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Energy-Efficient SINR-Based Routing for Multi-hop Wireless Networks

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Abstract—In this paper, we develop an energy-efficient routing scheme that takes into account the interference created by existing flows in the network. The routing scheme chooses a route such that the network expends the minimum energy satisfying with the minimum constraints of flows. Unlike previous works, we explicitly study the impact of routing a new flow on the energy consumption of the network. Under certain assumptions on how links are scheduled, we can show that our proposed algorithm is asymptotically (in time) optimal in terms of minimizing the average energy consumption. We also develop a distributed version of the algorithm. Our algorithm automatically detours around a congested area in the network, which helps mitigate network congestion and improve overall network performance. Using simulations, we show that the routes chosen by our algorithm (centralized and distributed) are more energy efficient than the state of the art.

Index Terms—wireless communication, routing protocols.

1 Introduction

Over the last several years, multi-hop wireless networks have received considerable attention. These networks are expected to have widespread applicability for the purpose of sensing, communications, and distributed computation [1], [2], [3]. The advantages of these networks are: (1) they do not require a sophisticated infrastructure and can be rapidly deployed; (2) they can be deployed in remote, hostile, or hard to reach areas; (3) they can be used to extend the reach of existing network infrastructure.

The evolution of broadband wireless technologies has the potential to significantly extend the scope of applicability of such networks [2], [3], [4]. For example, a wireless mesh network could substitute for a wireline infrastructure in urban areas and offer broadband Internet services. However, unlike wired networks, wireless systems need efficient power management, since transmission power is a precious resource. Even if a wireless system is connected to a power outlet, power is still important as it directly affects the amount of interference created in the network, and thus impacts the overall throughput that the network can sustain. Therefore, power management is a critical component in wireless networks.

To minimize the energy (or power) consumed in wireless networks various mechanisms such as power control [5], [6], [7], link scheduling, [8], [9], [10], [11], and energy-efficient routing algorithms [12], [13] have been studied. By appropriately scheduling links for transmission, one can potentially reduce the mutual interference imposed by concurrent transmissions in order to get better performance. Energy-efficient routing algorithms find the route that minimizes the overall energy consumption. Recently, there has also been an effort to use cross-layer information in order to optimize the efficiency of wireless networks [14], [15], [16], [17]. For example, opportunistic scheduling at the medium access control (MAC) layer can use physical layer information to maximize the overall throughput transmitted through the system [11], [18]. Routing and congestion control at the network layer can use physical layer information to satisfy quality of service (QoS) requirements or to minimize energy consumption [12], [19], [20].

In [6], [13], [21], [22], energy-efficient routing mechanisms have been developed in multi-hop wireless networks, but these do not account for the minimum signal-to-interference-and-noise (SINR) requirements at the different links. The authors in [12] study end-to-end QoS constraints, but do not consider the impact of routing a new flow on the interference and power requirements of the network, i.e., they do not consider how routing a new flow interferes with ongoing flows in the network. In [23], [24], [25], [26], the authors choose routes that use interference between links as metrics for routing. In [23], [24] routing algorithms are developed to minimize the amount of interference caused by a transmission, while in [25], [26], routing algorithms are developed to minimize the amount of interference encountered by a transmission. However, these algorithms may result in choosing energy inefficient routes because they do not explicitly consider energy efficiency, but only interference.

Here is an example to show how interference could influence the optimal routing decision in a wireless network. Suppose two flows with their own minimum quality requirements enter a multi-hop wireless network one by one and are served by two different links, L_1 and L_2. We assume that the service times of the two flows overlap with each other so that interference occurs. The first link L_1, complying

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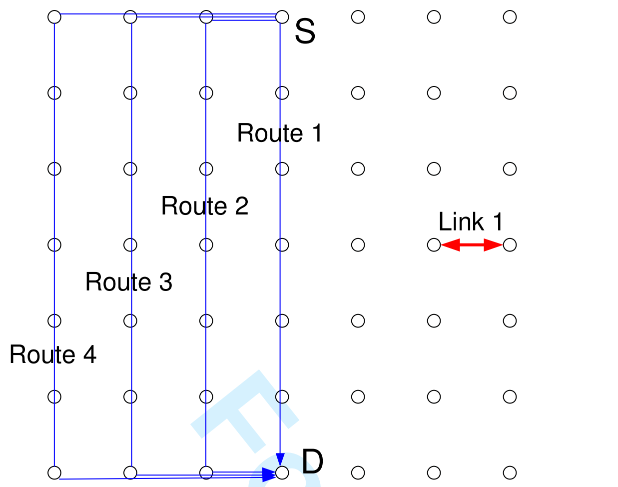

Fig. 1: The example of the impact of interference on selecting an optimal route: an existence of Link 1 affects to choose the energy-efficient rout between nodes S and D.

with its minimum link quality, will be set up based on the interference measured at the receiver node. The second link L_2 satisfying the minimum link quality will be also set up based on the observed interference at the receiver node of L_2. When L_2 is established to serve the second flow, the interference level at the receiver of L_1 will increase. To compensate the degraded link quality, L_1 will increase its transmission power. In response to the increase in L_1's transmission power, L_2 will also increase its transmission power. In the same way, the two links L_1 and L_2 iteratively increase their transmission power due to the interaction between links. Hence, the initially estimated transmission power of each link is different from the actual transmission power required on the flows that traverse the links.

The interaction between the links above affects the selection of energy-efficient routes. Suppose that a new flow comes into a multi-hop wireless network with source node S and destination node D, as shown in Figure 1. If there is no ongoing link in the network, Route 1 will be the optimal route for the new incoming flow between nodes S and D in terms of energy consumption. However, when there is ongoing communication on Link 1, as shown in Figure 1, due to the interference between links, other routes (Routes 2, 3 and 4 in Figure 1) could be more energy-efficient depending on the network environment. Thus, it is important to take into account the interference between links, when considering the routing problem.

In this paper, we make the following intellectual contributions. We develop an energy-efficient SINR-based routing algorithm for given minimum data rates on individual service flows. We explicitly study the impact of routing a new flow on the energy consumption of the network for a certain class of scheduling schemes. Our approach is to transform the problem defined with the minimum bandwidth requirements into a corresponding problem with the SINR constraints, and to find a solution in the SINR domain (as illustrated in Figure 2) by using matrix arithmetic. Throughout this paper, unless stated otherwise, we use boldface notation to denote either a matrix

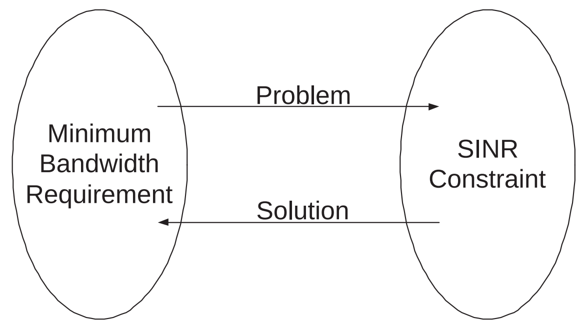

Fig. 2: An approach to find an energy efficient route

or a vector.

The rest of the paper is organized as follows. In Section 2, we describe the system model and state our basic assumptions. In Section 3, we develop an energy-efficient routing scheme using SINR and power control. In Section 4, we discuss a distributed version of our algorithm. In Section 5, we provide numerical results to study the efficacy of the scheme. We conclude in Section 6.

2 System Model and Power Control

2.1 System model

We consider a power-controlled wireless network that supports multi-hop routing, i.e., each node can control its transmission power. We further assume that the time required for power control to converge is negligible. The multi-hop wireless network is modeled as a directed graph $G = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ represents the set of nodes and $\mathcal{L}$ the set of edges that represent communication links between nodes in the network. Each link in $\mathcal{L}$ is identified by the ordered pair, i.e., the transmitting and receiving nodes. Links sharing the same frequency interfere with other links when simultaneously activated.

Assume that a new service with data ζ to be transmitted requires a fixed data rate Δq, so that the service flow has a fixed duration ν, such that $\nu = \frac{\zeta}{\Delta q}$. The service times for incoming flows are not known a priori. Such assumptions are typically valid for real-time services such as video and audio. In this paper, we assume that a flow will be routed over only a single route for the entire duration of the flow and that chosen routes of flows are maintained until the flows are completely served. Re-routing of flows, whenever flows arrive into or depart from the network, could help improve performance. However, in many network scenarios such re-routing may be impractical, hence, in this paper, we do not re-route existing flows.

We define, $E(l)$, the energy consumption of link l to be

$$E(l) = P(l)\nu^l,$$

where $P(l)$ is the transmission power of link l and ν^l is the amount of time it takes the flow to be served at link l.

2.2 Wireless link model

Due to the shared nature of the wireless medium, the wireless links interfere with each other. The impact of interference affects the available capacity of these links.

We define a function $g(\cdot)$ that maps the bandwidth $r(l)$ to the corresponding SINR $\theta(l)$ as follows:

$$\theta(l) = g(r(l)). \tag{1}$$

We assume that the function g is increasing and differentiable with respect to $r(l)$, almost surely.

In the case of the band-limited additive white Gaussian noise (AWGN) channel, the channel capacity (also called the Shannon's capacity [27]) at link l, $r(l)$, is given by

$$r(l) = W(l)\log(1+\theta(l)),$$

where $W(l)$ represents the bandwidth at link l. Hence, (1) becomes

$$\theta(l) = \exp\left(\frac{r(l)}{W(l)}\right) - 1. \tag{2}$$

In the low SINR region, the available capacity in (1) is often assumed to be a linear function of SINR, that is expressed as

$$\theta(l) \quad = \quad Kr(l), \tag{3}$$

where K is a constant. For simplicity we first develop our routing algorithm under (3) and then extend it to more general cases.

The SINR $\theta(l)$ at each link l is defined as

$$\begin{aligned}\theta(l) &= \frac{G(T(l),R(l))P(l)}{\sum_{m:m\neq l} G(T(m),R(l))P(m) + \sigma_{R(l)}} \quad (4)\\ &= \frac{G(T(l),R(l))P(l)}{\eta_{R(l)}},\end{aligned}$$

where $T(l)$ is the transmitting node of link l, $R(l)$ is the receiving node corresponding to link l, $\sigma_{R(l)}$ is the ambient noise at node $R(l)$, $P(l)$ is the transmission power at node $T(l)$, $G(T(m),R(l))$ is the path gain between transmitter $T(m)$ and receiver $R(l)$, and $\eta_{R(l)}$ is the sum of interference and noise at node $R(l)$.

The path gain is generally defined as the ratio of the received power to the transmit power, and many models are proposed for the path gain [14]. A frequently used model is expressed as

$$\begin{aligned}& 10\log_{10} G(T(m),R(l))(dB) \quad (5)\\ = \; & 10\log_{10} K_{T(m)R(l)} - 10\delta\log_{10} d_{T(m)R(l)} - \Psi_{dB},\end{aligned}$$

where $K_{T(m)R(l)}$ is the attenuation factor, $d_{T(m)R(l)}$ is the distance between nodes $T(m)$ and $R(l)$, δ is the path loss exponent that typically ranges between 2 and 6, and Ψ_{dB} is a Gaussian-distributed random variable with mean zero and variance $\sigma_{\Psi_{dB}}$ [28].

2.3 Problem formulation

We study the problem of energy-efficient routing from the perspective of a new service (or flow) arriving into a multi-hop wireless network. The problem is to find an energy-efficient route for the new service from source to destination that does not violate the resource constraints. To that end, the problem becomes finding a route that minimizes the total *energy increment* needed to serve the new arrival over the entire network for given SINR constraints.

Let the source and destination of a new flow be nodes i and j, respectively. Let $q(l)$ be the minimum required data rate for service flows at link l, i.e. $r(l) \geq q(l)$, for all $l \in \mathcal{L}$, where $r(l)$ is the real data rate served at link l and $\mathcal{L}$ is the set of links. Given link scheduling S, data to be transmitted ζ, maximum power limitation $P^{\max}(l)$, and constraint $q(l)$ for all links $l \in \mathcal{L}$, our problem can be formally expressed as

$$\begin{aligned}(A) \quad & \arg\min_R \quad \Delta E(P,S,R,\zeta)\\ & \text{subject to} \quad r(l) \geq q(l) \quad \forall l \in \mathcal{L}\\ & \qquad\qquad P^{\max}(l) \geq P(l) \geq 0 \quad \forall l \in \mathcal{L},\end{aligned}$$

where $\Delta E(P,S,R,\nu)$ is the additional energy expended by the network when route R is chosen as the new flow, and P is the transmission power.

To solve this problem, we convert the constraint domains from bandwidth to SINR as illustrated in Fig. 2, and develop an energy-efficient routing algorithm that complies with the minimum requirements in the SINR constraint domain. The map from the minimum bandwidth requirement to the corresponding minimum SINR constraint is defined by a wireless link model, as explained in the previous subsection.

Let $c(l)$ be the minimum SINR corresponding to the minimum required bandwidth at link l, i.e. $\theta(l) \geq c(l)$, for all $l \in \mathcal{L}$, where $\mathcal{L}$ is the set of links. Then, for a given link scheduling S, Problem (A) can be rewritten as

$$\begin{aligned}(B) \quad & \arg\min_{R\in R(i,j)} \quad \Delta E(P,S,R,\nu) && (6)\\ & \text{subject to} \quad \theta(l) \geq c(l) \quad \forall l \in \mathcal{L} && (7)\\ & \qquad\qquad P^{\max}(l) \geq P(l) \geq 0 \quad \forall l \in \mathcal{L}, && (8)\end{aligned}$$

where $c(l)$ is the SINR constraint for link l.

For example, let us define the minimum required bandwidth at link l to be 10 kbps in Problem (A). After converting it with (1), we specify the constraint at link l as 0.1 (-10 dB) SINR in Problem (B).

For simplicity, in our analysis, we do not consider the maximum transmission power constraint (8), i.e., we let $P^{\max}(l)$ go to infinity for all l. However, in Section 5 we use simulations to study a power-constrained system that incorporates admission control. In contrast to previous work, our goal is to find the optimal routes in terms of energy consumption over the entire network given a certain class of link scheduling schemes.

2.4 Power control

Recall that $\mathcal{L}$ is the set of links. We let $\mathbf{P}$ denote the power vector defined by $\mathbf{P} = (P(1),\cdots,P(L_{\mathcal{L}}))^T$, where $P(l)$ is the power of link l, and $L_{\mathcal{L}}$ is the number of links in set $\mathcal{L}$. Using (4), we can rewrite (7) in matrix form as

$$\mathbf{P} \geq \mathbf{F}\mathbf{P} + \mathbf{b}, \tag{9}$$

where $\mathbf{b} = (b(1),\cdots,b(L_{\mathcal{L}}))^T$ such that $b(l) = \frac{c(l)\sigma_{R(l)}}{G(T(l),R(l))}$, and $\mathbf{F}$ is the $L_{\mathcal{L}} \times L_{\mathcal{L}}$ matrix with (l,m) entry

$$F(l,m) = \begin{cases} \frac{G(T(m),R(l))c(l)}{G(T(l),R(l))} & , \; l \neq m\\ 0 & , \; l = m.\end{cases} \tag{10}$$

Matrix $\mathbf{F}$ defined by (10) has non-negative elements, and since the links interact with each other, it is also irreducible. Hence, we have the following theorem [29] from the Perron-Frobenius theorem and standard matrix theory [30].

Theorem 1: The following statements are equivalent:

1) $\rho_{\mathbf{F}} \leq 1$ where $\rho_{\mathbf{F}}$ is the Perron-Frobenius eigenvalue of $\mathbf{F}$.
2) There exists a vector $\mathbf{P} > 0$ such that $(\mathbf{I}-\mathbf{F})\mathbf{P} \geq \mathbf{b}$.
3) $(\mathbf{I}-\mathbf{F})^{-1}$ exists and is positive componentwise.

If there exists a positive feasible vector $\mathbf{P}$, it follows from Theorem 1 that $(\mathbf{I}-\mathbf{F})^{-1}$ exists. From (9) we can obtain

$$\mathbf{P} \geq (\mathbf{I}-\mathbf{F})^{-1}\mathbf{b}.$$

Hence, we have the Pareto optimal[1] solution $(\mathbf{I}-\mathbf{F})^{-1}\mathbf{b}$ that supports the network topology defined by links in $\mathcal{L}$ and their associated minimum requirements. One can use distributed power control algorithm [5], [29] to achieve this minimum power vector. More properties of Theorem 1 can be found in [5], [29], [31].

3 Routing with SINR Metric

In order to solve Problem (B), we divide *the solving procedure* into three steps.

- First, as a preliminary step, we study the impact that is caused by an individual link's power increment on the overall network to be able to support a new flow. In this step, we consider only the influence of the change of the link constraint on the network.
- Second, we consider the impact of admitting a new flow on the network as it traverses an entire route. Since the route consists of one or more links, there is interference between the links themselves that transport the flow. Based on the impact on the network energy consumption, we propose a new routing algorithm with SINR metrics in order to satisfy the minimum constraints of all the links and to minimize the energy consumption over the network.
- Finally, we consider the case when the links are not activated at the same time but scheduled according to some link scheduling mechanism. In the first two cases, we do not consider link scheduling over the network and we simply assume that there exist static matrices $\mathbf{F}$, $\mathbf{b}$ and $\mathbf{P}$. However, in practice, these matrices can change dynamically because of link scheduling.

3.1 Impact of a single link that is activated by a new flow

Assume that a new service arrives at an arbitrary link n in the network. The service has data ζ to be transmitted and needs a minimum data rate of Δq. The minimum data rate is assumed to be fixed during the service time over the network. Since the corresponding SINR constraint Δc and the flow duration of the service ν are expressed as $K\Delta q$ and $\frac{\zeta}{\Delta q}$, respectively, the constraint Δc and the flow duration ν are fixed as stated in the previous section. In the case when link n serves the new service flow, the minimum bandwidth for service flows at link n increases by Δq, i.e. the minimum SINR increases by Δc. As a result, the power requirement for link n to accommodate this flow increases from $P(n)$ to $P'(n)$. The additional energy $\Delta E(n)$ required to serve the flow at link n is expressed as

$$\begin{aligned}\Delta E(n) &= \Delta P(n)\nu \\ &= \left(P^{'}(n) - P(n)\right)\nu,\end{aligned}$$

where $\Delta P(n)$ is the additional transmission power required at link n. Since the flow duration, ν, is fixed, we only need to consider the transmission powers of the links in the network.

