On the Design of Large Scale Wireless Systems

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***Abstract*—In this paper, we consider the downlink of large OFDMA-based networks and study their performance bounds as a function of the number of - transmitters B, users K, and resource-blocks N. Here, a resource block is a collection of subcarriers such that all such collections, that are disjoint have associated independently fading channels. In particular, we analyze the expected achievable sum-rate as a function of above variables and derive novel upper and lower bounds for a general spatial geometry of transmitters, a truncated path-gain model, and a variety of fading models. We establish the associated scaling laws for dense and extended networks, and propose design guidelines for the regulators to guarantee various QoS constraints and, at the same time, maximize revenue for the service providers. Thereafter, we develop a distributed resource allocation scheme that achieves the same sum-rate scaling as that of the proposed upper bound for a wide range of K, B, N. Based on it, we compare low-powered peer-to-peer networks to high-powered single-transmitter networks and give an additional design principle. Finally, we also show how our results can be extended to the scenario where each of the B transmitters have $M(>1)$ co-located antennas.**

***Index Terms*—Bounded pathloss model, dense network, design principles, distributed algorithm, extended network, multiple antennas, orthogonal frequency division multiple access (OFDMA), resource allocation.**

I. Introduction

WITH the widespread usage of smart phones and an increasing demand for numerous mobile applications, wireless cellular/dense networks have grown significantly in size and complexity. Consequently, the decisions regarding the deployment of transmitters (base-stations, femtocells, picocells etc.), the maximum number of subscribers, the amount to be spent on purchasing more bandwidth, and the revenue model to choose have become much more complicated for service providers. Understanding the performance limits of large wireless networks and the optimal balance between the number of serving transmitters, the number of subscribers, the number of antennas used for physical-layer communication, and the amount of available bandwidth to achieve those limits are critical components of the decisions made. Given that the most significant fraction of the performance growth of wireless networks in the last few decades is associated [1] with cell sizes (that affect interference management schemes) and the amount of available bandwidth, the aforementioned issues become more important.

To answer some of the above questions, we analyze the expected achievable downlink sum-rate in large OFDMA systems as a function of the number of transmitters B, users K, available resource-blocks N, and/or co-located antennas at each transmitter M. Here, a resource block is a collection of subcarriers such that all such disjoint sub-collections have associated independently fading channels. Using our analysis, we make the following contributions:

- For a general spatial geometry of transmitters and the end users, we develop novel non-asymptotic upper and lower bounds on the average achievable rate as a function of K, B, and N.
- We consider asymptotic scenarios in two networks: dense and regular-extended, in which user nodes have a uniform spatial distribution. Under this setup, we evaluate our bounds for Rayleigh, Nakagami-m, Weibull, and LogNormal fading models along with a truncated path-gain model using various results from the *extreme value theory*, and specify the associated scaling laws in all parameters.
- With the developed bounds, we give four design principles for service providers/regulators. In the first scenario, we consider a *dense femtocell network* and develop an asymptotic *necessary* condition on K, B, and N to guarantee a non-diminishing rate for each user. In the second scenario, we consider an extended multicell network and develop asymptotic *necessary* conditions for K, B, and N to guarantee a minimum return-on-investment for the service provider while maintaining a minimum per-user throughput. In the third and fourth scenarios, we consider extended multicell networks and derive bounds for the choice of user-density K/B in order for the service provider to maximize the revenue per transmitter and, at the same time, keep the per-user rate above a certain limit.
- For dense and regular-networks, we find a distributed resource allocation scheme that achieves, for a wide range of $\{K, B, N\}$, a sum-rate scaling equal to that of the upper bound (on achievable sum-rate) that we developed earlier.
- Using the proposed achievability scheme, we show that the achievable sum-rate of peer-to-peer networks increases linearly with the number of coordinating transmit nodes B under fixed power allocation schemes only if $B = O\left(\frac{\log K}{\log \log K}\right)$. Our result extends the result in [2],

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wherein it was stated that if $B = \Omega(\log K)$, then a linear increase in achievable sum-rate w.r.t. B cannot be achieved. We end our discussion with a note on MISO (Multiple-Input Single-Output) systems, where there are a fixed number of co-located antennas at each transmitter, and obtain a similar distributed resource allocation problem as we found earlier towards achievability of expected achievable sum-rate.

