts the noise signal to inform the other sending nodes of the collision (R15) and to let them run a random back-off.

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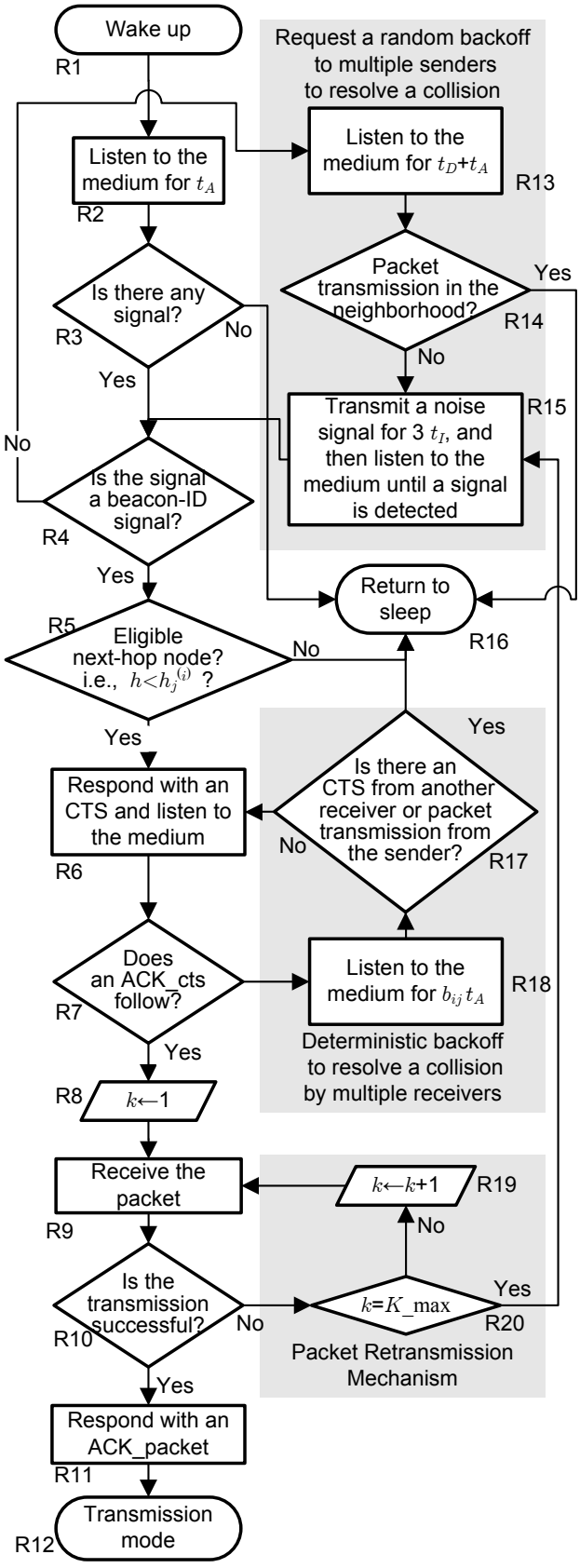

Fig. 5. Collision resolving protocol (Receiver)

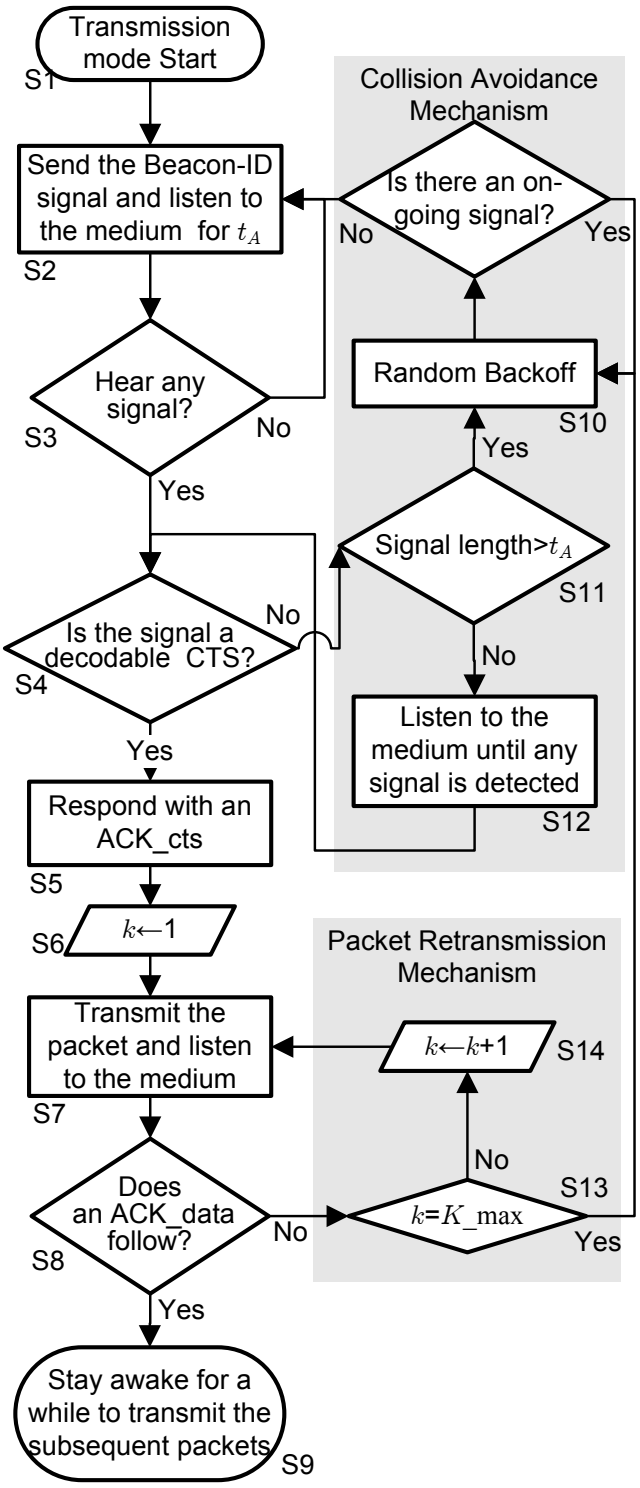

Fig. 6. Collision resolving protocol (Sender)