For the new flow, we define $\mathbf{F}^{'}$, $\mathbf{P}^{'}$, and $\mathbf{b}^{'}$ to be the matrices corresponding to $\mathbf{F}$, $\mathbf{P}$, and $\mathbf{b}$ in the new environment. When the constraint at link n changes, the additional power $\mathbf{\Delta P}_n$ required in the network can be expressed as

$$\begin{aligned}\mathbf{\Delta P}_n &= \mathbf{P}' - \mathbf{P} \\ &= (\mathbf{F}'\mathbf{P}' + \mathbf{b}') - (\mathbf{F}\mathbf{P} + \mathbf{b}) \\ &= (\mathbf{F}'(\mathbf{P} + \mathbf{\Delta P}_n) + \mathbf{b}') - (\mathbf{F}\mathbf{P} + \mathbf{b}) \\ &= (\mathbf{F}' - \mathbf{F})\mathbf{P} + \mathbf{F}'\mathbf{\Delta P}_n + \mathbf{b}' - \mathbf{b} \\ &= (\mathbf{F}' - \mathbf{F})\mathbf{P} + \mathbf{F}'\mathbf{\Delta P}_n + \mathbf{\Delta b}_n,\end{aligned}$$

where $\mathbf{P} = \mathbf{F}\mathbf{P} + \mathbf{b}$, $\mathbf{P}' = \mathbf{F}'\mathbf{P}' + \mathbf{b}'$, and $\mathbf{\Delta b}_n \triangleq \mathbf{b}' - \mathbf{b}$. Hence, when $(\mathbf{I}-\mathbf{F}')^{-1}$ exists,[2] by rearranging and premultiplying the resulting matrix equality by $(\mathbf{I}-\mathbf{F}')^{-1}$, we obtain

$$\mathbf{\Delta P}_n = (\mathbf{I}-\mathbf{F}')^{-1}((\mathbf{F}'-\mathbf{F})\mathbf{P} + \mathbf{\Delta b}_n). \quad (11)$$

We let $(\mathbf{I}-\mathbf{F})^{-1}_{(n)}$ denote the nth column vector of matrix $(\mathbf{I}-\mathbf{F})^{-1}$ and $(\mathbf{I}-\mathbf{F})^{-1}_{\sum^n}$ as the elementwise sum of matrix $(\mathbf{I}-\mathbf{F})^{-1}_{(n)}$. Then,

$$\begin{aligned}&\mathbf{\Delta P}_n \\ =\ & (\mathbf{I}-\mathbf{F}')^{-1}[\underbrace{0,\cdots,0}_{n-1}, \sum_{m\neq n}\frac{G(T(m),R(n))P(m)\Delta c}{G(T(n),R(n))} \\ & + \frac{\sigma_{R(n)}\Delta c}{G(T(n),R(n))}, \underbrace{0,\cdots,0}_{L_{\mathcal{L}}-n}]^T \\ =\ & (\mathbf{I}-\mathbf{F}')^{-1}[\underbrace{0,\cdots,0}_{n-1}, \frac{\eta_{R(n)}}{G(T(n),R(n))}\Delta c, \underbrace{0,\cdots,0}_{L_{\mathcal{L}}-n}]^T \\ =\ & (\mathbf{I}-\mathbf{F}')^{-1}_{(n)}\left(\frac{\eta_{R(n)}}{G(T(n),R(n))}\right)\Delta c, \quad (12)\end{aligned}$$

where $\eta_{R(n)} = \sum_{m\neq n} G(T(m),R(n))P(m) + \sigma_{R(n)}$.

Before taking the next step, one question to be asked is whether (12) really reflects all the increments of the nodes' power consumption over the entire network when link n's constraint is changed by Δc. This question can be explained by Theorem 1. If $\mathbf{P}'$ is feasible, then $(\mathbf{I}-\mathbf{F}')^{-1}$ is equal to

1. $\mathbf{P}^*$ is said to be Pareto optimal if $\mathbf{P}^*$ is feasible and any feasible $\mathbf{P}$ satisfies $\mathbf{P} \geq \mathbf{P}^*$ componentwise.

2. If this matrix is not invertible, then it implies that there is no solution, i.e., the power levels required to sustain the SINR go to infinity.

$\sum_{m=0}^{\infty} (\mathbf{F}')^m$. Hence, we can rewrite (12) as follows.

$$\begin{aligned}
&\mathbf{\Delta P}_n \\
= \;& (\mathbf{I}-\mathbf{F}')^{-1}[\underbrace{0,\cdots,0}_{n-1},\Delta P(n),\underbrace{0,\cdots,0}_{L_{\mathcal{L}}-n}]^T \\
= \;& (\mathbf{I}+\mathbf{F}'+(\mathbf{F}')^{\mathbf{2}}+\cdots)[\underbrace{0,\cdots,0}_{n-1},\Delta P(n),\underbrace{0,\cdots,0}_{L_{\mathcal{L}}-n}]^T \\
= \;& [\underbrace{0,\cdots,0}_{n-1},\Delta P(n),\underbrace{0,\cdots,0}_{L_{\mathcal{L}}-n}]^T+\mathbf{F}'_{(n)}\Delta P(n) \\
& +(\mathbf{F}')^2_{(n)}\Delta P(n)+(\mathbf{F}')^3_{(n)}\Delta P(n)+\cdots \\
= \;& [\underbrace{0,\cdots,0}_{n-1},\Delta P(n),\underbrace{0,\cdots,0}_{L_{\mathcal{L}}-n}]^T+\mathbf{F}'_{(n)}\Delta P(n) \\
& +\mathbf{F}'(\mathbf{F}'_{(n)}\Delta P(n))+(\mathbf{F}')^2(\mathbf{F}'_{(n)}\Delta P(n))+\cdots, \quad (13)
\end{aligned}$$

where $\Delta P(n) = \frac{\eta_{R(n)}}{G(T(n),R(n))}\Delta c$. The first term in (13), $[\underbrace{0,\cdots,0}_{n-1},\Delta P(n),\underbrace{0,\cdots,0}_{L_{\mathcal{L}}-n}]^T$, represents the amount of additional power at link n when the constraint of link n is changed by Δc. The second term, $\mathbf{F}'_{(n)}\Delta P(n)$, is equal to the amount of additional power of each node in the network when link n increases its power by $\Delta P(n)$. The third term, $\mathbf{F}'(\mathbf{F}'_{(n)}\Delta P(n))$, is the amount of additional power in the network when the network increases its power by $\mathbf{F}'_{(n)}\Delta P(n)$ at each link. Thus, each term represents the increment of power consumption iterated back and forth over the whole network.

The energy increment of each link in the network, when a new flow with additional constraint Δc and duration ν arrives at node n, can be expressed as

$$\begin{aligned}
\mathbf{\Delta E}_n &= \mathbf{\Delta P}_n\ \nu \\
&= (\mathbf{I}-\mathbf{F}')^{-1}_{(n)}\left(\frac{\eta_{R(n)}}{G(T(n),R(n))}\right)\Delta c\ \nu.
\end{aligned}$$

3.2 Impact of a route on the network when a new flow traverses the route

In this subsection, we consider the impact that a flow traversing a route (rather than a single link) has on the energy requirement of the network. In contrast to the previous subsection, one or more links in the route may simultaneously transmit the flow. Hence, when transmitting this flow, a link can interfere with other links that transport the flow over the route.

To study the case when more than one link in a route is simultaneously activated, we first consider the power increment needed when there are two activated links, m and n, which deliver the new flow at the same time. As in the previous subsection, we define $\mathbf{\Delta P}_{m,n}$ and $\mathbf{\Delta b}_{m,n}$ to be $\mathbf{\Delta P}_{m,n} \triangleq \mathbf{P}'-\mathbf{P}$ and $\mathbf{\Delta b}_{m,n} \triangleq \mathbf{b}'-\mathbf{b}$, respectively, where $\mathbf{P}=\mathbf{FP}+\mathbf{b}$ and $\mathbf{P}'=\mathbf{F}'\mathbf{P}'+\mathbf{b}'$. Hence, as in (12), we have

$$\begin{aligned}
&\mathbf{\Delta P}_{m,n} \\
= \;& (\mathbf{I}-\mathbf{F}')^{-1}((\mathbf{F}'-\mathbf{F})\mathbf{P}+\mathbf{\Delta b}_{m,n}) \\
= \;& (\mathbf{I}-\mathbf{F}')^{-1}[\underbrace{0,\cdots,0}_{m-1},\frac{\eta_{R(m)}}{G(T(m),R(m))}\Delta c,\underbrace{0,\cdots,0}_{n-m-1}, \\
& \frac{\eta_{R(n)}}{G(T(n),R(n))}\Delta c,\underbrace{0,\cdots,0}_{L_{\mathcal{L}}-n}]^T \\
= \;& (\mathbf{I}-\mathbf{F}')^{-1}[\underbrace{0,\cdots,0}_{m-1},\Delta P(m),\underbrace{0,\cdots,0}_{n-m-1},\Delta P(n),\underbrace{0,\cdots,0}_{L_{\mathcal{L}}-n}]^T \\
= \;& (\mathbf{I}-\mathbf{F}')^{-1}[\underbrace{0,\cdots,0}_{m-1},\Delta P(m),\underbrace{0,\cdots,0}_{L_{\mathcal{L}}-m}]^T \\
& +(\mathbf{I}-\mathbf{F}')^{-1}[\underbrace{0,\cdots,0}_{n-1},\Delta P(n),\underbrace{0,\cdots,0}_{L_{\mathcal{L}}-n}]^T \\
= \;& (\mathbf{I}-\mathbf{F}')^{-1}_{(m)}\Delta P(m)+(\mathbf{I}-\mathbf{F}')^{-1}_{(n)}\Delta P(n),
\end{aligned}$$

where $\Delta P(m) = \frac{\eta_{R(m)}}{G(T(m),R(m))}\Delta c$ and $\Delta P(n) = \frac{\eta_{R(n)}}{G(T(n),R(n))}\Delta c$. In the same way, when a flow is served by a set of links Λ at a given time slot, the increased power consumption $\mathbf{\Delta P}_\Lambda$ can be *decomposed* as

$$\mathbf{\Delta P}_\Lambda = \sum_{n\in\Lambda}(\mathbf{I}-\mathbf{F}')^{-1}_{(n)}\Delta P(n),$$

where $\Delta P(n) = \frac{\eta_{R(n)}}{G(T(n),R(n))}\Delta c$.

Here, we study how much additional energy is consumed when a new flow with Δc and ν arrives at the network and route R is chosen to serve the flow between source i and destination j. Let T_R be the set of time slots over which the flow is served by route R. We define ν_t^l an indicator function such that it is one if link l serves the flow at time slot t, and zero otherwise. Hence, $\sum_{t\in T_R}\nu_t^l$ is equal to the total duration ν. Then, the additional energy consumed by the network through route R, $\Delta E_R^{Network}$, is

$$\begin{aligned}
&\Delta E_R^{Network} \\
= \;& \sum_{t\in T_R}\Delta E_R^t \\
= \;& \sum_{t\in T_R}\sum_{l\in R}(\mathbf{I}-\mathbf{F}')^{-1}_{\sum^l}\Delta P(l)\nu_t^l \\
= \;& \sum_{t\in T_R}\sum_{l\in R}(\mathbf{I}-\mathbf{F}')^{-1}_{\sum^l}\frac{\eta_{R(m)}}{G(T(m),R(m))}\Delta c\ \nu_t^l \\
= \;& \sum_{l\in R}\left(\sum_{t\in T_R}\nu_t^l\right)(\mathbf{I}-\mathbf{F}')^{-1}_{\sum^l}\frac{\eta_{R(m)}}{G(T(m),R(m))}\Delta c \\
= \;& \left(\sum_{l\in R}(\mathbf{I}-\mathbf{F}')^{-1}_{\sum^l}\frac{\eta_{R(m)}}{G(T(m),R(m))}\right)\Delta c\ \nu,
\end{aligned}$$

where ΔE_R^t is the energy additionally consumed by the network at time t when route R is chosen for the flow. Since $\Delta c\ \nu$ can be rewritten by $K\zeta$ that is fixed, we have

$$\begin{aligned}
&\arg\min_{R\in R(i,j)}\Delta E_R^{Network} \\
= \;& \arg\min_{R\in R(i,j)}\sum_{l\in R}(\mathbf{I}-\mathbf{F}')^{-1}_{\sum^l}\frac{\eta_{R(m)}}{G(T(m),R(m))},
\end{aligned}$$

where $R(i,j)$ is the set of possible routes from node i to node j. Moreover, when Δc goes to 0, $\mathbf{F}'$ converges to $\mathbf{F}$ componentwise.

If we ignore link scheduling and the additional required constraint is infinitesimally small, our energy-efficient routing algorithm to solve Problem (B) becomes

$$\begin{aligned} \arg \min_{R \in R(i,j)} \quad & \sum_{l \in R} \left((\mathbf{I}-\mathbf{F})^{-1}_{\sum^l} \left(\frac{\eta_{R(l)}}{G(T(l),R(l))} \right) \right) \quad (14) \\ \text{subject to} \quad & \theta(l) \geq c(l) \quad \forall l \in \mathcal{L}. \\ & P(l) \geq 0 \quad \forall l \in \mathcal{L}, \end{aligned}$$

where $R(i,j)$ is the set of possible routes from node i to node j. From (14), instead of the exact value of the network energy increment of a route, $\Delta E_R^{Network}$, we can choose a minimum energy route from the interference measured at $R(l)$, $\frac{\eta_{R(l)}}{G(T(l),R(l))}$, and minimum requirements acquired from other nodes, $(\mathbf{I}-\mathbf{F})^{-1}_{\sum^l}$, as described below.

Construct a directed graph $G = (\mathcal{N}, \mathcal{L})$.
For an incoming flow, check if resources are available.
If yes,
  Measure the interference strength at all nodes in $\mathcal{N}$.
  Calculate $(\mathbf{I}-\mathbf{F})^{-1}$ based on path loss and constraints.
  Calculate link cost $(\mathbf{I}-\mathbf{F})^{-1}_{\sum^l} \left(\frac{\eta_{R(l)}}{G(T(l),R(l))} \right) \forall l \in \mathcal{L}$.
  Apply a shortest path algorithm, e.g. Dijkstra algorithm or Bellman-Ford algorithm, to find the minimum cost route.
Otherwise,
  Reject the incoming flow.
  Notify the rejection to the source.

The normalized interference, $\frac{\eta_{R(l)}}{G(T(l),R(l))}$, in (14) can be locally measured. Hence, if $\mathbf{F}$ is computed at each node, a fully distributed algorithm can be implemented since the link cost $(\mathbf{I}-\mathbf{F})^{-1}_{\sum^l} \left(\frac{\eta_{R(l)}}{G(T(l),R(l))} \right)$ can be locally computed at each node. We will discuss the distributed algorithm in Section 4.

3.3 When links in the network are scheduled

In a multi-hop wireless network, link scheduling may be used in order to reduce the interference and to improve network performance. The link scheduling problem over a multi-hop network is NP-complete [10] even if scheduling allows only half-duplex links per node. In the case when a system uses a maximal matching scheduling scheme [19], [32], [33], then the scheduling at each time slot can be probabilistically independent. Although we do not require that the scheduling scheme is independent, we do require that it satisfy a much weaker "mixing" property to be defined soon.

Let $\mathbf{S}_t$ be an $L_\mathcal{L} \times L_\mathcal{L}$ scheduling matrix of links at time slot t with (l,m) entry defined as

$$S_t(l,m) = \begin{cases} 1, & \text{if link } l \text{ is active and } l = m \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

and $\mathbf{F}_t$ be an $L_\mathcal{L} \times L_\mathcal{L}$ matrix with (l,m) entry defined as

$$F_t(l,m) = \begin{cases} \frac{1_{t,(l,m)} G(T(m),R(l)) c(l)}{G(T(l),R(l))} & , m \neq l \\ 0 & , m = l \end{cases} \quad (16)$$

where $1_{t,(l,m)}$ is one, if links l and m are active at time t, otherwise zero.

As mentioned earlier, we assume that the scheduling $\mathbf{S}_t$ has a mixing property with respect to mapping φ such that $\varphi^k(\mathbf{S}_t) = \mathbf{S}_{t+k}$, i.e. $\lim_{k\to\infty} \Pr(A \cap \varphi^{-k} B) = \Pr(A)\Pr(B)$ for all random variables A and B in a σ-field generated by $\mathbf{S}_t$, where $\Pr(\cdot)$ stands for a probability operator. The mixing assumption is more practical and weaker than that of time independence (although even time independence satisfies the opportunistic scheduling schemes for certain cases in [34]). Since mixing implies ergodicity [35], [36], it follows from the ergodic theorem that

$$\lim_{k\to\infty} \frac{1}{k} \sum_{n=1}^{k} \varphi^{n-1}(\mathbf{S}_t) = \mathbf{\Pi}. \quad (17)$$

Here $\mathbf{\Pi}$ is an $L_\epsilon \times L_\epsilon$ diagonal matrix such that $\mathbf{\Pi}$ is the expected value of $\mathbf{S}_t$. Each (l,m) entry of $\mathbf{\Pi}$ denoted by $\Pi(l,m)$ is defined to be equal to π_l, if $l = m$, or zero otherwise, where π_l is the expected value of $S_t(l,l)$. Similarly, let $\hat{\mathbf{F}}$ be $\hat{\mathbf{F}} = \lim_{k\to\infty} \frac{1}{k} \sum_{t=1}^{k} \mathbf{F}_t$ with (l,m) entry defined as $\hat{F}(l,m)$ is $\frac{\pi_{l,m} G(T(m),R(l)) c(l)}{G(T(l),R(l))}$, if $l \neq m$, zero otherwise, where $\pi_{l,m}$ [3] is the expected value of $1_{t,(l,m)}$.

For time slot t, (9) becomes

$$\mathbf{S}_t \mathbf{P} \quad \geq \quad \mathbf{F}_t \mathbf{P} + \mathbf{S}_t \mathbf{b} \; \forall t \in T, \quad (18)$$

where T is a set of time slots. In the case of a half-duplex link, $\mathbf{S}_t - \mathbf{F}_t$ may not be a full rank matrix. For each time t, even if the rank of $\mathbf{S}_t - \mathbf{F}_t$ in (18) is less than $L_\mathcal{L}$, the non-negative optimal solution can be found by a Moore-Penrose inverse matrix [37]. In the solution, the elements of $\mathbf{P}$ that correspond to the scheduled (active) links are positive at time t, and zero otherwise. Over all t in T, since all the vectors of (18) are componentwise non-negative, it follows from (17) that

$$\begin{aligned} \sum_{t=1}^{k} \mathbf{S}_t \mathbf{P} \quad & \geq \quad \sum_{t=1}^{k} (\mathbf{F}_t \mathbf{P} + \mathbf{S}_t \mathbf{b}) \\ \frac{1}{k} \sum_{t=1}^{k} \mathbf{S}_t \mathbf{P} \quad & \geq \quad \frac{1}{k} \sum_{t=1}^{k} (\mathbf{F}_t \mathbf{P} + \mathbf{S}_t \mathbf{b}) \\ \lim_{k\to\infty} \frac{1}{k} \sum_{t=1}^{k} \mathbf{S}_t \mathbf{P} \quad & \geq \quad \lim_{k\to\infty} \frac{1}{k} \sum_{t=1}^{k} (\mathbf{F}_t \mathbf{P} + \mathbf{S}_t \mathbf{b}) \\ \mathbf{\Pi P} \quad & \geq \quad \hat{\mathbf{F}} \mathbf{P} + \mathbf{\Pi b}. \end{aligned} \quad (19)$$

From the above result (19), we have the following lemma.