We now discuss related work. Calculation of achievable performance of wireless networks has been a challenging, and yet an extremely popular problem in the literature. The performance of large networks has been mainly analyzed in the asymptotic regimes and the results have been in the form of scaling laws [2]–[11] following the seminal work by Gupta and Kumar [3]. Various channel and propagation models (e.g., distance-based power-attenuation models and fading) have been incorporated in the scaling law analyses of wireless networks in [12]–[14]. The path-gain model used by these studies are based on far-field assumption, which is developed to model long-distance electro-magnetic wave propagation. These models can be problematic [15], [16] for random networks, since the singularity of the channel gain at zero distance leads to an unrealistic scenario at small distances in which the received signal power becomes greater than the transmitted signal power. Furthermore, it affects the asymptotic behavior of achievable rates significantly. Indeed, the capacity scaling law of $\Theta(\log K)$ found in [13], [14] arises due to the unboundedly increasing channel-gains of the users close to the transmitter, whereas, under a fixed path-gain, the scaling law changes to $\Theta(\log \log K)$ [2].

Unlike the aforementioned studies, we provide *non-asymptotic* bounds[1] (in Theorem 1) for multicellular wireless networks. To develop our bounds, we assume a truncated path-gain model that eliminates the singularity of unbounded path-gain models. Our path-gain model belongs to the category of bounded path-gain models [16]. At large distances, it behaves similar to the well-studied unbounded path-gain models. At the same time, it does not suffer from their limitation of received power becoming greater than the transmitted power at small distances. In our study, we also take into account the bandwidth and number of transmitters (and/or antennas) in large networks, and provide a distributed scheme that achieves a performance, which scales identical to the optimal performance with the number of users, the number of resource blocks, and the number of transmitters.

The rest of the paper is organized as follows. In Section II, we introduce our system model. In Section III, we give general upper and lower bounds on expected achievable sum-rate. We also give, for the cases of dense and regular-extended networks, associated sum-rate scaling laws and four network-design principles. In Section IV, we find a deterministic power allocation scheme that governs the proposed distributed achievability scheme, followed by an analysis of peer-to-peer networks. In Section V, we provide details of another

[1]Even though Theorem 1 is non-asymptotic, the subsequent analyses focus on scaling laws, which we derive based on Theorem 1. However, we also discuss how to evaluate/simplify our bounds, so that they can provide further insights into the achievable performance in various non-asymptotic scenarios.

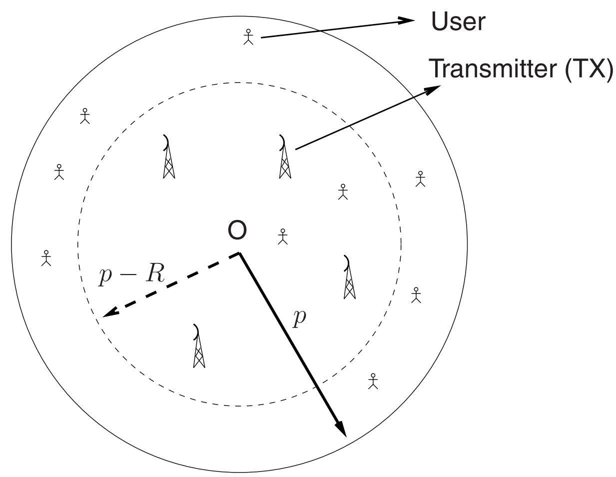

Fig. 1. OFDMA downlink system with K users and B transmitters. O is assumed to be the origin.

achievability scheme, similar to that developed in Section IV, for MISO systems. Finally, we conclude in Section VI.

II. System Model

We consider a time-slotted OFDMA-based downlink network of B transmitters (or base-stations or femtocells or geographically distributed antennas) and K active users, as shown in Fig. 1. The transmitters (TX) lie in a disc of radius $p - R$ ($p > R > 0$), and the users are distributed according to some spatial distribution in a concentric disc of radius p. Under such general settings, Theorem 1 gives bounds on the expected achievable sum-rate of the system. In the sequel, however, we assume for simplicity that the transmitter locations are arbitrary and deterministic and the users are uniformly distributed. This model too is quite general and can be applied to several network configurations. For example, it models a *dense network* when transmitter locations are random and the network radius p is fixed. Similarly, it models a *multi-cellular regular extended network* when the transmitters (or base-stations) are located on a regular hexagonal grid with a fixed grid-size, i.e., $p \propto \sqrt{B}$.

Let us denote the coordinates of TX i ($1 \le i \le B$) by (a_i, b_i), and the coordinates of user k ($1 \le k \le K$) by (x_k, y_k). Therefore, (a_i, b_i) are assumed to be known for all i, and (x_k, y_k) is governed by the following probability density function (pdf):

$$f_{(x_k,y_k)}(x,y) = \begin{cases} \frac{1}{\pi p^2} & \text{if } x^2+y^2 \le p^2 \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

We now describe the channel model. We assume that the OFDMA subchannels are grouped into N independently-fading resource blocks [17], across which the transmitters (TXs) schedule users for downlink data-transmission. We denote the complex-valued channel gain over resource-block n ($1 \le n \le N$) between user k and TX i by $h_{i,k,n}$, and assume that it is defined as

$$h_{i,k,n} \triangleq \beta R_{i,k}^{-\alpha} \nu_{i,k,n}. \quad (2)$$

Here, $\beta R_{i,k}^{-\alpha}$ denotes the path-gain,

$$R_{i,k} = \max\{r_0, \sqrt{(x_k-a_i)^2+(y_k-b_i)^2}\} \quad (3)$$

for positive constants α, β, r_0 ($\alpha > 1, r_0 < R$), and the fading factor $\nu_{i,k,n}$ is a complex-valued random variable that is i.i.d. across all (i, k, n). Note that r_0 is the truncation parameter that eliminates singularity in the path-gain model. Currently, we keep the distribution of $\nu_{i,k,n}$ general. Specific assumptions on the fading model $\{\nu_{i,k,n}\}$ will be made in subsequent sections. Assuming unit-variance AWGN, the channel Signal-to-Noise Ratio (SNR) between user k and TX i across resource-block n can now be defined as

$$\gamma_{i,k,n} \triangleq |h_{i,k,n}|^2 = \beta^2 R_{i,k}^{-2\alpha} |\nu_{i,k,n}|^2. \tag{4}$$

We initially assume that perfect knowledge of the users' channel-SNRs from all TXs is available at every transmitter[2]. We also assume that the transmitters do not coordinate to send data to a particular user. Therefore, if a user is being served by more than one transmitter, then while decoding the signal from a given TX, it treats the signals from all other TXs as noise. This assumption is restrictive since one may achieve a higher performance by allowing coordination among TXs to send data to users. However, as will be explained after Theorem 1 in Section III, our results and design principles also hold for a class of networks wherein coordination among TXs is allowed.

The maximum achievable sum-rate of our system can now be written as

$$\mathcal{C}_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{\nu}}(\boldsymbol{U}, \boldsymbol{P}) \triangleq \sum_{i=1}^{B} \sum_{n=1}^{N} \log\left(1 + \frac{P_{i,n}\,\gamma_{i,U_{i,n},n}}{1 + \sum_{j\neq i} P_{j,n}\,\gamma_{j,U_{i,n},n}}\right) \tag{5}$$

where $\boldsymbol{x} := \{x_k \text{ for all } k\}$, $\boldsymbol{y} := \{y_k \text{ for all } k\}$, $\boldsymbol{\nu} := \{\nu_{i,k,n} \text{ for all } i,k,n\}$, $\boldsymbol{U} := \{U_{i,n} \text{ for all } i,n\}$, and $\boldsymbol{P} := \{P_{i,n} \text{ for all } i,n\}$. Here, $U_{i,n}$ is the sum-rate maximizing user scheduled by TX i across resource-block n, and $P_{i,n}$ is the corresponding allocated power. We assume that, in each time-slot, the total power allocated by each TX is upper-bounded by P_{con}. Therefore, $\sum_n P_{i,n} \leq P_{\mathsf{con}}$ for all i. One may also write (5) as