Lemma 1: Let $\mathbf{S}_t$ be a scheduling scheme that satisfies (17). Assume that matrix $\mathbf{\Pi}$ has full rank and $\bar{\mathbf{F}}$ is irreducible, where $\bar{\mathbf{F}}$ is equal to $\mathbf{\Pi}^{-1}\hat{\mathbf{F}}$. If the Perron-Frobenius eigenvalue of $\bar{\mathbf{F}}$ is smaller than one, then there exists a minimum average power vector $\bar{\mathbf{P}}$ such that $\bar{\mathbf{P}} = \mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}\mathbf{b}$ and $\bar{\mathbf{P}}$ is positive componentwise. If not, there is no minimum average power vector that satisfies the constraints, for the given link scheduling, $\mathbf{S}_t$.

Proof: Since $\mathbf{\Pi}$ is a full rank diagonal matrix, its inverse matrix, $\mathbf{\Pi}^{-1}$, exists. Hence, (19) becomes $\mathbf{P} \geq \mathbf{\Pi}^{-1}\hat{\mathbf{F}}\mathbf{P} + \mathbf{b}$. If $(\mathbf{I}-\bar{\mathbf{F}})^{-1}$ exists where $\bar{\mathbf{F}} = \mathbf{\Pi}^{-1}\hat{\mathbf{F}}$, then the minimum average power $\bar{\mathbf{P}}$ is equal to $\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})\mathbf{b}$ from Theorem 1. □

3. $\pi_{l,m}$ means the probability that links l and m are active simultaneously.

Based on Lemma 1, we henceforth assume feasibility that there exists $\bar{\mathbf{P}}$. As in (12), when link n has a new flow, the power increment in the networks is given by

$$\begin{aligned}
& \mathbf{\Delta}\bar{\mathbf{P}}_n \\
= \; & \mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}((\bar{\mathbf{F}}'-\bar{\mathbf{F}})\mathbf{P}+\mathbf{\Delta b}_n) \\
= \; & \mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}[\underbrace{0,\cdots,0}_{n-1},\sum_{m\neq n}\frac{G(T(m),R(n))\pi_{m|n}P(m)}{G(T(n),R(n))}\Delta c \\
& +\frac{\sigma_{R(n)}}{G(T(n),R(n))}\Delta c,\underbrace{0,\cdots,0}_{L_{\mathcal{L}}-n}]^T \\
= \; & \left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}\right)_{(n)}\left(\frac{\bar{\eta}_{R(n)}}{G(T(n),R(n))}\right)\Delta c,
\end{aligned}$$

where $\bar{\eta}_{R(n)}$ is the average of the interference and noise measured at the receiving node of link n when link n is active, and $\pi_{m|n}$ is defined as $\frac{\pi_{m,n}}{\pi_n}$, where $\pi_{m|n}$ is the conditional probability that link m is active given link n that is active. Since the flow duration ν and additional constraint Δc are fixed, for a given link scheduling S, the routing algorithm for minimizing the average energy consumption can be expressed as follows.

$$\arg\min_{R\in R(i,j)} \quad \sum_{l\in R}\left(\frac{\bar{\eta}_{R(l)}}{G(T(l),R(l))}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}\right)_{\sum l}\right) \tag{20}$$

$$\begin{aligned}
\text{subject to} \quad & \theta(l)\geq c(l) \quad \forall l\in\mathcal{L}, \\
& P(l)\geq 0 \quad \forall l\in\mathcal{L}.
\end{aligned}$$

The algorithm in procedure is similar to that of the previous subsection, and is shown below.

Construct a directed graph $G=(\mathcal{N},\mathcal{L})$.
For an incoming flow, check if resources are available.
If yes,
  Measure the *average* interference strength at all nodes.
  Calculate the time *average* of link scheduling matrix, $\mathbf{\Pi}$.
  Calculate $(\mathbf{I}-\bar{\mathbf{F}})^{-1}$ based on path loss and correlation between links.
  Calculate link cost $\frac{\bar{\eta}_{R(l)}}{G(T(l),R(l))}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}\right)_{\sum l}$ $\forall l\in\mathcal{L}$.
  Apply a shortest path algorithm to find the minimum cost route.
Otherwise,
  Reject the incoming flow.
  Notify the rejection to the source.

3.4 When the data rate is a nonlinear function of SINR

Extending our earlier solution to the case when the data rate is a more general function of SINR turns out to be quite straightforward. To do this, we develop our algorithm with a function $g(\cdot)$ defined in (1), which maps the bandwidth constraint in Problem (A) to the SINR constraint in Problem (B). Similarly to the previous subsection, the impact of a new route for a new service flow on network energy, $\Delta\bar{E}_R^{Network}$, can now be expressed as:

$$\begin{aligned}
& \Delta\bar{E}_R^{Network} \\
= \; & \sum_{l\in R}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}{}_{\sum l}\right)\Delta\bar{P}(l)\nu^l \\
= \; & \sum_{l\in R}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}{}_{\sum l}\right)\Delta\bar{P}(l)\left(\frac{\zeta}{\Delta q}\right) \\
= \; & \sum_{l\in R}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}{}_{\sum l}\right)\frac{\bar{\eta}_{R(l)}\Delta c}{G(T(l),R(l))}\left(\frac{\zeta}{\Delta q}\right) \\
= \; & \sum_{l\in R}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}{}_{\sum l}\right)\frac{\bar{\eta}_{R(l)}}{G(T(l),R(l))}\left(\frac{\Delta c}{\Delta q}\right)\zeta,
\end{aligned}$$

where $\Delta c = g(q(l)+\Delta q)-g(q(l))$ and ζ is the total data to be transmitted by the new flow. In the case when the required bandwidth Δq is infinitesimal, $\bar{F}'$ and $\frac{\Delta c}{\Delta q}$ $\left(=\frac{g(q(l)+\Delta q)-g(q(l))}{\Delta q}\right)$ become $\bar{F}$ and $g'(q(l))$, respectively, where $g'(q(l))$ is a derivative of $g(q(l))$ with respect to minimum required bandwidth $q(l)$ at link l. Since ζ is fixed, our algorithm is formally expressed by

$$\begin{aligned}
\arg\min_{R\in R(i,j)} \quad & \sum_{l\in R}\left(\frac{\bar{\eta}_{R(l)}g'(q(l))}{G(T(l),R(l))}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}\right)_{\sum l}\right) \\
\text{subject to} \quad & \theta(l)\geq c(l) \quad \forall l\in\mathcal{L}, \\
& P(l)\geq 0 \quad \forall l\in\mathcal{L}.
\end{aligned}$$

The algorithm for the non-linear case is also similar to the algorithm that we have previously discussed, except that each node additionally computes the gradients $g'(q(l))$ for the links associated with the node.

4 Discussion

In this section, we discuss some of the issues that we face when implementing our algorithm.

4.1 Simplification and Complexity

A practical issue to be discussed is how to reduce the computational complexity. This simplification is also necessary in the development of a distributed algorithm. In (20), the term $\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}$ corresponds to global information in the network. Clearly, it is inefficient to gather information over all links in a large wireless network. Firstly, transmissions that are distant from a link will barely interfere with it, since each element in (10) is multiplied by weighting factor $G(T(m),R(l))$ that is proportional to $d_{T(m)R(l)}^{-\delta}$, where δ is between 2 and 6, and m is a link in $\mathcal{L}$. Also, in practice, there could be a large time delay in gathering this information from the network. This is especially true, when the wireless network is large and the operating algorithms are distributed, as the information from distant nodes may become too stale to be useful. Hence, for all nodes k in the network, we define $\mathcal{N}_k$ to be the set including node k and its neighbor nodes in a certain range called the *information range*, $\mathcal{L}_k$. The number of links in $\mathcal{N}_k$ is denoted by $L_{\mathcal{L}_k}$. Then, instead of matrices $\bar{\mathbf{F}}$ and $\mathbf{\Pi}$, the link weights associated with node k in $\mathcal{N}$

uses $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{\Pi}}$ reduced to $L_{\mathcal{L}_k} \times L_{\mathcal{L}_k}$ matrices in order to reduce the computational complexity at each node and control messages over the network. Since the number of links in the information range, $L_{\mathcal{L}_k}$, at node $k \in \mathcal{N}$, the complexity order for computing link weights at each node is $O(1)$ for the simplified version, but $O(n^4)$ for (20), where n is the number of nodes in the network. Even though distant links are removed from $\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}$ in (20), the influence of these links on links l is still included in the average SINR, $\frac{\overline{\eta_{R(l)}}}{G(T(l),R(l))}$ that is measured at node $R(l)$. We next provide a distributed solution using the above described simplification.

4.2 Distributed algorithm

We assume that each node knows the path gains to its neighboring nodes within its information range. The path gains can be measured at each node by using a variety of techniques such as the receiving power [28]. We assume that each node uses some method to disseminate its messages to its neighbors, e.g., using a control channel or pyggybacking.

When we use a distributed shortest path algorithm such as the Dijkstra algorithm or the Bellman-Ford algorithm [38], the issue is how to define the weights of links. Instead of $\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}$ in (20), we use $\tilde{\mathbf{\Pi}}(\mathbf{I}-\tilde{\mathbf{F}})^{-1}$ that was discussed in the previous subsection. To obtain the matrix $\tilde{\mathbf{\Pi}}(\mathbf{I}-\tilde{\mathbf{F}})^{-1}$ at each node, node k in $\mathcal{N}$ needs to obtain the conditional probability $\pi_{m|l}$, where node k is $R(l)$, and normalized additional constraint defined as $\frac{\Delta c(m)}{G(T(m),R(m))}$ for all $m \in \mathcal{L}_k$. When transmitting and receiving data through link l, nodes $R(l)$ and $T(l)$ disseminate slot times occupied by link l and $\frac{\Delta c(l)}{G(T(l),R(l))}$ to the neighborhood in sets $\mathcal{N}_{R(l)}$ and $\mathcal{N}_{T(l)}$, respectively. After receiving the information from the neighboring nodes, each node generates a matrix $\tilde{\mathbf{\Pi}}(\mathbf{I}-\tilde{\mathbf{F}})^{-1}$. In the case when the link scheduling is time independent, the transmitter nodes of the links can send link activated probabilities (time average) to their neighborhood since $\pi_{m|n}$ becomes π_m, meaning that the probability that link m is active and can be measured at nodes $R(m)$ and $T(m)$.

4.3 Ongoing service protection

We need an admission control mechanism to prevent incomig flows from a significant degradation of the quality of service of ongoing flows. We next outline how to control incoming flows.

Calculate the maximum eigenvalue $\rho_{\mathbf{F}}$ ($\rho_{\tilde{\mathbf{F}}}$) of $\mathbf{F}$ ($\tilde{\mathbf{F}}$).
If $\rho_{\mathbf{F}}$ ($\rho_{\tilde{\mathbf{F}}}$) ≥ 1,
  Reject the incoming flow.
  Notify the denial to the previous node.
Otherwise,
  Calculate power consumption when the new flow is admitted
  If the power consumption $\geq$ threshold
    Reject the new flow.
    Notify the denial to the previous node.
  Otherwise
    Admit the incoming flow.

Even after flows are admitted, because of variable link conditions resulting from random scheduling, instantaneous transmission power could exceed power constraints. In a distributed setting, inaccuracies due to local information and information dissemination delay, could also lead to incorrect flow admission decisions. To prevent the quality of ongoing flows from being degraded due to such cases, nodes check if the required transmission power exceeds a given threshold, as in the previous admission control. When a violation happens, the node rejects the latest admitted flows and notifies all the rejection to the previous relay node.

5 Simulations

In this section we use simulations to verify the performance of our algorithm. We call our algorithm *OptSINR*. We compare the performance of this algorithm to other routing algorithms that are used in the literature. These algorithms are based on a shortest path idea, where the minimum transmission energy or the minimum interference is chosen as the cost over each link. In the first algorithm called the minimum energy algorithm (ME), the cost over each link does not take interference into account. In contrast, the second algorithm called PwrOpt, considers interference at each node. Based on the interference level, the PwrOpt chooses the minimum transmission power to meet the SINR requirement at each like l in the network, i.e

$$\Delta P(l) = \frac{\eta_{R(l)}}{G(T(l),R(l))}\Delta c \quad \forall l \in \mathcal{L}, \tag{21}$$

where $\eta_{R(l)}$, $G(T(l),R(l))$, Δc represent the sum of interference and noise at link l, the path loss of link l, and the corresponding SINR requirement of a new service, respectively. The algorithm with (21) becomes the "minimum interference routing algorithm" of [25] in the case when $G(T(l),G(R(l)))$ for all $l \in \mathcal{L}$ is a fixed constant. Since the minimum interference routing algorithm belongs to this PwrOpt category, we do not consider it separately.

We also compare our algorithm to the Least Interference Routing (LIR) algorithm in [23] and [24]. In contrast to minimum interference routing [25], LIR chooses a route that focuses only on minimizing the total interference induced by the route. In the simulation, we calculate the interference precisely using global information and the exact distance between the nodes.

For comparison purposes, we consider two different versions of our algorithm depending on the information range used to develop matrix $\bar{F}$ in (20). The first algorithm, OptSINR, uses global information so that the performance of the algorithm can be the upperbound for the different version of our algorithm. In our second algorithm called OptSINRd2.0, we assume that each node can have only local information about neighboring nodes that are located within a distance of two units.

For all the algorithms compared, we assume that the nodes employ power control. In each case, when sending flows through the routes chosen by the algorithm, each node adjusts

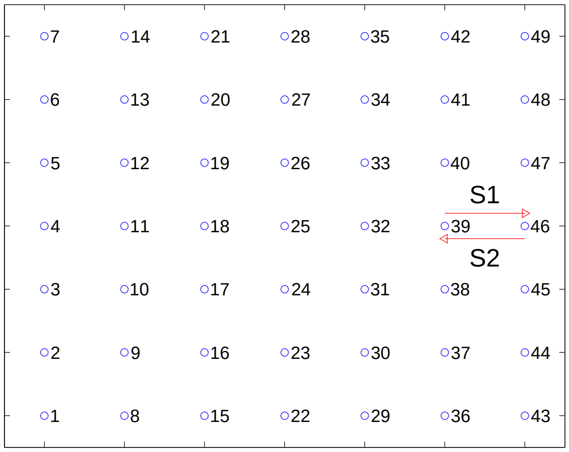

Fig. 3: system environment for simulations in a seven by seven grid network.

its transmission power to satisfy the new minimum constraints at the links. For the simulations, we use a seven by seven grid static network *with 49 homogeneous nodes*, as shown in Fig. 3, and the separation between adjacent nodes in the $x-$ and $y-$ coordinates is one unit of distance. We fix the path loss exponent at three, the attenuation factor at one, the ambient noise at one, and the random variable that represents shadow fading at zero in (5). We assume that all ambient noise is identical and that each link is directional. We assume that wireless links are linear[4], as defined in (3) and the required SINR of a new service flow is fixed at 0.1 (-10dB).

In order to scrutinize the difference in the algorithms, we first study the "microscopic" behavior of the incoming flows. For this study, we compare our algorithm with ME for various scenarios. By studying the microscopic behavior, we understand the dynamics of our algorithm. Further, we validate our algorithm against other algorithms in the case when services randomly arrive into the system. In this environment, we compare our algorithms to ME and PwrOpt in order to validate the performance of our algorithms. *All the simulations are conducted by our own simulator developed in MATLAB.*

5.1 Impact of an ongoing link on a new route

First, we consider how an ongoing link acts on the route of a new flow for different source-destination pairs. To do this, we set one ongoing link to continuously transmit data. The ongoing link can be interpreted as a congested area (hot spot) in a wireless network. In the case of a hybrid network, congested areas occur around base stations [39] (or gateways [40]). In the case when homogeneous nodes are uniformly distributed in a symmetric area and source-destination pairs are randomly chosen, the hot spot usually occurs at the center of the deployed area [41]. For the environment, we establish link $S1$ from nodes 39 to 46 to continuously transmit data, as shown in Fig. 3, and fix $S1$'s SINR at 3 (4.8 dB). To investigate the impact of the ongoing link on a route, we vary the positions of the source and destination nodes.

4. We seperately consider a non-linear case in Section 5.4.

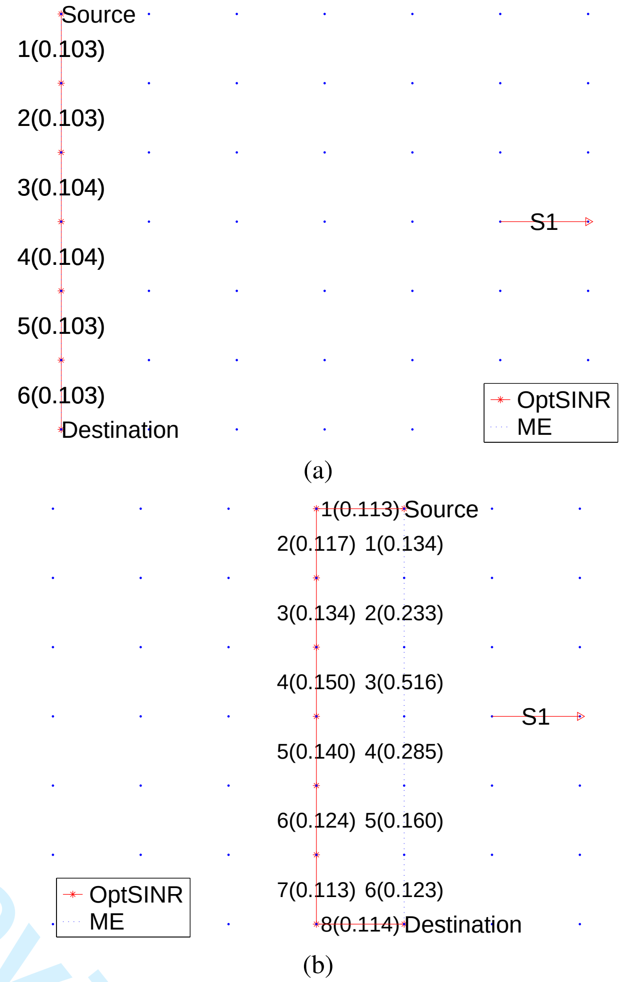

Fig. 4: Comparison of routes when the source-destination pair is differently located from link $S1$: (a) a minimum energy route is far away from link $S1$ (b) a minimum energy route is close to link $S1$.

Fig. 4(a) shows the simulation result when the route of a new incoming flow is located far away from the ongoing link. In the figure, the numbers on the routes stand for the order in which the links are activated in the routing procedure, and the number in each parenthesis represents the additional power consumption over the whole network when the link is activated. Since link $S1$ is far away from the route taken by the new flow, it barely affects the new flow. Thus, our algorithm chooses the same route as the ME algorithm.

When the source-destination pairs are such that a route for the new flow is close to the ongoing link, the interference due to the active link is now too significant to be ignored. Hence, in Fig. 4, our algorithm automatically chooses a different route from the ME algorithm, and takes a detour around the congested area. Compared with ME, the route chosen by our algorithm needs two more hops, but saves about 30% energy over the whole network. Hence, to reduce the interference, our algorithm chooses a longer route than the ME algorithm, but results in consuming a smaller amount of total energy. Thus, energy efficiency comes at the cost of requiring a larger number of hops.

5.2 Impact of average power of existing links on a new route

In the previous subsection, we studied the impact of the power of an ongoing link on a new route. Here, we study the impact of the average power generated by multiple ongoing links that could be transmitting randomly at different times on the route that a new flow will take. To study how this case impacts our algorithm, we consider two-way communication with directional links $S1$ and $S2$ between nodes 39 and 46, as shown in Fig. 3. The two links are mutually exclusive and activated in each time slot with probabilities α for link $S1$ and β for link $S2$ such that $\alpha + \beta = 1$. The scheduling is independent of other links. We fix the source and destination nodes of a new flow at nodes 35 and 29, respectively.

We first vary the probabilities α and β, and fix the minimum SINR constraints of links $S1$ and $S2$ at 1 and 7, respectively, in order to study the impact of the average interference on the route chosen. Fig. 5 shows that our routing algorithm depends on the average power of the ongoing links but the ME algorithm is independent of the ongoing links' environment. In the figure, the numbers on the routes denote the order in which the links are activated in the routing procedure, and the number in each parentheses represents the average power increment expended by the whole network when the link is activated. When the average power of the ongoing links is small (β is small), our algorithm and the ME algorithm work identically, as shown in Fig. 5 (a). However, as the average power of the ongoing links increases (β increases), our algorithm automatically detours farther away from the links to save total network energy consumption, as shown in Fig. 5 (b).

To study the efficiency of our algorithm, we change the probabilities α and β at different minimum SINR constraints for link $S2$ when the minimum constraint for link $S1$ is fixed at one. Fig. 6 depicts the energy improvement of our routing algorithm as compared to the ME routing algorithm. We define here the energy improvement E as $\frac{(B-A)}{B} \times 100$, where A and B correspond to the energy consumption in the network when routes are chosen for a new flow by our algorithm and the ME algorithm, respectively. When the minimum requirements of links $S1$ and $S2$ are fixed, and the probability β changes from 0 to 1, the energy improvement increases, since the average power of the ongoing links is increased. When the minimum SINR of link $S1$ increases, the route computed with SINR metrics is also more energy-efficient than the route computed without SINR metrics.

5.3 Impact of the randomness of flows on the routing

In this subsection, we study the performance of our algorithm for the case when flows randomly arrive at the network and the source-destination pairs are randomly chosen. We consider a time-slotted system. For this simulation, we use two scheduling schemes. Since each node has four adjacent nodes in our simulation environment, we divide each time-slot into eight sub-time-slots and assign them to each link at each node as follows. We assume that reception and transmission do not occur simultaneously at each node. In the first scheduling

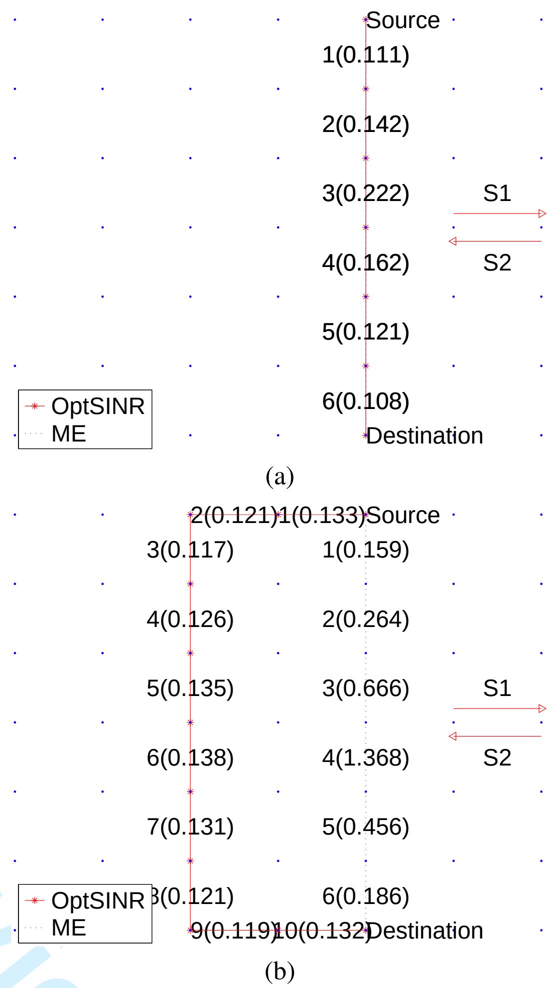

Fig. 5: Comparison of routes when the probability links change: (a) routes does not depend on SINR when the probability of link $S1$ is 0.01 and the probability of link $S2$ is 0.99 (b) the energy efficiency of the proposed route is 58.89% when the probability of link $S1$ is 0.99 and the probability of link $S2$ is 0.01.

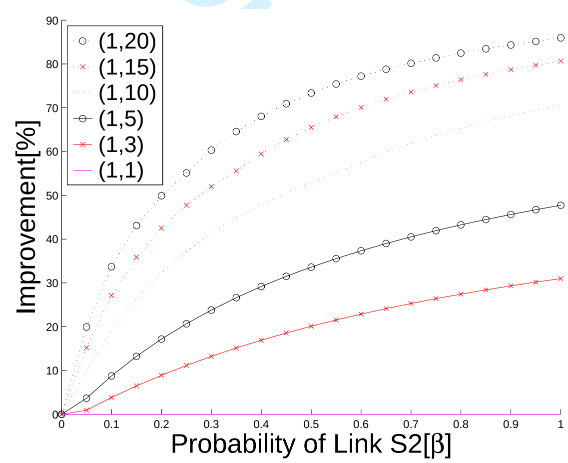

Fig. 6: The energy improvement of OptSINR compared to ME.

scheme, we use fixed and periodic link scheduling as proposed in [42]. Each node periodically transmits (or receives) data to its neighbors at fixed sub-time-slots. For example, node 25 (11) in Fig. 3 is scheduled to transmit data to node 32 (18) every first sub-time-slot and to receive data from node 32 (18) every second sub-time-slot.

In the second scheduling scheme, we use random link scheduling that models IEEE 802.11 wireless LAN. In the random scheduling scheme, each node can randomly choose a link at each sub-slot. Since the unevenly distributed number of links over the sub-time-slots may affect the comparison of both schemes, we constrain the number of scheduled links to be even over all the sub-slots. To compute the statistics of links, each node stores the previous 10 slot information of scheduled links and interference signal strength for random link scheduling.

First, we simulate how many flows can be concurrently accommodated in the network for different values of total power used in the network to study the impact of interference on the performance of routing algorithms. The metric depends on a routing algorithm even in the same system setting and is related to the blocking rate of flows when an admission control is employed to protect the existing flows. For the simulation, we randomly choose source-destination pairs of flows incoming into the network and fix their duration at infinity.

Fig. 7 describes the difference between the performance of the various routing algorithms considered. As the available maximum network power increases, for all the routing algorithms, the number of simultaneously served flows increases together until a maximum number of flows is reached. In both link scheduling schemes, our routing algorithms (OptSINR and OptSINRd2.0) perform better than previous algorithms. The ME algorithm is the most susceptible to the number of flows, since it does not account for interference. PwrOpt and LIR perform better than ME since they consider the impact of interference on a transmission power. The performance of OptSINRd2.0 is in fact quite close to OptSINR, even though these algorithms only use local information. As expected from [42], the routing algorithms are more robust to interference in the fixed and periodic link scheduling environment than in the random link scheduling environment. In the case of the periodic link scheduling, our routing algorithm can simultaneously serve up to 204 flows in the network. However, in case of random link scheduling, up to 118 flows can be served at the same time.

We next consider flow dynamics. Service flows arrive to the system according to a Poisson process with rate 0.33. We fix the holding time at 600 time units for fixed and periodic link scheduling (400 time units for random scheduling) so that the average number of flows in the network is 200 (133), which, from Fig. 7, is close to the maximum number of flows served by our routing algorithm. Since the excessive power consumption may result in call drop (block) in the case of limited power resource, to avoid dropping existing flows, we employ an admission control mechanism in this network, as we discussed in Section 4. The admission control blocks an incoming flow when admitting the incoming flow would result in exceeding a certain threshold of power.

Fig. 8 shows the performance of the routing algorithms. Since our routing algorithms have more capabilities to simultaneously accommodate the number of flows in the network, as shown in Fig. 7, OptSINR and OptSINRd2.0 block fewer number of flows than the other algorithms (ME, LIR, and PwrOpt).

5.4 Impact of system parameters on routing

Here, we study the impact of system parameters such as non-linear SINR wireless channel, the inaccuracy of routing information, etc., on performance. Since, in the previous subsection, the performance comparisons of routing algorithms show similar graphs in both periodic and random scheduling schemes, in this subsection we devise a fixed and periodic scheduling scheme. For simulations, as in the previous subsection, nodes employ admission control and regulate flows when the required power exceeds the maximum transmission power at any link over the chosen route for the flow.

First, we study the impact of flow arrival rates. Fig. 9 depicts the performance in terms of the number of transmitted flows when these arrival rates are different and the maximum network power is fixed at 500 power units. In the case when the arrival rate is low, the average number of simultaneously served flows in the network is low. Hence, all algorithms serve all flows without any drops, as shown in Fig. 9 (a), when the arrival rate is small (0.1 flow per time slot). However, the performance of the various algorithms is quite different when the arrival rate is increased to 0.33 flow per time slot, as shown in Fig. 9 (b). Fig. 10 shows the comparison of the number of transmitted flows when the arrival rate of incoming flows varies and a thousand of flows are generated.

In a distributed set-up, the inaccuracy of the information due to infrequent updates of the information can deteriorate routing performance. We assume that the flow arrival is Poisson with rate 0.33 and that the service time is exponentially distributed with mean 600 slots. We vary the time delay for updating information and study the impact of the delay on routing. Since OptSINRd2.0 uses neighboring nodes' information within a two unit distance, we vary the update frequency from 0 (OptSINRd2.0: immediately update) to 3 (OptSINRd2.0-3: update after three slot time). Fig. 11 shows the impact of information update delay. The information discrepancy results in negligible performance degradation, as shown in Fig. 11.

In reality, the available bandwidth is not a linear function of SINR, as has been assumed in the previous subsections. To study the impact of non-linearity of SINR, we assume that the minimum data rate for each flow is 10 kbps and that the mapping in (1) from the minimum data rate at link l to the minimum SINR constraint $\theta(l)$ at the link is $10 \exp \frac{r(l)}{10^6} - 10$. We assume that the arrival rate of flows is 0.33 and that the random data length has an exponential distribution with average 600. The impact of the function of SINR on routing algorithms is shown in Fig. 12. The shape is similar to the linear SINR case.

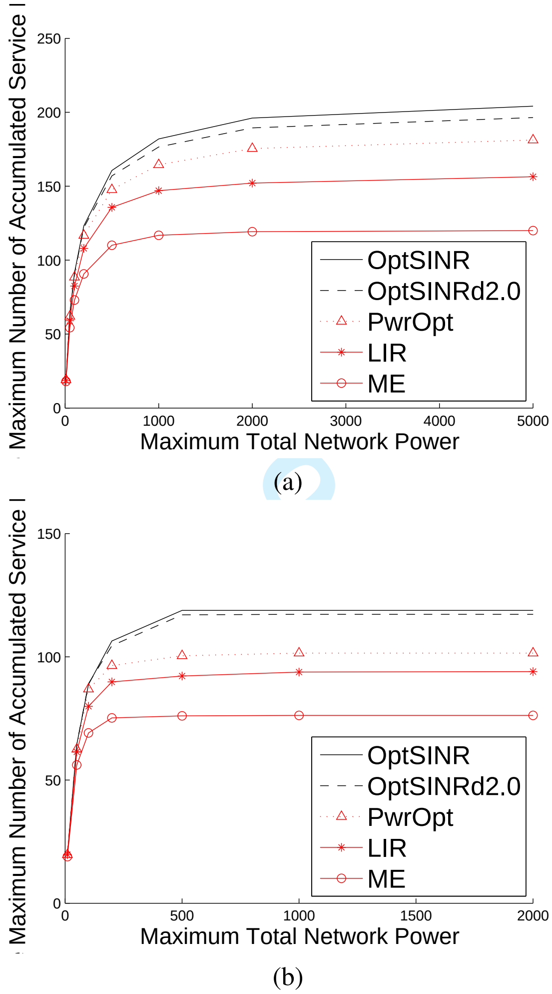

Fig. 7: Comparison of the number of simultaneously served service flows when the total network power varies: (a) Fixed and periodic link scheduling (b) Random link scheduling.

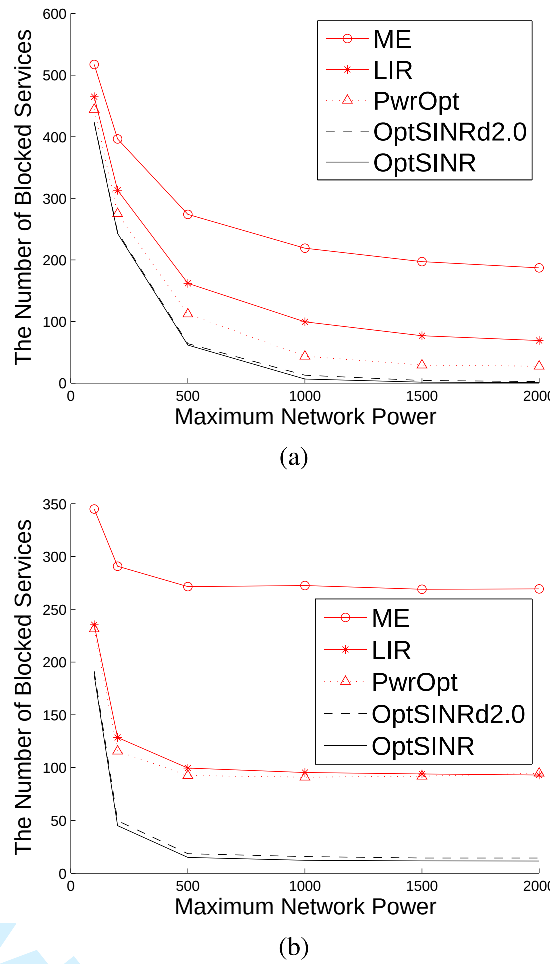

Fig. 8: Comparison of blocked service flows when new flows randomly arrive with rate 3 at the network: (a) Fixed and periodic link scheduling and data length 600 (b) Random link scheduling and data length 400

6 Conclusion

In this paper, we have developed an energy-efficient routing algorithm for given bandwidth constraints of incoming flows in multi-hop wireless networks. The objective is to minimize the total energy consumed in the network under minimum data rate constraints. To do this, we have converted a routing problem defined in the bandwidth domain into the corresponding SINR domain and developed a cross layer routing algorithm that exploits both the SINR (physical layer information) and power control (MAC layer). A nice feature of this routing algorithm is that it automatically routes around congested areas, and thus results in mitigating the overall congestion in the network. We show that for a given class of link scheduling schemes, this algorithm is asymptotically optimal in the sense of average energy consumption. We then develop a distributed version of this algorithm that uses local information and requires a substantial reduction in computational overhead. We find via simulation results that both distributed and centralized versions of the algorithm perform very well, and result in substantial energy savings over the state-of-the-art.

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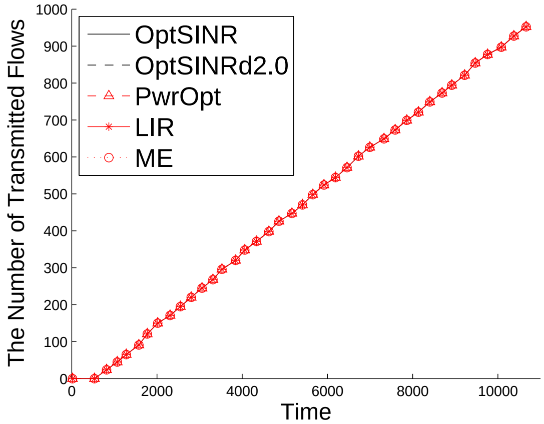
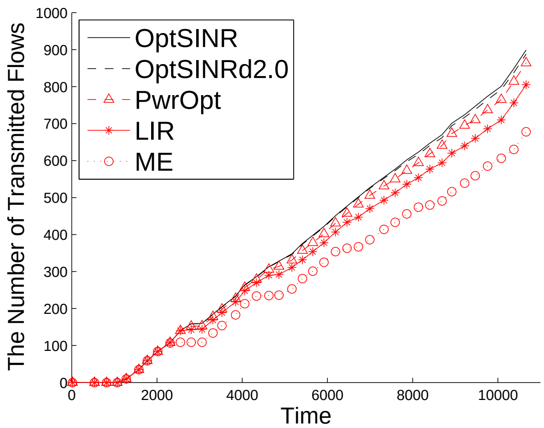


(a)



(b)

Fig. 9: Comparison of the number of transmitted flows when new flows randomly arrive at the network: (a) Poisson arrival rate = 0.1 (b) Poisson arrival rate = 0.33.