$$\mathcal{C}_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{\nu}}(\boldsymbol{U}, \boldsymbol{P}) = \max_{\boldsymbol{u}\in\mathcal{U},\,\boldsymbol{p}\in\mathcal{P}} \sum_{i=1}^{B} \sum_{n=1}^{N} \log\left(1 + \frac{p_{i,n}\,\gamma_{i,u_{i,n},n}}{1 + \sum_{j\neq i} p_{j,n}\,\gamma_{j,u_{i,n},n}}\right), \tag{6}$$

where $\boldsymbol{u} \triangleq \{u_{i,n} \text{ for all } i,n\}$, $\boldsymbol{p} \triangleq \{p_{i,n} \text{ for all } i,n\}$, and $\{\mathcal{U}, \mathcal{P}\}$ are the sets of feasible user allocations and power allocations. In particular,

$$\begin{aligned}\mathcal{U} &\triangleq \{\{u_{i,n}\} : 1 \leq u_{i,n} \leq K \text{ for all } i,n\} \text{ and}\\ \mathcal{P} &\triangleq \Big\{\{p_{i,n}\} : p_{i,n} \geq 0 \text{ for all } i,n,\\ &\qquad \text{and } \sum_n p_{i,n} \leq P_{\mathsf{con}} \text{ for all } i\Big\}.\end{aligned} \tag{7}$$

In the next section, we derive novel upper and lower bounds on the expected value of $\mathcal{C}_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{\nu}}(\boldsymbol{U}, \boldsymbol{P})$ that are later used to determine the scaling laws and develop various network-design guidelines. To state the scaling laws, we use the following notations: for two non-negative functions $f(t)$ and $g(t)$, we write $f(t) = O(g(t))$ if there exists constants $c_1 \in \mathbb{R}^+$ and $r_1 \in \mathbb{R}$ such that $f(t) \leq c_1\, g(t)$ for all $t \geq r_1$. Similarly, we write $f(t) = \Omega(g(t))$ if there exists constants $c_2 \in \mathbb{R}^+$ and $r_2 \in \mathbb{R}$ such that $f(t) \geq c_2\, g(t)$ for all $t \geq r_2$. In other words, $g(t) = O(f(t))$. Finally, we write $f(t) = \Theta(g(t))$ if $f(t) = O(g(t))$ and $f(t) = \Omega(g(t))$.

[2]This can be achieved via a back-haul network that enables sharing of users' channel-state information. Later, we will propose a distributed resource allocation scheme that does not require any sharing of CSI among the transmitters and its sum-rate scales at the same rate as that of an upper bound on the optimal centralized resource allocation scheme for a wide range of network parameters.

III. Proposed General Bounds on Achievable Sum-Rate

The expected achievable sum-rate of the system can be written, using (5), as

$$\mathcal{C}^* = \mathrm{E}\left\{\mathcal{C}_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{\nu}}(\boldsymbol{U}, \boldsymbol{P})\right\}, \tag{8}$$

where the expectation is over the SNRs $\{\gamma_{i,k,n}$ for all $i,k,n\}$. The following theorem gives bounds on (8) that depend only on the sum-power constraint and the exogenous channel-SNRs.