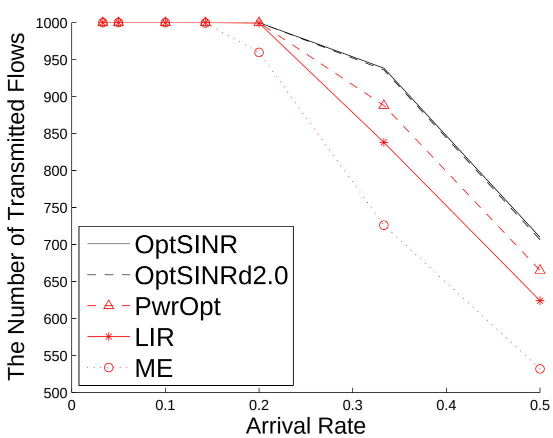


Fig. 10: Comparison of the number of successfully transmitted flows when the arrival rate of new flows varies

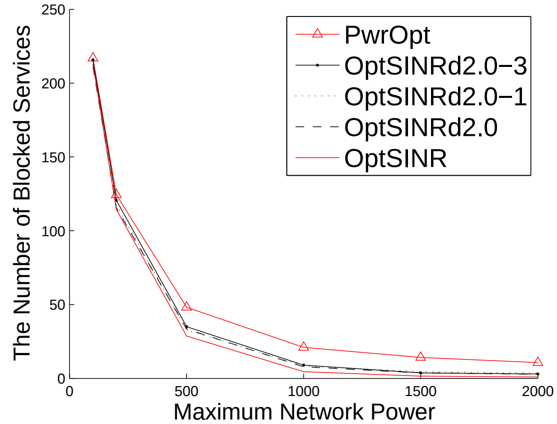


Fig. 11: Comparison of blocked service flows when load information update at each node is delayed. An incoming flow with random data length (average =600) randomly arrives with rate 0.33 at the network.

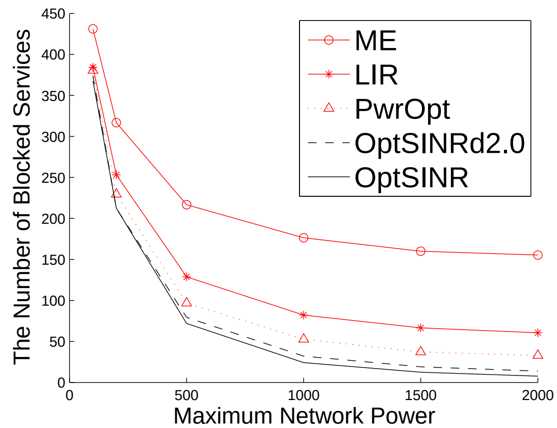


Fig. 12: Comparison of blocked service flows when new flows with random data length randomly arrive with rate 0.33 at the network and data rate is a exponential function of SINR. The average data length is 600.

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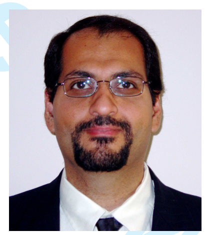

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Dr. Shroff has received numerous awards for his networking research, including the NSF CAREER award, the best paper awards for IEEE INFOCOM'06 and IEEE INFOCOM'08, the best paper award for IEEE IWQoS'06, the best paper of the year award for the Computer Networks journal, and the best paper of the year award for the Journal of Communications and Networks (JCN) (his IEEE INFOCOM'05 paper was one of two runner-up papers).

Response to the Reviewer Comments on "Energy-Efficient SINR-Based Routing for Multi-Hop Wireless Networks"

We thank the reviewers for their helpful comments on our paper. We have revised the paper to take into account the reviewers' questions. The changes are highlighted by using italic text in the manuscript.

We address the specific comments that the reviewers have made. For the reviewers' convenience, we have retyped the reviewers' comments in italics followed by our response.

Reviewer 1

R1-(1) *The authors incorporate interference in energy efficient routing and data dissemination. The problem formulation, discussion, and analysis of results are easy to read. However, it would be nice to add more details regarding the simulation environment. What simulation tool was used to implement 802.11?*

We have developed our own simulator using MATLAB.

R1-(2) *What is the rationale behind choosing the following numbers?*
R1-(2a) *Page 9: lines 24-29: We fix the path loss exponent at three, the attenuation factor at one, the ambient noise at one, and the random variable that represents shadow fading at zero in (5). We assume that all ambient noise is identical and that each link is directional.*

Typically, the path loss exponent can vary from 2 to 4 (we state this in Section 2.2). We fix the exponent to 3 because on page 3 and fixed it at three because it is between this range. Our system model is static and the nodes are homogeneous so we ignore fast fading and set the shadow fading at zero. To clarify we have emphasized the homogeneity of the nodes in the simulation section

R1-(2b) *Page 11, lines 49-55: Service flows arrive to the system according to a Poisson process with rate 0.33, holding time at 600 time units for fixed and periodic link scheduling (400 time units for random scheduling).*

Figure 7 shows that the network can accommodate up to around 200 services simultaneously in the case of periodic link scheduling and 133 services in the case of random link scheduling. As we have explained in the manuscript, we set the arrival rate and service time to be close to the maximal capacity of the network, rate 0.33 and 600 (400) units for service time in the case of the periodic (random) link scheduling. If the arrival rate and service time is too small, all the algorithms can serve all the incoming services and we cannot compare the algorithms. Thus, the arrival rates were chosen such that the system would operate in a high enough load setting to test the efficacy of our solutions. Please see Figures 9 and 10 to understand what happen as the arrival rate varies.

R1-(3) *Minor typos or suggestions:*
R1-(3a) *Page 9, line 45: "To do this, we set one ongoing link to continuously transmits data."*
R1-(3b)*Page 11, line 15: "system parameters such as non-linear SINR, wireless channel the inaccuracy of routing information, etc., on performance."*
R1-(3c) *In Equation (4), keep P on the same side of G(T,R) in both numerator and denominator.*

Thank you. We have fixed all of these.

Reviewer 2

R2-(1) *The authors solves the SINR power control problem to get a solution to the minimum-energy routing problem. While this approach is valid, some justifications/explanations are due here. Is it hard to solve the original problem directly? Or it is just convenient to solve the SINR power control problem?*

While direct solutions can and indeed have been studied, due to the interaction between links, previous studies use approximation, or provide complicated solutions. By making the appropriate transformation, in this paper, we are able to provide a novel and effective approach to find a solution even in the case of non-linear SINR wireless link since we can use simple matrix operation and solve a problem more conveniently with the transformed problem. In a sense one can view the approach in the same way one uses Fourier transforms. While problems can be solved in the time domain, the solution can often be obtained more effectively in the frequency domain after making the appropriate transformation.

R2-(2) *For the dynamic traffic problem (one new connection request at a time), it is harder to deal with the intra-flow interference than to deal with the inter-flow interference. Section 3.1 discusses this effect. The following paper presented a (centralized) linear programming based solution to deal with intra-flow interference in a different (but related) problem: J. Tang, G. Xue and W. Zhang, " Interference-aware topology control and QoS routing in multi-channel wireless mesh networks", ACM MobiHoc'2005, pp. 68-77.*

Thank you for the reference. However, there is substantial difference between our work and this paper. In Section 3.1 we analyze the general case of the impact of a link on the network. The analysis is not confined to only the intra-flow but also the inter-flow. Due to the difference, we could not include the paper in the reference list, but another paper written by the same is more relevant and has been referred to in our reference list.

R2-(3) *The paper still contains a few typos that need to be corrected. The following are some examples.*
R2-(3a) *Page 2, 2nd column, Line 32: "represents" ==> represent*
R2-(3b) *Page 2, 2nd column, Line 35: "interfere with other links" ==> interfere with each other*
R2-(3c)*Page 4, 1st column, Line 24: "we divide it into three steps" ==> It is not clear what "it" means. Please rephrase here.*

Thank you. We now rephrase the sentence (R2-3c) as "We divide the problem into three steps" in the revised manuscript, and have fixed other typos. Thank you.

Energy-Efficient Interference-Based Routing for Multi-hop Wireless Networks

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***Abstract*— In this paper, we develop an energy efficient routing scheme that takes into account the interference created by existing flows in the network. Unlike previous works, we explicitly study the impact of routing a new flow on the energy consumption of the network. Under certain assumptions on how links are scheduled, we can show that our proposed algorithm is asymptotically (in time) optimal in terms of minimizing the average energy consumption. We also develop a distributed version of the algorithm. Our algorithm automatically detours around a congested area in the network, which helps mitigate network congestion and improve overall network performance. Using simulations, we show that the routes chosen by our algorithm (centralized and distributed) are more energy efficient than the state of the art.**

***Index Terms*— multi-hop wireless network, energy efficient routing, SINR, cross layer, simulations.**

I. Introduction

Over the last several years, multi-hop wireless networks have received considerable attention. These networks are expected to have widespread applicability for the purpose of sensing, communications, and distributed computation [1], [2], [3]. The advantages of these networks are: (1) they do not require a sophisticated infrastructure and can be rapidly deployed; (2) they can be deployed in remote, hostile, or hard to reach areas; (3) they can be used to extend the reach of existing network infrastructure.

The evolution of broadband wireless technologies has the potential to significantly extend the scope of applicability of such networks [2], [3], [4]. For example, a wireless mesh network could substitute for a wireline infrastructure in urban areas and offer broadband Internet services. Hence, multi-hop wireless networks may also need to support real-time services that have quality of service (QoS) requirements, as in their wired counterparts. However, unlike wired networks, wireless systems need efficient power management, since transmission power is a precious resource. Even if a wireless system is connected to a power outlet, power is still important as it directly affects the amount of interference created in the network, and thus impacts the overall throughput that the network can sustain. Therefore, power management is a critical component in wireless networks.

To minimize the energy (or power) consumed in wireless networks various mechanisms such as power control [5], [6], [7], link scheduling, [8], [9], [10], [11], and energy-efficient routing algorithms [12], [13] have been studied. By appropriately scheduling links for transmission, one can potentially reduce the mutual interference imposed by concurrent transmissions in order to get better performance. Energy-efficient routing algorithms find the route that minimizes the overall energy consumption. Recently, there has also been an effort to use cross-layer information in order to optimize the efficiency of wireless networks [14]. For example, opportunistic scheduling at the medium access control (MAC) layer can use physical layer information to maximize the overall throughput transmitted through the system [11], [15]. Routing and congestion control at the network layer can use physical layer information to satisfy QoS requirements or to minimize energy consumption [12], [16], [17].

In [6], [13], [18], [19], energy-efficient routing mechanisms have been developed in multi-hop wireless networks, but these do not account for the minimum signal-to-interference-and-noise (SINR) requirements at the different links. The authors in [12] study end-to-end QoS constraints, but do not consider the impact of routing a new flow on the interference and power requirements of the network, i.e., they do not consider how routing a new flow interferes with ongoing flows in the network. In [20], [21], [22], [23], the authors choose routes that use interference between links as metrics for routing. In [20], [21] routing algorithms are developed to minimize the amount of interference caused by a transmission, while in [22], [23], routing algorithms are developed to minimize the amount of interference encountered by a transmission. However, these algorithms may result in choosing energy inefficient routes because they do not explicitly consider energy efficiency, but only interference.

In this paper, we make the following intellectual contributions. We develop an energy-efficient interference-based routing algorithm for given minimum data rates on individual service flows. We explicitly study the impact of routing a new flow on the energy consumption of the network for a certain

This work has been partially supported by the NSF grant ANI-0207728 and a grant from the Indiana 21st Century Fund.

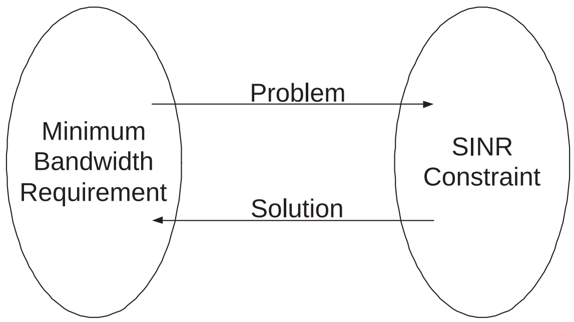

Fig. 1. An approach to find an energy efficient route

class of scheduling schemes. Our approach is to transform the problem defined in the frequency domain into a corresponding problem in the SINR domain, and find a solution in the SINR domain, as illustrated in Figure 1, by using matrix arithmetic.

The rest of the paper is organized as follows. In Section II, we describe the system model and state our basic assumptions. In Section III, we develop an energy-efficient routing scheme using SINR and power control. In Section IV, we discuss a distributed version of our algorithm. In Section V, we provide numerical results to study the efficacy of the scheme. We conclude in Section VI.

II. System Model and Power Control

A. System model

We consider a power-controlled wireless network that supports multi-hop routing, i.e., each node can control its transmission power. We further assume that the time required for power control to converge is negligible. The multi-hop wireless network is modeled as a directed graph $G = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ represents the set of nodes and $\mathcal{L}$ the set of edges that represents communication links between nodes in the network. Each link in $\mathcal{L}$ is identified by the ordered pair, i.e., the transmitting and receiving nodes. Links sharing the same frequency interfere with other links when simultaneously activated. We further assume that the path gain is time invariant, i.e., the attenuation factor is constant and the path gain depends only on the distance between the transmitter and receiver. A new service with data ζ to be transmitted is assumed to require a fixed data rate Δq, so the service flow has a fixed duration μ, such that $\mu = \frac{\zeta}{\Delta q}$. For this paper, we assume that a flow will be routed over only a single route for the entire duration of the flow. We define, $E(l)$, the energy consumption of link l to be

$$E(l) = P(l)\mu^l,$$

where $P(l)$ is the transmission power of link l and μ^l is the amount of time it takes the flow to be served at link l.

B. Wireless link model

Due to the shared nature of the wireless medium, the wireless links interfere with each other. The impact of interference affects the available capacity of these links.

We define a function $g(\cdot)$ that maps the bandwidth $r(l)$ to the corresponding SINR $\theta(l)$ as follows:

$$\theta(l) = g(r(l)). \tag{1}$$

We assume that the function g is increasing and differentiable with respect to $r(l)$, almost surely.

In the case of the band-limited additive white Gaussian noise (AWGN) channel, the channel capacity (also called the Shannon's capacity [24]) at link l, $r(l)$, is given by

$$r(l) = W(l)\log\left(1 + \theta(l)\right), \tag{2}$$

where W represents the bandwidth at link l. Hence, (1) becomes

$$\theta(l) = \exp\left(\frac{r(l)}{W(l)}\right) - 1.$$

In the low SINR region, the available capacity in (1) is often assumed to be a linear function of SINR, that is expressed as

$$\theta(l) \quad = \quad Kr(l), \tag{3}$$

where K is a constant. For simplicity we first develop our routing algorithm under (3) and then extend our algorithm to the more general cases.

The SINR $\theta(l)$ at each link l is defined as

$$\begin{aligned} \theta(l) &= \frac{G(T(l), R(l))P(l)}{\sum_{m:m\neq l} P(m)G(T(m), R(l)) + \sigma_{R(l)}} \\ &= \frac{G(T(l), R(l))P(l)}{\eta_{R(l)}}, \end{aligned} \tag{4}$$

where $T(l)$ is the transmitting node of link l, $R(l)$ is the receiving node corresponding to link l, $\sigma_{R(l)}$ is the ambient noise at node $R(l)$, $P(l)$ is the transmission power at node $T(l)$, $G(T(m), R(l))$ is the path gain between transmitter $T(m)$ and receiver $R(l)$, and $\eta_{R(l)}$ is the sum of interference and noise at node $R(l)$. The path gain $G(T(m), R(l))$ is modeled as

$$G(T(m), R(l)) = K_{T(m)R(l)} d_{T(m)R(l)}^{-\delta},$$

where $K_{T(m)R(l)}$ is the attenuation factor that models power loss due to shadowing, $d_{T(m)R(l)}$ is the distance between nodes $T(m)$ and $R(l)$, and δ is the path loss exponent that typically ranges between 2 and 6 [25].

C. Problem formulation

We study the problem of energy efficient routing from the perspective of a new service (or flow) arriving into a multi-hop wireless network. The problem is to find an energy efficient route for the new service from source to destination that does not violate the resource constraints. To that end, the problem becomes finding a route that minimizes the total *energy increment* needed to serve the new arrival over the entire network for given SINR constraints.

Let the source and destination of a new flow be nodes i and j, respectively. Let $q(l)$ be the minimum required data rate for service flows at link l, i.e. $r(l) \geq q(l)$, for all $l \in \mathcal{L}$, where $r(l)$ is the real data rate served at link l and $\mathcal{L}$ is the set of links. Given link scheduling S, data to be transmitted ζ, maximum power limitation $P^{\max}(l)$, and constraint $q(l)$ for all links $l \in \mathcal{L}$, our problem can be formally expressed as

$$\begin{aligned} (A) \quad & \arg\min_R \quad \Delta E(P,S,R,\zeta) \\ & \text{subject to} \quad r(l) \geq q(l) \quad \forall l \in \mathcal{L} \\ & \qquad P^{\max}(l) \geq P(l) \geq 0 \quad \forall l \in \mathcal{L}, \end{aligned}$$

where $\Delta E(P,S,R,\mu)$ is the additional energy expended by the network when route R is chosen as the new flow, and P is the transmission power.

To solve this problem, we convert the constraint domains from bandwidth to SINR as illustrated in Fig. 1, and develop an energy efficient routing algorithm that complies with the minimum requirements in the SINR constraint domain. The map from the minimum bandwidth requirement to the corresponding minimum SINR constraint is defined by a wireless link model as explained in the previous subsection.

Let $c(l)$ be the minimum SINR corresponding to the minimum required bandwidth at link l, i.e. $\theta(l) \geq c(l)$, all $l \in \mathcal{L}$, where $\mathcal{L}$ is the set of links. Then, for a given link scheduling S, Problem (A) becomes

$$\begin{aligned} (B) \quad & \arg\min_{R\in R(i,j)} \quad \Delta E(P,S,R,\mu) && (5) \\ & \text{subject to} \quad \theta(l) \geq c(l) \quad \forall l \in \mathcal{L} && (6) \\ & \qquad P^{\max}(l) \geq P(l) \geq 0 \quad \forall l \in \mathcal{L}, && (7) \end{aligned}$$

where $c(l)$ is the SINR constraint for link l.

For simplicity, in our analysis, we do not consider the maximum transmission power constraint (7), i.e., we let $P^{\max}(l)$ go to infinity for all l. However, in Section V we use simulations to study a power-constrained system that incorporates admission control. In contrast to previous work, our goal is to find the optimal routes in terms of energy consumption over the entire network given a certain class of link scheduling schemes.

D. Power control

Recall that $\mathcal{L}$ is the set of links. We let $\mathbf{P}$ denote the power vector defined by $\mathbf{P} = (P(1), \cdots, P(L_{\mathcal{L}}))^T$, where $P(l)$ is the power of link l, and $L_{\mathcal{L}}$ is the number of links in set $\mathcal{L}$. Using (4), we can rewrite (6) in matrix form as

$$\mathbf{P} \geq \mathbf{F}\mathbf{P} + \mathbf{b}, \qquad (8)$$

where $\mathbf{b} = (b(1), \cdots, b(L_{\mathcal{L}}))^T$ such that $b(l) = \frac{c(l)\sigma_{R(l)}}{G(T(l),R(l))}$, and $\mathbf{F}$ is the $L_{\mathcal{L}} \times L_{\mathcal{L}}$ matrix with (l,m) entry

$$F(l,m) = \begin{cases} \frac{G(T(m),R(l))c(l)}{G(T(l),R(l))} & , \ l \neq m \\ 0 & , \ l = m. \end{cases} \qquad (9)$$

Matrix $\mathbf{F}$ defined by (9) has non-negative elements, and since the links interact with each other, it is also irreducible. Hence, we have the following theorem [26] from the Perron-Frobenius theorem and standard matrix theory [27].

Theorem 1: The following statements are equivalent:

1) $\rho_{\mathbf{F}} \leq 1$ where $\rho_{\mathbf{F}}$ is the Perron-Frobenius eigenvalue of $\mathbf{F}$.
2) There exists a vector $\mathbf{P} > 0$ such that $(\mathbf{I} - \mathbf{F})\mathbf{P} \geq \mathbf{b}$.
3) $(\mathbf{I} - \mathbf{F})^{-1}$ exists and is positive componentwise.

If there exists a positive feasible vector $\mathbf{P}$, it follows from Theorem 1 that $(\mathbf{I} - \mathbf{F})^{-1}$ exists. From (8) we can obtain

$$\mathbf{P} \geq (\mathbf{I} - \mathbf{F})^{-1}\mathbf{b}.$$

Hence, we have the Pareto optimal[1] solution $(\mathbf{I} - \mathbf{F})^{-1}\mathbf{b}$ that supports the network topology defined by links in $\mathcal{L}$ and their associated minimum requirements. One can use distributed power control algorithm [5], [26] to achieve this minimum power vector.

III. Routing with SINR Metric

In order to solve Problem (B), we divide it into three steps.

- First, as a preliminary step, we study the impact that is caused by an individual link's power increment on the overall network to be able to support a new flow. In this step, we consider only the influence of the change of the link constraint on the network.
- Second, we consider the impact of admitting a new flow on the network as it traverses an entire route. Since the route consists of one or more links, there is interference between the links themselves that transport the flow. Based on the impact on the network energy consumption, we propose a new routing algorithm with SINR metrics in order to satisfy the minimum constraints of all the links and to minimize the energy consumption over the network.
- Finally, we consider the case when the links are not activated at the same time but scheduled according to some link scheduling mechanism. In the first two cases, we do not consider link scheduling over the network and we simply assume that there exist static matrices $\mathbf{F}$, $\mathbf{b}$ and $\mathbf{P}$. However, in practice, these matrices can change dynamically because of link scheduling.

A. Impact of a single link that is activated by a new flow

Assume that a new service arrives at an arbitrary link n in the network. The service has data ζ to be transmitted and needs a minimum data rate of Δq. The minimum data rate is assumed to be fixed during the service time over the network. Since the corresponding SINR constraint Δc and the flow duration of the service μ are expressed as $K\Delta q$ and $\frac{\zeta}{\Delta q}$, respectively, the constraint Δc and the flow duration μ are fixed. In the case when link n serves the new service flow, the minimum bandwidth for service flows at link n increases by Δq, i.e. the minimum SINR increases by Δc. As a result, the power requirement for link n to accommodate this flow increases from $P(n)$ to $P'(n)$. The additional energy required to serve the flow at link n, $\Delta E(n)$, is expressed as

$$\begin{aligned} \Delta E(n) &= \Delta P(n)\mu \\ &= \left(P^{'}(n) - P(n)\right)\mu, \end{aligned}$$

[1] $\mathbf{P}^*$ is said to be Pareto optimal if $\mathbf{P}^*$ is feasible and any feasible $\mathbf{P}$ satisfies $\mathbf{P} \geq \mathbf{P}^*$ componentwise.

where $\Delta P(n)$ is the additional transmission power required at link n. Since the flow duration, μ, is fixed, we only need to consider the transmission powers of the links in the network.

For the new flow, we define $\mathbf{F}^{'}$, $\mathbf{P}^{'}$, and $\mathbf{b}^{'}$ to be the matrices corresponding to $\mathbf{F}$, $\mathbf{P}$, and $\mathbf{b}$ in the new environment. When the constraint at link n changes, the additional power required in the network $\mathbf{\Delta P}_n$ can be expressed as

$$
\begin{aligned}
\mathbf{\Delta P}_n &= \mathbf{P}' - \mathbf{P} \\
&= (\mathbf{F}'\mathbf{P}' + \mathbf{b}') - (\mathbf{F}\mathbf{P} + \mathbf{b}) \\
&= (\mathbf{F}'(\mathbf{P} + \mathbf{\Delta P}_n) + \mathbf{b}') - (\mathbf{F}\mathbf{P} + \mathbf{b}) \\
&= (\mathbf{F}' - \mathbf{F})\mathbf{P} + \mathbf{F}'\mathbf{\Delta P}_n + \mathbf{b}' - \mathbf{b} \\
&= (\mathbf{F}' - \mathbf{F})\mathbf{P} + \mathbf{F}'\mathbf{\Delta P}_n + \mathbf{\Delta b}_n,
\end{aligned}
$$

where $\mathbf{P} = \mathbf{F}\mathbf{P} + \mathbf{b}$, $\mathbf{P}' = \mathbf{F}'\mathbf{P}' + \mathbf{b}'$, and $\mathbf{\Delta b}_n \triangleq \mathbf{b}' - \mathbf{b}$. Hence, when $(\mathbf{I} - \mathbf{F}')^{-1}$ exists,[2] by rearranging and premultiplying the resulting matrix equality by $(\mathbf{I} - \mathbf{F}')^{-1}$, we obtain

$$
\mathbf{\Delta P}_n = (\mathbf{I} - \mathbf{F}')^{-1}((\mathbf{F}' - \mathbf{F})\mathbf{P} + \mathbf{\Delta b}_n). \quad (10)
$$

We let $(\mathbf{I} - \mathbf{F})^{-1}_{(n)}$ denote the nth column vector of matrix $(\mathbf{I} - \mathbf{F})^{-1}$ and $(\mathbf{I} - \mathbf{F})^{-1}_{\sum^n}$ as the elementwise sum of matrix $(\mathbf{I} - \mathbf{F})^{-1}_{(n)}$. Then,

$$
\begin{aligned}
&\mathbf{\Delta P}_n \\
= &(\mathbf{I} - \mathbf{F}')^{-1}[\underbrace{0, \cdots, 0}_{n-1}, \sum_{m \neq n} \frac{G(T(m), R(n))P(m)\Delta c}{G(T(n), R(n))} \\
&+ \frac{\sigma_{R(n)}\Delta c}{G(T(n), R(n))}, \underbrace{0, \cdots, 0}_{L_{\mathcal{L}}-n}]^T \\
= &(\mathbf{I} - \mathbf{F}')^{-1}[\underbrace{0, \cdots, 0}_{n-1}, \frac{\eta_{R(n)}}{G(T(n), R(n))}\Delta c, \underbrace{0, \cdots, 0}_{L_{\mathcal{L}}-n}]^T \\
= &(\mathbf{I} - \mathbf{F}')^{-1}_{(n)} \left(\frac{\eta_{R(n)}}{G(T(n), R(n))} \right) \Delta c, \quad (11)
\end{aligned}
$$

where $\eta_{R(n)} = \sum_{m \neq n} G(T(m), R(n))P(m) + \sigma_{R(n)}$.

Before taking the next step, one question to be asked is whether (11) really reflects all the increments of the nodes' power consumption over the entire network when link n's constraint is changed by Δc. This question can be explained by Theorem 1. If $\mathbf{P}'$ is feasible, then $(\mathbf{I} - \mathbf{F}')^{-1}$ is equal to $\sum_{m=0}^{\infty} (\mathbf{F}')^m$. Hence, we can rewrite (11) as follows.

$$
\begin{aligned}
&\mathbf{\Delta P}_n \\
= &(\mathbf{I} - \mathbf{F}')^{-1}[\underbrace{0, \cdots, 0}_{n-1}, \Delta P(n), \underbrace{0, \cdots, 0}_{L_{\mathcal{L}}-n}]^T \\
= &(\mathbf{I} + \mathbf{F}' + (\mathbf{F}')^2 + \cdots)[\underbrace{0, \cdots, 0}_{n-1}, \Delta P(n), \underbrace{0, \cdots, 0}_{L_{\mathcal{L}}-n}]^T \\
= &[\underbrace{0, \cdots, 0}_{n-1}, \Delta P(n), \underbrace{0, \cdots, 0}_{L_{\mathcal{L}}-n}]^T + \mathbf{F}'_{(n)}\Delta P(n) \\
&+ (\mathbf{F}')^2_{(n)}\Delta P(n) + (\mathbf{F}')^3_{(n)}\Delta P(n) + \cdots \\
= &[\underbrace{0, \cdots, 0}_{n-1}, \Delta P(n), \underbrace{0, \cdots, 0}_{L_{\mathcal{L}}-n}]^T + \mathbf{F}'_{(n)}\Delta P(n) \\
&+ \mathbf{F}'(\mathbf{F}'_{(n)}\Delta P(n)) + (\mathbf{F}')^2(\mathbf{F}'_{(n)}\Delta P(n)) + \cdots, \quad (12)
\end{aligned}
$$

where $\Delta P(n) = \frac{\eta_{R(n)}}{G(T(n), R(n))}\Delta c$. The first term in (12), $[\underbrace{0, \cdots, 0}_{n-1}, \Delta P(n), \underbrace{0, \cdots, 0}_{L_{\mathcal{L}}-n}]^T$, represents the amount of additional power at link n when the constraint of link n is changed by Δc. The second term, $\mathbf{F}'_{(n)}\Delta P(n)$, is equal to the amount of additional power of each node in the network when link n increases its power by $\Delta P(n)$. The third term, $\mathbf{F}'(\mathbf{F}'_{(n)}\Delta P(n))$, is the amount of additional power in the network when the network increases its power by $\mathbf{F}'_{(n)}\Delta P(n)$ at each link. Thus, each term represents the increment of power consumption iterated back and forth over the whole network.

The energy increment of each link in the network, when a new flow with additional constraint Δc and duration μ arrives at node n, can be expressed as

$$
\begin{aligned}
\mathbf{\Delta E}_n &= \mathbf{\Delta P}_n \ \mu \\
&= (\mathbf{I} - \mathbf{F}')^{-1}_{(n)} \left(\frac{\eta_{R(n)}}{G(T(n), R(n))} \right) \Delta c \ \mu.
\end{aligned}
$$

B. Impact of a route on the network when a new flow traverses the route

In this subsection, we consider the impact that a flow traversing a route has on the energy requirement of the network. In contrast to the previous subsection, one or more links in the route may simultaneously transmit the flow. Hence, when transmitting this flow, a link can interfere with other links that transport the flow over the route.

To study the case when more than one link in a route is simultaneously activated, we first consider the power increment needed when there are two activated links, m and n, which deliver the new flow at the same time. As in the previous subsection, we define $\mathbf{\Delta P}_{m,n}$ and $\mathbf{\Delta b}_{m,n}$ to be $\mathbf{\Delta P}_{m,n} \triangleq \mathbf{P}' - \mathbf{P}$ and $\mathbf{\Delta b}_{m,n} \triangleq \mathbf{b}' - \mathbf{b}$, respectively, where $\mathbf{P} = \mathbf{F}\mathbf{P} + \mathbf{b}$ and $\mathbf{P}' = \mathbf{F}'\mathbf{P}' + \mathbf{b}'$. Hence, as in (11), we have

$$
\begin{aligned}
&\mathbf{\Delta P}_{m,n} \\
= &(\mathbf{I} - \mathbf{F}')^{-1}((\mathbf{F}' - \mathbf{F})\mathbf{P} + \mathbf{\Delta b}_{m,n}) \\
= &(\mathbf{I} - \mathbf{F}')^{-1}[\underbrace{0, \cdots, 0}_{m-1}, \frac{\eta_{R(m)}}{G(T(m), R(m))}\Delta c, \underbrace{0, \cdots, 0}_{n-m-1}, \\
&\frac{\eta_{R(n)}}{G(T(n), R(n))}\Delta c, \underbrace{0, \cdots, 0}_{L_{\mathcal{L}}-n}]^T \\
= &(\mathbf{I} - \mathbf{F}')^{-1}[\underbrace{0, \cdots, 0}_{m-1}, \Delta P(m), \underbrace{0, \cdots, 0}_{n-m-1}, \Delta P(n), \underbrace{0, \cdots, 0}_{L_{\mathcal{L}}}]^T \\
= &(\mathbf{I} - \mathbf{F}')^{-1}[\underbrace{0, \cdots, 0}_{m-1}, \Delta P(m), \underbrace{0, \cdots, 0}_{L_{\mathcal{L}}}]^T \\
&+ (\mathbf{I} - \mathbf{F}')^{-1}[\underbrace{0, \cdots, 0}_{n-1}, \Delta P(n), \underbrace{0, \cdots, 0}_{L_{\mathcal{L}}}]^T \\
= &(\mathbf{I} - \mathbf{F}')^{-1}_{(m)}\Delta P(m) + (\mathbf{I} - \mathbf{F}')^{-1}_{(n)}\Delta P(n),
\end{aligned}
$$

[2] If this matrix is not invertible, then it implies that there is no solution, i.e., the power levels required to sustain the SINR go to infinity.

where $\Delta P(m) = \frac{\eta_{R(m)}}{G(T(m),R(m))}\Delta c$ and $\Delta P(n) = \frac{\eta_{R(n)}}{G(T(n),R(n))}\Delta c$. In the same way, when a flow is served by a set of links Λ at a given time slot, the increased power consumption $\mathbf{\Delta P}_\Lambda$ can be *decomposed* as

$$\mathbf{\Delta P}_\Lambda = \sum_{n\in\Lambda} (\mathbf{I}-\mathbf{F}')^{-1}_{(n)}\Delta P(n),$$

where $\Delta P(n) = \frac{\eta_{R(n)}}{G(T(n),R(n))}\Delta c$.

Here, we study how much additional energy is consumed when a new flow with Δc and μ arrives at the network and route R is chosen to serve the flow between source i and destination j. Let T_R be the set of time slots over which the flow is served by route R. We define μ_t^l an indicator function such that it is one if link l serves the flow at time slot t, and zero otherwise. Hence, $\sum_{t\in T_R}\mu_t^l$ is equal to the total duration μ. Then, the additional energy consumed by the network through route R, $\Delta E_R^{Network}$, is

$$\begin{aligned}
&\Delta E_R^{Network}\\
=\ & \sum_{t\in T_R}\Delta E_R^t\\
=\ & \sum_{t\in T_R}\sum_{l\in R}(\mathbf{I}-\mathbf{F}')^{-1}_{\sum^l}\Delta P(l)\mu_t^l\\
=\ & \sum_{t\in T_R}\sum_{l\in R}(\mathbf{I}-\mathbf{F}')^{-1}_{\sum^l}\frac{\eta_{R(m)}}{G(T(m),R(m))}\Delta c\ \mu_t^l\\
=\ & \sum_{l\in R}\left(\sum_{t\in T_R}\mu_t^l\right)(\mathbf{I}-\mathbf{F}')^{-1}_{\sum^l}\frac{\eta_{R(m)}}{G(T(m),R(m))}\Delta c\\
=\ & \left(\sum_{l\in R}(\mathbf{I}-\mathbf{F}')^{-1}_{\sum^l}\frac{\eta_{R(m)}}{G(T(m),R(m))}\right)\Delta c\ \mu,
\end{aligned}$$

where ΔE_R^t is the energy additionally consumed by the network at time t when route R is chosen for the flow. Since $\Delta c\ \mu$ can be rewritten by $K\zeta$ that is fixed, we have

$$\begin{aligned}
&\arg\min_{R\in R(i,j)}\Delta E_R^{Network}\\
=\ & \arg\min_{R\in R(i,j)}\sum_{l\in R}(\mathbf{I}-\mathbf{F}')^{-1}_{\sum^l}\frac{\eta_{R(m)}}{G(T(m),R(m))},
\end{aligned}$$

where $R(i,j)$ is the set of possible routes from node i to node j. Moreover, when Δc goes to 0, $\mathbf{F}'$ converges to $\mathbf{F}$ componentwise.

If we ignore link scheduling and the additional required constraint is infinitesimally small, our energy efficient routing algorithm to solve Problem (B) becomes

$$\begin{aligned}
\arg\min_{R\in R(i,j)}\quad & \sum_{l\in R}\left((\mathbf{I}-\mathbf{F})^{-1}_{\sum^l}\left(\frac{\eta_{R(l)}}{G(T(l),R(l))}\right)\right) \qquad (13)\\
\text{subject to}\quad & \theta(l)\ge c(l) \quad \forall l\in\mathcal{L}.\\
& P(l)\ge 0 \quad \forall l\in\mathcal{L},
\end{aligned}$$

where $R(i,j)$ is the set of possible routes from node i to node j. From (13), instead of the exact value of the network energy increment of a route, $\Delta E_R^{Network}$, we can choose a minimum energy route from the interference measured at $R(l)$, $\frac{\eta_{R(l)}}{G(T(l),R(l))}$, and minimum requirements acquired from other nodes, $(\mathbf{I}-\mathbf{F})^{-1}_{\sum^l}$, as described below.

Construct a directed graph $G=(\mathcal{N},\mathcal{L})$.
For an incoming flow, check if resources are available.
If yes,
  Measure the interference strength at all nodes in $\mathcal{N}$.
  Calculate $(\mathbf{I}-\mathbf{F})^{-\mathbf{1}}$ based on path loss and constraints.
  Calculate link cost $(\mathbf{I}-\mathbf{F})^{-1}_{\sum^l}\left(\frac{\eta_{R(l)}}{G(T(l),R(l))}\right)\ \forall l\in\mathcal{L}$.
  Apply a shortest path algorithm, e.g. Dijkstra's algorithm or Bellman-Ford algorithm, to find the minimum cost route.
Otherwise,
  Reject the incoming flow.
  Notify the rejection to the source.