Theorem 1 (General bounds). *The expected achievable sum-rate of the system, $\mathcal{C}^*$, can be bounded as:*

$$\sum_{i,n} \mathrm{E}\left\{\frac{\log\left(1 + P_{\mathsf{con}}\,\gamma_{i,k^*,n}\right)}{N + P_{\mathsf{con}} \sum_{j\neq i} \gamma_{j,k^*,n}}\right\} \leq \mathcal{C}^* \leq \sum_{i,n} \mathrm{E}\left\{\log\left(1 + P_{\mathsf{con}} \max_k \gamma_{i,k,n}\right)\right\}, \tag{9}$$

where k^* in the lower bound is a function of TX i and resource-block n and is identical to $\mathrm{argmax}_k\, \gamma_{i,k,n}$. *Moreover, an alternate upper bound on $\mathcal{C}^*$ obtained via Jensen's inequality over powers is:*

$$\mathcal{C}^* \leq N \sum_i \mathrm{E}\left\{\log\left(1 + \frac{P_{\mathsf{con}}}{N} \max_{n,k} \gamma_{i,k,n}\right)\right\}. \tag{10}$$

Proof: See Appendix A for proof sketch. For detailed proof, see [18, Theorem 1]. Note that the upper bounds in (9) and (10) can be further simplified via Jensen's inequality by taking the expectation over $\{\gamma_{i,k,n}\}$ inside the logarithm and can be evaluated easily for finite K. ■

The upper bounds in Theorem 1 are obtained by ignoring interference, and the lower bound is obtained by allocating equal powers $\frac{P_{\mathsf{con}}}{N}$ to every resource-block by every TX. As mentioned earlier, our bounds, which assume an uncoordinated system, also serve as bounds (up to a constant scaling factor) for the expected max-sum-rate of a class of networks wherein the number of transmitters coordinating to send data to any user on any resource block is bounded. This can be explained using the following argument. Let S transmitters coordinate to send data to user k on resource block n and let $\{\gamma_{1,k,n}, \ldots, \gamma_{S,k,n}\}$ be the corresponding instantaneous exogenous Signal-to-Noise ratios. Then, an upper bound on the sum-rate of those S transmitters across resource block n is $\log\left(1 + \left(\sum_{s=1}^{S} \sqrt{P_{s,n}\gamma_{s,k,n}}\right)^2\right)$ [19], where $P_{s,n}$ is the power allocated by transmitter s across resource block n. However, this term is upper bounded by $S \sum_{s=1}^{S} \log(1 + P_{s,n}\gamma_{s,k,n})$, which is S times the upper bound on sum-rate obtained by ignoring interference in a completely uncoordinated system (same as that used in Theorem 1). Since S is bounded, our subsequent scaling laws for the upper bound and the resulting design principles remain unchanged for this level of coordination. Now, our lower bound assumes no coordination and allocates equal power to every TX and every resource-block. Clearly, by coordinating among transmitters, one can achieve better performance. The above arguments, coupled with the fact that Theorem 1 does not assume any specific channel-fading process or any specific distribution on transmitter and user-locations, make our bounds valid for a wide variety of coordinated and uncoordinated networks. In the next subsection, Section III-A, we evaluate the bounds in Theorem 1 under asymptotic situations in two classes of networks – *dense* and *regular-extended* – using extreme-value theory, and then provide interesting design principles based on them.

A. Scaling Laws and Their Applications in Network Design

We first present an analysis of dense networks, followed by an analysis of regular-extended networks. In particular, we use extreme-value theory and Theorem 1 to obtain performance bounds and associated scaling laws. Our results hold good for uncoordinated systems and a class of coordinated systems in which the number of transmitters coordinating to send data to any user across any resource-block is bounded.

1) Dense Networks: Dense networks contain a large number of transmitters that are distributed over a fixed area. Typically, such networks occur in dense-urban environments and in dense femtocell deployments. In our system-model, a dense network corresponds to the case in which p is fixed, and K, B, N are allowed to grow. The following two lemmas use extreme-value theory and Theorem 1 to give bounds on the achievable sum-rate of the system for various fading channels.

Lemma 1. *For dense networks with large number of users K and Rayleigh fading channels, i.e., $\nu_{i,k,n} \sim \mathcal{CN}(0,1)$ for all i,k,n,*

$$\begin{aligned}\big(\log(1+P_{\mathsf{con}}l_K)+O(1)\big)BNf_{\mathsf{lo}}^{\mathsf{DN}}(r,B,N)\\ \le \mathcal{C}^* \le \big(\log(1+P_{\mathsf{con}}l_K)+O(1)\big)BN, \qquad (11)\end{aligned}$$

where $r>0$ is a constant, $l_K = \beta^2 r_0^{-2\alpha}\log\frac{Kr_0^2}{p^2}$, and $f_{\mathsf{lo}}^{\mathsf{DN}}(r,B,N) = \frac{r^2}{(1+r^2)(N+P_{\mathsf{con}}\beta^2 r_0^{-2\alpha}(1+r)B)}$. Moreover, the upper bound on $\mathcal{C}^*$ obtained via (10) gives $\mathcal{C}^* \le \Big(\log\big(1+\frac{P_{\mathsf{con}}}{N}l_{KN}\big)+O(1)\Big)BN$, where $l_{KN}=\beta^2 r_0^{-2\alpha}\log\frac{KNr_0^2}{p^2}$. The following scaling laws result from (11):

$$\begin{aligned}\mathcal{C}^* &= O(BN\log\log K), \text{ and}\\ \mathcal{C}^* &= \Omega(\min\{B,N\}\log\log K). \qquad (12)\end{aligned}$$

Proof: For proof sketch, see Appendix B. For complete proof, see [18, Theorem 2]. ∎

Similar results under different fading models are summarized in the following lemma.

Lemma 2. *If $|\nu_{i,k,n}|$ belongs to either Nakagami-m, Weibull, or LogNormal family of distributions, then, for dense networks, the $\mathcal{C}^*$ satisfies, for large K,*

For Nakagami-(m,w):	$\mathcal{C}^* = \Omega(\min\{B,N\}\log\log K)$
For Weibull(λ,t):	$\mathcal{C}^* = \Omega(\min\{B,N\}\log\log^{\frac{2}{t}} K)$
For LogNormal(a,ω):	$\mathcal{C}^* = \Omega(\min\{B,N\}\sqrt{\log K})$,

and

For Nakagami-(m,w):	$\mathcal{C}^* = O(BN\log\log K)$
For Weibull(λ,t):	$\mathcal{C}^* = O(BN\log\log^{\frac{2}{t}} K)$
For LogNormal(a,ω):	$\mathcal{C}^* = O(BN\sqrt{\log K})$.

Proof: For proof sketch, see Appendix C. For complete proof, see [18, Theorem 3]. ∎

Based on Lemma 1, we now propose a design principle for large dense networks. In the sequel, we call our system *scalable* under a certain condition, if the condition is not violated as the number of users $K\to\infty$.

Principle 1. *In dense femtocell deployments, with the condition that the per-user throughput remains above a certain lower bound, for the system to be scalable, the total number of independent resources BN must scale as $\Omega\big(\frac{K}{\log\log K}\big)$.*

We use the dense-network abstraction for a dense femtocell deployment [20] where the service provider wants to maintain a minimum throughput per user. In such cases, based on the upper bound on $\mathcal{C}^*$ in (12), the *necessary* condition that the service provider must satisfy is:

$$\frac{BN\log\log K}{K} = \Omega(1). \qquad (13)$$

Therefore, the total number of independent resources BN, i.e., the product of number of transmitters and the number of resource blocks (or bandwidth), must scale no slower than $\frac{K}{\log\log K}$. Otherwise, then the system is not scalable and a minimum per-user throughput requirement cannot be maintained.

Next, we consider another class of networks, namely regular-extended networks, and find performance bounds that motivate the subsequent design guidelines for such networks.

2) Regular Extended Networks: In extended networks, the area of the network grows with the number of transmitter nodes, keeping the transmitter density (number of transmitters per unit area) fixed. The users are then distributed in the network. Here we study *regular* extended networks, in which the TXs lie on a regular hexagonal grid as shown in Fig. 2 and the users are distributed uniformly in the network. The distance between two neighboring transmitters is $2R$. Hence, the radius of the network $p=\Theta(R\sqrt{B})$.

The following two lemmas use Theorem 1 and extreme-value theory to give performance bounds and associated scaling laws for regular extended networks under various fading channels.