The normalized interference, $\frac{\eta_{R(l)}}{G(T(l),R(l))}$, in (13) can be locally measured. Hence, if $\mathbf{F}$ is computed at each node, a fully distributed algorithm can be implemented since the link cost $(\mathbf{I}-\mathbf{F})^{-1}_{\sum^l}\left(\frac{\eta_{R(l)}}{G(T(l),R(l))}\right)$ can be locally computed at each node. We will discuss the distributed algorithm in Section IV.

C. *When links in the network are scheduled*

In a multi-hop wireless network, link scheduling may be used in order to reduce the interference and to improve network performance. The link scheduling problem over a large network is NP-complete [10] even if scheduling allows only half-duplex links per node. In the case when each node opportunistically uses the wireless environment as in [11], [15], the scheduling is distributed and independent. Although we do not require that the scheduling scheme is independent, we do require that it satisfy a much weaker "mixing" property to be defined soon.

Let $\mathbf{S}_t$ be an $L_\mathcal{L}\times L_\mathcal{L}$ scheduling matrix of links at time slot t with (l,m) entry defined as

$$S_t(l,m) = \begin{cases} 1, & \text{if link } l \text{ is active and } l=m\\ 0, & \text{otherwise}\end{cases} \qquad (14)$$

and $\mathbf{F}_t$ be an $L_\mathcal{L}\times L_\mathcal{L}$ matrix with (l,m) entry defined as

$$F_t(l,m) = \begin{cases} \frac{1_{t,(l,m)}G(T(m),R(l))c(l)}{G(T(l),R(l))} & ,\ m\ne l\\ 0 & ,\ m=l\end{cases} \qquad (15)$$

where $1_{t,(l,m)}$ is one, if links l and m are active at time t, otherwise zero.

As mentioned earlier, we assume that the scheduling $\mathbf{S}_t$ has a mixing property with respect to mapping φ such that $\varphi^k(\mathbf{S}_t)=\mathbf{S}_{t+k}$, i.e. $\lim_{k\to\infty}\Pr(A\cap\varphi^{-k}B)=\Pr(A)\Pr(B)$ for all random variables A and B in a σ-field generated by $\mathbf{S}_t$, where $\Pr(\cdot)$ stands for a probability operator. The mixing assumption is more practical and weaker than that of time independence (although even time independence satisfies the opportunistic scheduling schemes in [11], [15]). Since mixing implies ergodicity [28], [29], it follows from the ergodic theorem that

$$\lim_{k\to\infty}\frac{1}{k}\sum_{n=1}^{k}\varphi^{n-1}(\mathbf{S}_t)=\mathbf{\Pi}. \tag{16}$$

Here $\mathbf{\Pi}$ is an $L_\epsilon \times L_\epsilon$ diagonal matrix such that $\mathbf{\Pi}$ is the expected value of $\mathbf{S}_t$. Each (l,m) entry of $\mathbf{\Pi}$ denoted by $\Pi(l,m)$ is defined to be equal to π_l, if $l=m$, otherwise zero, where π_l is the expected value of $S_t(l,l)$. Similarly, let $\hat{\mathbf{F}}$ be $\hat{\mathbf{F}}=\lim_{k\to\infty}\frac{1}{k}\sum_{t=1}^{k}\mathbf{F}_t$ with (l,m) entry defined as $\hat{F}(l,m)$ is $\frac{\pi_{l,m}G(T(m),R(l))c(l)}{G(T(l),R(l))}$, if $l\neq m$, otherwise zero, where $\pi_{l,m}$ [3] is the expected value of $1_{t,(l,m)}$.

For time slot t, (8) becomes

$$\mathbf{S}_t\mathbf{P} \;\; \geq \;\; \mathbf{F}_t\mathbf{P}+\mathbf{S}_t\mathbf{b}\;\forall t\in T, \tag{17}$$

where T is a set of time slots. In the case of a half-duplex link, $\mathbf{S}_t-\mathbf{F}_t$ may not be a full rank matrix. For each time t, even if the rank of $\mathbf{S}_t-\mathbf{F}_t$ in (17) is less than $L_{\mathcal{L}}$, the non-negative optimal solution can be found by a Moore-Penrose inverse matrix [30]. In the solution, the elements of $\mathbf{P}$ that correspond to the scheduled (active) links are positive at time t, and zero otherwise. Over all t in T, since all the vectors of (17) are componentwise non-negative, it follows from (16) that

$$\begin{aligned}
\sum_{t=1}^{k}\mathbf{S}_t\mathbf{P} &\geq \sum_{t=1}^{k}(\mathbf{F}_t\mathbf{P}+\mathbf{S}_t\mathbf{b})\\
\frac{1}{k}\sum_{t=1}^{k}\mathbf{S}_t\mathbf{P} &\geq \frac{1}{k}\sum_{t=1}^{k}(\mathbf{F}_t\mathbf{P}+\mathbf{S}_t\mathbf{b})\\
\lim_{k\to\infty}\frac{1}{k}\sum_{t=1}^{k}\mathbf{S}_t\mathbf{P} &\geq \lim_{k\to\infty}\frac{1}{k}\sum_{t=1}^{k}(\mathbf{F}_t\mathbf{P}+\mathbf{S}_t\mathbf{b})\\
\mathbf{\Pi}\mathbf{P} &\geq \hat{\mathbf{F}}\mathbf{P}+\mathbf{\Pi}\mathbf{b}.
\end{aligned} \tag{18}$$

From the above result (18), we have the following lemma.

Lemma 1: Let $\mathbf{S}_t$ be a scheduling scheme that satisfies (16). Assume that matrix $\mathbf{\Pi}$ has full rank and $\bar{\mathbf{F}}$ is irreducible, where $\bar{\mathbf{F}}$ is equal to $\mathbf{\Pi}^{-1}\hat{\mathbf{F}}$. If the Perron-Frobenius eigenvalue of $\bar{\mathbf{F}}$ is smaller than one, then there exists a minimum average power vector $\bar{\mathbf{P}}$ such that $\bar{\mathbf{P}}=\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}\mathbf{b}$ and $\bar{\mathbf{P}}$ is positive componentwise. If not, there is no minimum average power vector that satisfies the constraints, for the given link scheduling, $\mathbf{S}_t$.

Proof: Since $\mathbf{\Pi}$ is a full rank diagonal matrix, its inverse matrix, $\mathbf{\Pi}^{-1}$, exists. Hence, (18) becomes $\mathbf{P}\geq\mathbf{\Pi}^{-1}\hat{\mathbf{F}}\mathbf{P}+\mathbf{b}$. If $(\mathbf{I}-\bar{\mathbf{F}})^{-1}$ exists where $\bar{\mathbf{F}}=\mathbf{\Pi}^{-1}\hat{\mathbf{F}}$, then the minimum average power $\bar{\mathbf{P}}$ is equal to $\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})\mathbf{b}$ from Theorem 1. ■

Based on Lemma 1, we henceforth assume feasibility that there exists $\bar{\mathbf{P}}$. As in (11), when link n has a new flow, the power increment in the networks is given by

$$\begin{aligned}
&\mathbf{\Delta}\bar{\mathbf{P}}_n\\
=\;& \mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}((\bar{\mathbf{F}}'-\bar{\mathbf{F}})\mathbf{P}+\mathbf{\Delta}\mathbf{b}_n)\\
=\;& \mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}[\underbrace{0,\cdots,0}_{n-1},\sum_{m\neq n}\frac{G(T(m),R(n))\pi_{m|n}P(m)}{G(T(n),R(n))}\Delta c\\
&+\frac{\sigma_{R(n)}}{G(T(n),R(n))}\Delta c,\underbrace{0,\cdots,0}_{L_{\mathcal{L}}-n}]^T\\
=\;& \left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}\right)_{(n)}\left(\frac{\bar{\eta}_{R(n)}}{G(T(n),R(n))}\right)\Delta c,
\end{aligned}$$

where $\bar{\eta}_{R(n)}$ is the average of the interference and noise measured at the receiving node of link n when link n is active, and $\pi_{m|n}$ is defined as $\frac{\pi_{m,n}}{\pi_n}$, where $\pi_{m|n}$ is the conditional probability that link m is active given link n that is active. Since the flow duration μ and additional constraint Δc are fixed, for a given link scheduling S, the routing algorithm for minimizing the average energy consumption can be expressed as follows.

$$\arg\min_{R\in R(i,j)} \quad \sum_{l\in R}\left(\frac{\bar{\eta}_{R(l)}}{G(T(l),R(l))}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}\right)_{\sum^l}\right) \tag{19}$$

$$\text{subject to} \quad \theta(l)\geq c(l) \quad \forall l\in\mathcal{L},$$
$$P(l)\geq 0 \quad \forall l\in\mathcal{L}.$$

The algorithm in procedure is similar to that of the previous subsection, and is shown below.

Construct a directed graph $G=(\mathcal{N},\mathcal{L})$.
For an incoming flow, check if resources are available.
If yes,
- Measure the *average* interference strength at all nodes.
- Calculate the time *average* of link scheduling matrix, $\mathbf{\Pi}$.
- Calculate $(\mathbf{I}-\bar{\mathbf{F}})^{-1}$ based on path loss and correlation between links.
- Calculate link cost $\frac{\bar{\eta}_{R(l)}}{G(T(l),R(l))}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}\right)_{\sum^l}$ $\forall l\in\mathcal{L}$.
- Apply a shortest path algorithm to find the minimum cost route.

Otherwise,
- Reject the incoming flow.
- Notify the rejection to the source.

D. When the data rate is a nonlinear function of SINR

Extending our earlier solution to the case when the data rate is a more general function of SINR turns out to be quite straightforward. To do this, we develop our algorithm with a function $g(\cdot)$ defined in (1), which maps the bandwidth constraint in Problem (A) to the SINR constraint in Problem (B). Similarly to the previous subsection, the impact of a new route for a new service flow on network energy, $\Delta\bar{E}_R^{Network}$, can

[3] $\pi_{l,m}$ means the probability that links l and m are active simultaneously.

now be expressed as:

$$
\begin{aligned}
&\Delta \bar{E}_R^{Network} \\
&= \sum_{l\in R}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}{}_{\Sigma^l}\right)\Delta\bar{P}(l)\mu^l \\
&= \sum_{l\in R}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}{}_{\Sigma^l}\right)\Delta\bar{P}(l)\left(\frac{\zeta}{\Delta q}\right) \\
&= \sum_{l\in R}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}{}_{\Sigma^l}\right)\frac{\bar{\eta}_{R(l)}\Delta c}{G(T(l),R(l))}\left(\frac{\zeta}{\Delta q}\right) \\
&= \sum_{l\in R}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}}')^{-1}{}_{\Sigma^l}\right)\frac{\bar{\eta}_{R(l)}}{G(T(l),R(l))}\left(\frac{\Delta c}{\Delta q}\right)\zeta,
\end{aligned}
$$

where $\Delta c = g(q(l)+\Delta q) - g(q(l))$ and ζ is the total data to be transmitted by the new flow. In the case when the required bandwidth Δq is infinitesimal, $\bar{F}'$ and $\frac{\Delta c}{\Delta q}$ $\left(= \frac{g(q(l)+\Delta q)-g(q(l))}{\Delta q}\right)$ become $\bar{F}$ and $g'(q(l))$, respectively, where $g'(q(l))$ is a derivative of $g(q(l))$ with respect to minimum required bandwidth $q(l)$ at link l. Since ζ is fixed, our algorithm is formally expressed by

$$
\begin{aligned}
\arg\min_{R\in R(i,j)} \quad & \sum_{l\in R}\left(\frac{\bar{\eta}_{R(l)}g'(q(l))}{G(T(l),R(l))}\left(\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}\right)_{\Sigma^l}\right) \\
\text{subject to} \quad & \theta(l) \geq c(l) \quad \forall l \in \mathcal{L}, \\
& P(l) \geq 0 \quad \forall l \in \mathcal{L}.
\end{aligned}
$$

The algorithm for the non-linear case is also similar to the algorithm that we have previously discussed, except that each node additionally computes the gradients $g'(q(l))$ for the links associated with the node.

IV. Discussion

In this section, we discuss how to simplify of our algorithm and develop a distributed version.

A. Simplification

A practical issue to be discussed is how to reduce the computational complexity. This simplification is also necessary in the development of a distributed algorithm. In (19), the term $\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}$ corresponds to global information in the network. Clearly, it is inefficient to gather information over all links in a large wireless network. Firstly, transmission that are distant from link will barely interfere with it, since each element in (9) is multiplied by weighting factor $G(T(m),R(l))$ that is proportional to $d_{T(m)R(l)}^{-\delta}$, where δ is between 2 and 6, and m is a link in $\mathcal{L}$. Also, in practice, there could be a large time delay in gathering this information from the network. This is especially true, when the wireless network is large and the operating algorithms are distributed, as the information from distant nodes may become too stale to be useful. Hence, for all nodes k in the network, we define $\mathcal{N}_k$ to be the set including node k and its neighbor nodes in a certain range called the *information range*, $\mathcal{L}_k$. The number of links in $\mathcal{N}_k$ is denoted by $L_{\mathcal{L}_k}$. Then, instead of matrices $\bar{\mathbf{F}}$ and $\mathbf{\Pi}$, the link weights associated with node k in $\mathcal{N}$ uses $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{\Pi}}$ reduced to $L_{\mathcal{L}_k} \times L_{\mathcal{L}_k}$ matrices in order to reduce the computational complexity at each node and control messages over the network. Even though distant links are removed from $\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}$ in (19), the influence of these links on links l is still included in the average SINR $\frac{\bar{\eta}_{R(l)}}{G(T(l),R(l))}$ that is measured at node $R(l)$. We next provide a distributed solution using the above described simplification.

B. Distributed algorithm

We assume that each node knows the path gains to its neighboring nodes within its information range. The path gains can be measured at each node by using a variety of techniques such as the receiving power [25]. We assume that each node uses some method to disseminate its messages to its neighbors, e.g., using a control channel or pyggybacking.

When we use a distributed shortest path algorithm such as Dijkstra's algorithm or Bellman-Ford algorithm [31], the issue is how to define the weights of links. Instead of $\mathbf{\Pi}(\mathbf{I}-\bar{\mathbf{F}})^{-1}$ in (19), we use $\tilde{\mathbf{\Pi}}(\mathbf{I}-\tilde{\mathbf{F}})^{-1}$ that was discussed in the previous subsection. To obtain the matrix $\tilde{\mathbf{\Pi}}(\mathbf{I}-\tilde{\mathbf{F}})^{-1}$ at each node, node k in $\mathcal{N}$ needs to obtain the conditional probability $\pi_{m|l}$, where node k is $R(l)$, and normalized additional constraint defined as $\frac{\Delta c(m)}{G(T(m),R(m))}$ for all $m \in \mathcal{L}_k$. When transmitting and receiving data through link l, nodes $R(l)$ and $T(l)$ disseminate slot times occupied by link l and $\frac{\Delta c(l)}{G(T(l),R(l))}$ to the neighborhood in sets $\mathcal{N}_{R(l)}$ and $\mathcal{N}_{T(l)}$, respectively. After receiving the information from the neighboring nodes, each node generates a matrix $\tilde{\mathbf{\Pi}}(\mathbf{I}-\tilde{\mathbf{F}})^{-1}$. In the case when the link scheduling is time independent, the transmitter nodes of the links can send link activated probabilities (time average) to their neighborhood since $\pi_{m|n}$ becomes π_m, meaning that the probability that link m is active and can be measured at nodes $R(m)$ and $T(m)$.

V. Simulations

In this section we use simulations to verify the performance of our algorithm. We call our algorithm *OptSINR*. We compare the performance of this algorithm to other routing algorithms that are used in the literature. These algorithms are based on a shortest path idea, where the minimum transmission energy or the minimum interference is chosen as the cost over each link. In the first algorithm called the minimum energy algorithm (ME), the cost over each link does not take interference into account. In contrast, the second algorithm called PwrOpt, considers interference at each node. Based on the interference level, the PwrOpt chooses the minimum transmission power to meet the SINR requirement at each like l in the network, i.e

$$\Delta P(l) = \frac{\eta_{R(l)}}{G(T(l),R(l))}\Delta c \quad \forall l \in \mathcal{L}, \tag{20}$$

where $\eta_{R(l)}$, $G(T(l),R(l))$, Δc represent the sum of interference and noise at link l, the path loss of link l, and the corresponding SINR requirement of a new service, respectively. The algorithm with (20) becomes the "minimum interference routing algorithm" of [22] in the case when $G(T(l),G(R(l)))$ for all $l \in \mathcal{L}$ is a fixed constant. Since the minimum interference routing algorithm belongs to this PwrOpt category, we do not consider it separately.

We also compare our algorithm to the Least Interference Routing (LIR) algorithm in [20] and [21]. In contrast to the minimum interference routing [22], LIR chooses that route which minimizes the total interference induced by the route. In the simulation, we calculate the interference precisely using global information and the exact distance between the nodes.

For comparison purposes, we consider three different versions of our distributed algorithm depending on the information range used to develop matrix $\bar{F}$ in (19). First, OptSINR uses global information so that the performance of the algorithm can be the upperbound for the different versions of our algorithm. Second, we assume that each node can have only local information about neighboring nodes that are located within 1.5 units distance. We call this algorithm OptSINRd1.5. Finally, we use OptSINRd2.0 whose information range is 2 units.

For all the algorithms compared, we assume that the nodes can employ power control. In each case, when sending flows through the routes chosen by the algorithm, each node adjusts its transmission power to satisfy the new minimum constraints at the links. For the simulations, we use a seven by seven grid network, as shown in Fig. 2, and the separation between adjacent nodes in the $x-$ and $y-$ coordinates is one unit of distance. We fix the path loss exponent at three, the attenuation factor at one, and the ambient noise at one. We assume that all ambient noise is identical and that each link is directional. We assume that wireless links are linear, as defined in (3) and the required SINR of a new service flow is fixed at 0.1.

In order to scrutinize how differently our algorithms work, we first study the "microscopic" behavior of the incoming flows. For this study, we compare our algorithm with ME for various scenarios. By studying the microscopic behavior, we understand the dynamics of our algorithm. Further, we validate our algorithm against other algorithms in the case when services randomly arrive into the system. In this environment, we compare our algorithms to ME, PwrOpt and LIR in order to validate the performance of our algorithms.