Lemma 3. *For regular extended networks ($p^2 \approx R^2B$) with large K and Rayleigh fading channels, i.e., ($\nu_{i,k,n}\sim\mathcal{CN}(0,1)$), we have*

$$\begin{aligned}\big(\log(1+P_{\mathsf{con}}l_K)+O(1)\big)BNf_{\mathsf{lo}}^{\mathsf{EN}}(r,N)\\ \le \mathcal{C}^* \le \big(\log(1+P_{\mathsf{con}}l_K)+O(1)\big)BN, \qquad (14)\end{aligned}$$

where $l_K=\beta^2 r_0^{-2\alpha}\log\frac{Kr_0^2}{BR^2}$, $f_{\mathsf{lo}}^{\mathsf{EN}}(r,N)=\frac{(1+r^2)^{-1}r^2}{N+(1+r)c_0}$, and $c_0=\frac{P_{\mathsf{con}}\beta^2 r_0^{2-2\alpha}}{R^2}\Big(4+\frac{\pi}{\sqrt{3}(2\alpha-2)}\Big)$. Moreover, the upper bound on $\mathcal{C}^*$ obtained via (10) gives $\mathcal{C}^*\le\Big(\log\big(1+\frac{P_{\mathsf{con}}}{N}l_{KN}\big)+$

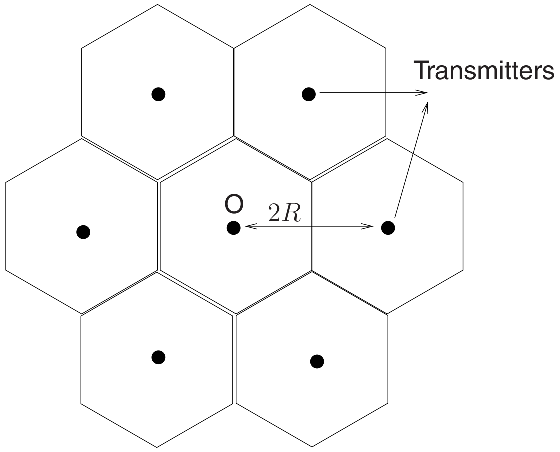

Fig. 2. A regular extended network setup.

$O(1)\Big)BN$, where $l_{KN} = \beta^2 r_0^{-2\alpha} \log \frac{KNr_0^2}{BR^2}$. The scaling laws associated with (14) are:

$$\begin{aligned} \mathcal{C}^* &= O\Big(BN \log\log \frac{K}{B}\Big), \text{ and} \\ \mathcal{C}^* &= \Omega\Big(B \log\log \frac{K}{B}\Big). \end{aligned} \tag{15}$$

Proof: For proof sketch, see Appendix B. For complete proof, see [18, Theorem 4]. ■

Lemma 4. *If $|\nu_{i,k,n}|$ belongs to either Nakagami-m, Weibull, or LogNormal family of distributions, then, for regular extended networks, the scaling laws for the upper bounds are:*

For Nakagami-(m, w):	$\mathcal{C}^* = \Omega(B \log\log \frac{K}{B})$
For Weibull(λ, t):	$\mathcal{C}^* = \Omega\Big(B \log\log^{\frac{2}{t}} \frac{K}{B}\Big)$
For LogNormal(a, ω):	$\mathcal{C}^* = \Omega\Big(B\sqrt{\log \frac{K}{B}}\Big)$,

and

For Nakagami-(m, w):	$\mathcal{C}^* = O\Big(BN \log\log \frac{K}{B}\Big)$
For Weibull(λ, t):	$\mathcal{C}^* = O\Big(BN \log\log^{\frac{2}{t}} \frac{K}{B}\Big)$
For LogNormal(a, ω):	$\mathcal{C}^* = O\Big(BN\sqrt{\log \frac{K}{B}}\Big)$.

Proof: For proof sketch, see Appendix C. For complete proof, see [18, Theorem 5]. ■

Using Lemma 3, we now propose three design principles.