A. *Impact of an ongoing link on a new route*

First, we consider how an ongoing link affects the route of a new flow for different source-destination pairs. To do this, we set one ongoing link to continuously transmits data. This ongoing link is represented in Fig. 2(a) by link $S1$ between nodes 39 and 46. The ongoing link can be interpreted as a congested area (hot spot) in a wireless network. In the case of a hybrid network, congested areas occur around base stations [32] (or gateways [33]). In the case when homogeneous nodes are uniformly distributed in a symmetric area and source-destination pairs are randomly chosen, the hot spot usually occurs at the center of the deployed area [34]. We fix the SINR for link $S1$ at three and vary the positions of the source and destination nodes.

Fig. 3(a) shows the simulation result when the route of a new incoming flow is located far away from the ongoing link. In the figure, the numbers on the routes stand for the order in which the links are activated in the routing procedure, and the number in each parenthesis represents the additional power consumption over the whole network when the link is activated. Since link $S1$ is far away from the route taken by the new flow, it barely affects the new flow. Thus, our algorithm chooses the same route as the ME algorithm.

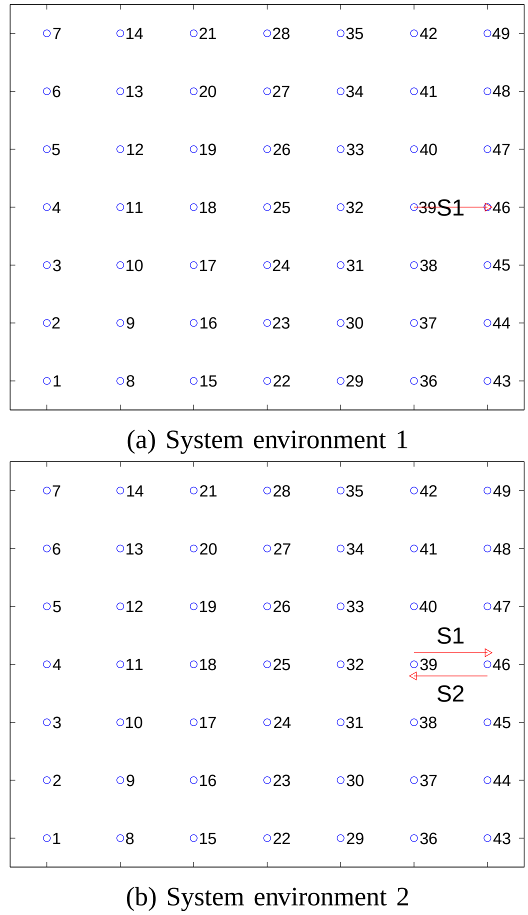

Fig. 2. Two system environments for simulations in a seven by seven grid network.

When the source-destination pairs are such that a route for the new flow is close to the ongoing link (e.g. as in Fig. 3 (a)), the interference due to the active link is now too significant to be ignored. Hence, in Fig. 3(b), our algorithm automatically chooses a different route from the ME algorithm, and takes a detour around the congested area. Compared with ME, the route chosen by our algorithm needs two more hops, but saves about 30% energy over the whole network. Hence, to reduce the interference, our algorithm chooses a longer route than the ME algorithm, but results in consuming a smaller amount of total energy. Thus, energy efficiency comes at the cost of requiring a larger number of hops.

B. *Impact of average power of existing links on a new route*

In the previous subsection, we studied the impact of the power of an ongoing link on a new route. Here, we study the

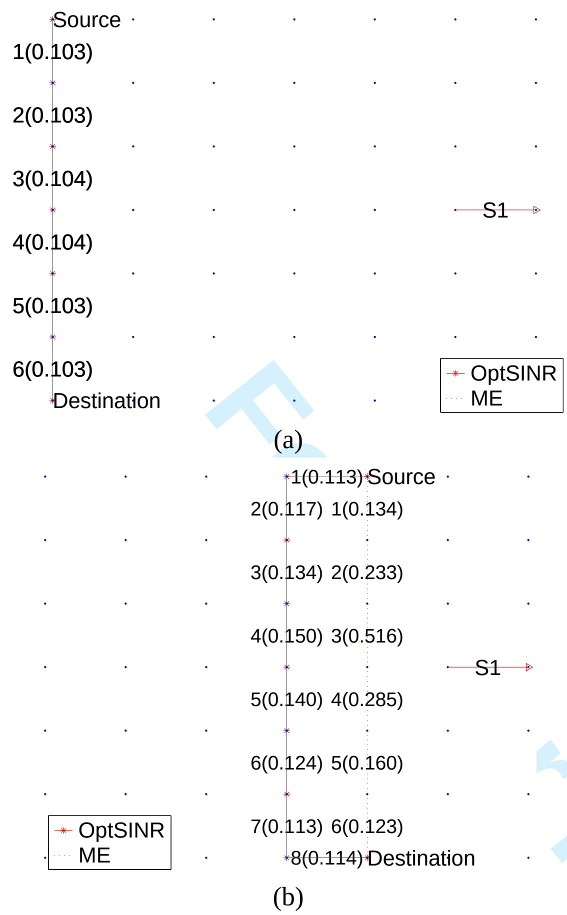

Fig. 3. Comparison of routes when the source-destination pair is differently located from link $S1$: (a) a minimum energy route is far away from link $S1$ (b) a minimum energy route is close to link $S1$.

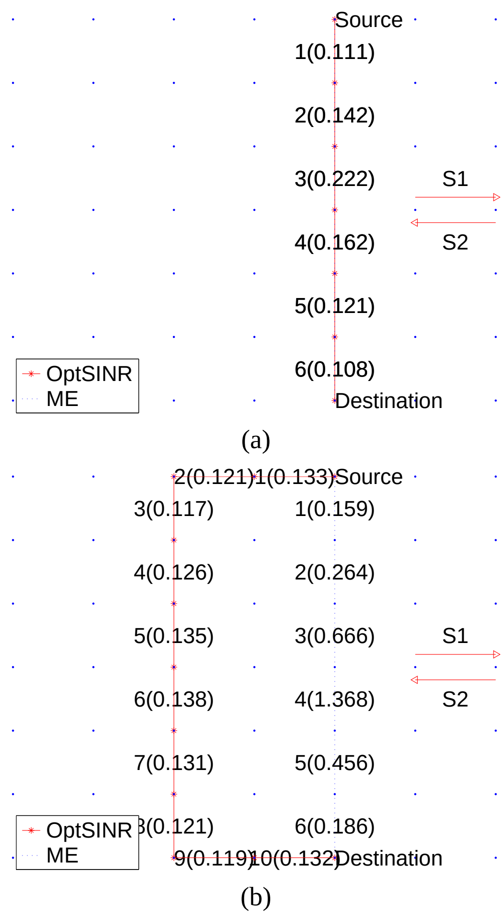

Fig. 4. Comparison of routes when the probability links change: (a) routes does not depend on SINR when the probability of link $S1$ is 0.01 and the probability of link $S2$ is 0.99 (b) the energy efficiency of the proposed route is 58.89% when the probability of link $S1$ is 0.99 and the probability of link $S2$ is 0.01.

impact of the average power generated by multiple ongoing links that could be transmitting randomly at different times on the route that a new flow will take. To study how this case impacts our algorithm, we consider two-way communication with directional links $S1$ and $S2$ between nodes 39 and 46, as shown in Fig. 2(b). The two links are mutually exclusive and activated in each time slot with probabilities α for link $S1$ and β for link $S2$ such that $\alpha + \beta = 1$. The scheduling is independent of other links. We fix the source and destination nodes of a new flow at nodes 35 and 29, respectively.

We first vary the probabilities α and β, and fix the minimum SINR constraints of links $S1$ and $S2$ at 1 and 7, respectively, in order to study the impact of the average interference on the route chosen. Fig. 4 shows that our routing algorithm depends on the average power of the ongoing links but the ME algorithm is independent of the ongoing links' environment. In the figure, the numbers on the routes denote the order in which the links are activated in the routing procedure, and the number in each parentheses represents the average power increment expended by the whole network when the link is activated. When the average power of the ongoing links is small (β is small), our algorithm and the ME algorithm work identically, as shown in Fig. 4 (a). However, as the average power of the ongoing links increases (β increases), our algorithm automatically detours farther away from the links to save total network energy consumption, as shown in Fig. 4 (b).

To study the efficiency of our algorithm, we change the probabilities α and β at different minimum SINR constraints for link $S2$ when the minimum constraint for link $S1$ is fixed at one. Fig. 5 depicts the energy improvement of our routing algorithm as compared to the ME routing algorithm. We define here the energy improvement E as $\frac{(B-A)}{B} \times 100$, where A and B correspond to the energy consumption in the network when routes are chosen for a new flow by our algorithm and the ME algorithm, respectively. When the minimum requirements of links $S1$ and $S2$ are fixed, and the probability β changes from 0 to 1, the energy improvement increases, since the average power of the ongoing links is increased. When the minimum SINR of link $S1$ increases, the route computed with SINR metrics is also more energy efficient than the route computed without SINR metrics.

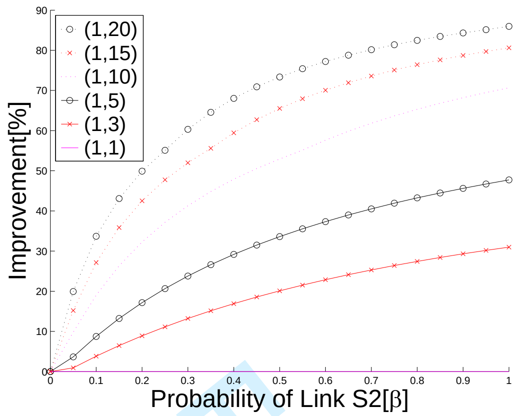

Fig. 5. The energy improvement of OptSINR compared to ME.

C. Impact of the randomness of flows on the routing

In this subsection, we study the performance of our algorithm for the case when flows randomly arrive at the network and the source-destination pairs are randomly chosen. For the simulations, we use a scheduling scheme such that reception and transmission do not occur simultaneously at each node. Since each node has four adjacent nodes in our simulation environment, we divide eight time slots and appropriately assign them to each link at each node.

To study the impact of the simultaneously served flows on the power consumption of the network, we assume that the flows with source-destination pairs chosen randomly arrive at the network in turn and, for simplicity, their duration is set to infinity. Fig. 6 describes the difference between the performance of the various routing algorithms considered. When the number of flows are small, the power consumption of all the routing algorithms are identical. However, as the number of simultaneously served flows increases, the power consumptions of the various routes are markedly different. The ME algorithm is the most susceptible to the number of flows, since it does not account for interference. PwrOpt and LIR perform better than ME since they consider the impact of interference on a transmission power. However, our algorithm, OptSINR, as well as its distributed counterparts OptSINRd1.5 and OptSINRd2.0 outperforms the others. The performance of OptSINRd1.5 and OptSINRd2.0 is in fact quite close to OptSINR, even though these algorithms only use local information. The ME algorithm cannot simultaneously accommodate more than 119 flows into the network even though there is no constraint on the transmission power. On the other hand, OptSINR can serve up to 201 flows at the same time with the same simulation settings, as shown in Fig. 6.

We next consider flow dynamics. Service flows arrive to the system according to a Poisson process and the holding time is fixed at 600 time slots. As shown in Fig. 6, the network power depends on the number of simultaneously served flows by the network. The excessive power consumption may result in call drop (block) in the case of limited power resource. To avoid dropping existing flows, we employ an admission control mechanism in this network, as in [26]. The admission control blocks an incoming flow when admitting the incoming flow would result in exceeding a certain threshold of power.

Fig. 7 depicts the performance in terms of the number of transmitted flows when these arrival rates are different and the maximum network power is fixed at 500 units power. In this simulation, the system blocks a new service flow in order to protect ongoing service flows when the required power to accommodate a new flow exceeds the maximum transmission power at any link over the chosen route for the flow. In the case when the arrival rate is low, the average number of simultaneously served flows in the network is low. Hence, all algorithms serve all flows without any drops, as shown in Fig. 7 (a), when the arrival rate is small (0.1 flows per time slot). However, the performance of various algorithms is quite different when the arrival rate is increased to 0.33 flows per time slot, as shown in Fig. 7 (b).

In Fig. 8, by varying the maximum network power and fixing the arrival rate at 0.2 flows per time slot, we compare the performance of the routing algorithms. In the case of ME and PwrOpt, more than 100 out of the 1000 incoming flows are blocked. However, the system devised by our algorithm using the same setup, can serve almost all 1000 service flows, even using local information (OptSINRd1.5 and OptSINRd2.0) when the maximum network power is more than 500 units. Although the systems are physically identical, the routing algorithms achieve different system capacities, as shown in Fig. 6. Hence, the blocking probabilities of the previous routing algorithms are greater than those of our routing algorithms.

VI. Conclusion

In this paper, we have developed an energy efficient routing algorithm for given SINR constraints in multi-hop wireless networks. The objective is to minimize the total energy consumed in the network. To do this, we have developed a cross layer routing algorithm that exploits both the SINR (physical layer information) and power control (MAC layer). A nice feature of this routing algorithm is that it automatically routes around congested areas, and thus results in mitigating the overall congestion in the network. We show that for a given class of link scheduling schemes, this algorithm is asymptotically optimal in the sense of average energy consumption. We then develop a distributed version of this algorithm that uses local information and requires a substantial reduction in computational overhead. We find via simulation results that both distributed and centralized versions of the algorithm perform very well, and result in substantial energy savings over the state-of-the-art.

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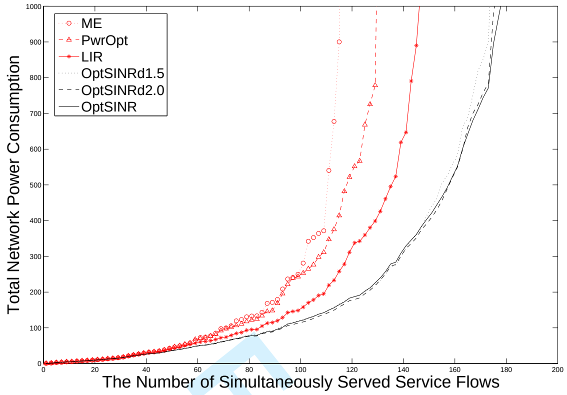
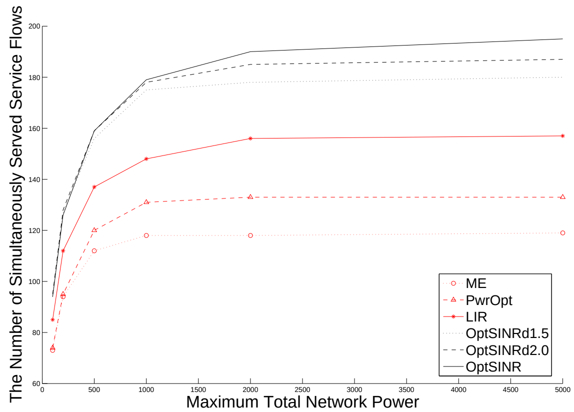
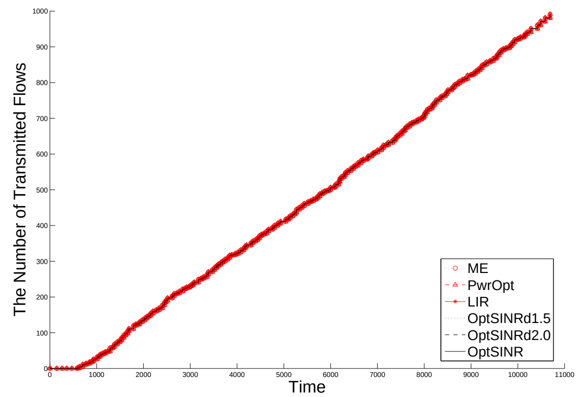


(a)

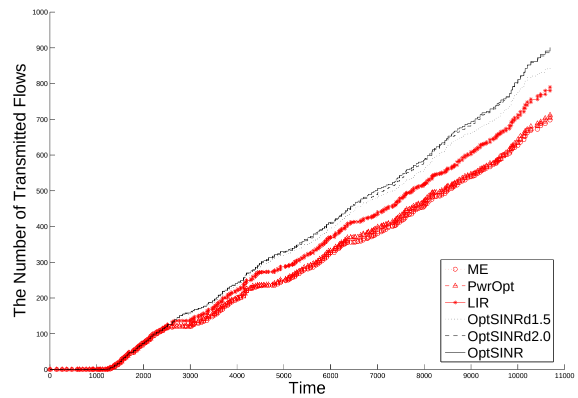


(b)

Fig. 6. Performance comparison: (a) Total network power versus simultaneously served service flows (b) The number of simultaneously served service flows versus the total network power.



(a)



(b)

Fig. 7. Comparison of the number of transmitted flows when new flows randomly arrive at the network: (a) Poisson arrival rate = 0.1 (b) Poisson arrival rate = 0.33.

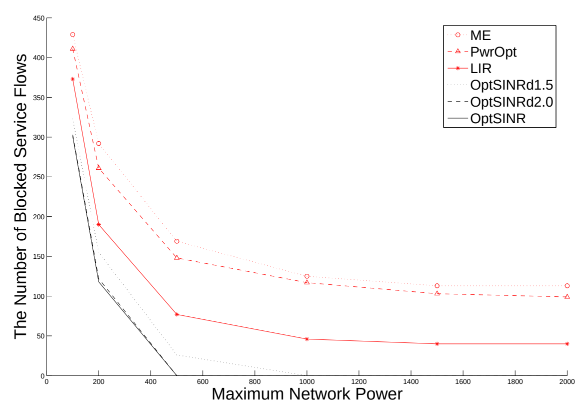


Fig. 8. Comparison of blocked service flows when new flows randomly arrive at the network.

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Summary of Differences

The early version of this manuscript has been presented at INFOCOM '06 and the paper title was "Energy-efficient interference-based routing for multi-hop wireless networks". The manuscript submitted to IEEE Transactions on Mobile Computing has been improved and revised as follows:

1. **Section 1**: we have added an example and Fig.1 for better presentation.

2. **Section 2**: we have clarified and revised the system model and assumptions.

3. **Section 3**: we have updated unclear expressions.

4. **Section 4**: we have further discussed the simplification of our algorithm. Furthermore, we have explained an admission control we have employed to prevent incoming flows from a significant degradation of the quality of service of ongoing flows in Section 4.3.

5. **Section 5**: We have focused on more practical environments for simulations. In the Infocom paper, we considered only fixed link a scheduling scheme, but in practice random link scheduling may be used such as IEEE 802.11 based systems, which is considered in this manuscript. We have also devoted some discussion to the impact of scheduling schemes not provided in the Infocom submission.

In this manuscript, we also have conducted simulations with a non-linear function, and varied the arrival rates and used random service time in the simulations, which were not performed in the Infocom paper.

The manuscript includes all the simulations.