Principle 2. *In regular extended networks, if a) the users are charged based on the number of bits they download; b) there is a unit cost for each TX installed and a cost c_N for unit resource block incurred by the service provider; c) the return-on-investment must remain above a certain lower bound; then for fixed B, the system is scalable only if $N = O(\log K)$, and for fixed N, the system is scalable only if $B = O(K)$. In addition, if a minimum per-user throughput requirement is also required to be met, then the system is scalable for fixed N only if $B = \Theta(K)$, and not scalable for fixed B.*

Consider the case of a regular extended network with large K. Using the upper bound in Lemma 3 obtained via (10), we have

$$\begin{aligned} \mathcal{C}^* &\le \Big(\log\Big(1 + \frac{P_{\mathsf{con}}}{N} l_{KN}\Big) + O(1)\Big)BN \\ &\approx BN \log\Big(\frac{P_{\mathsf{con}}}{N} l_{KN}\Big), \text{ for large } \frac{P_{\mathsf{con}} l_{KN}}{N}, \end{aligned} \tag{16}$$

where $l_{KN} = \beta^2 r_0^{-2\alpha} \log \frac{KNr_0^2}{BR^2}$. For simplicity of analysis, let $P_{\mathsf{con}} = \beta = r_0 = R = 1$ (in their respective SI units). If the service provider wants to maintain a minimum level of return-on-investment, then

$$\frac{BN}{B + c_N N} \log\Big(\frac{1}{N} \log \frac{KN}{B}\Big) > \bar{s}, \tag{17}$$

for some $\bar{s} > 0$. The above equation implies $N = O(\log K)$ for fixed B, and $B = O(K)$ for fixed N. In addition, if a minimum per-user throughput is also required, then the service provider must also satisfy $\frac{BN}{K} \log\big(\frac{1}{N} \log \frac{KN}{B}\big) > \hat{s}$ for some $\hat{s} > 0$. This yields that the system is not scalable under fixed B, and for fixed N, the system is scalable only if $B = \Theta(K)$.

Principle 3. *In a large extended multi-cellular network, if the users are charged based on the number of bits they download and there is a unit cost for each TX incurred by the service provider, then there is a finite range of values for the user-density $\frac{K}{B}$ in order to maximize return-on-investment of the service provider while maintaining a minimum per-user throughput.*

Consider a regular extended network with fixed number of resource blocks N. In this case, we have $\mathcal{C}^* = \Theta\big(B \log\log \frac{K}{B}\big)$. Assuming a revenue model wherein the service provider charges per bit provided to the users, the total return-on-investment of the service provider is proportional to the achievable sum-rate per TX. Therefore, in large scale systems (large K), one must solve:

$$\begin{aligned} \max_{K,B} \; & c \log\Big(1 + P_{\mathsf{con}} \beta^2 r_0^{-2\alpha} \log \frac{Kr_0^2}{BR^2}\Big) \\ \text{s.t. } & \frac{cB \log(1 + P_{\mathsf{con}} \beta^2 r_0^{-2\alpha} \log \frac{Kr_0^2}{BR^2})}{K} \ge \bar{s}, \end{aligned} \tag{18}$$

for some $\bar{s} > 0$, where c is a constant bounded according to (14)-(15). For simplicity, let $\beta = r_0 = R = P_{\mathsf{con}} = 1$ (in respective SI units). By variable-transformation, the above problem becomes convex in $\rho \triangleq \frac{K}{B}$. Solving it via dual method, the Karush-Kuhn-Tucker condition is

$$\rho = \frac{(\lambda + 1)10}{(1 + \log \rho)\lambda}, \tag{19}$$

where $\lambda \ge 0$ is the Lagrange multiplier. The plots of LHS and RHS of (19) along with the constraint curve as a function of ρ are plotted for $\lambda = 0.1, 1, \infty$ in Fig. 3. There, the constraint curve (see the constraint in (18)) is given by $\frac{c}{\bar{s}} \log(1 + \log \rho)$. Note that according to (18), the constraint is satisfied only when the constraint curve (in Fig. 3) lies above the LHS curve, i.e., when $\rho \in [1.1, 12.7]$. Therefore, the optimal ρ lies in the set $[1.1, 12.7]$. In Fig. 3, the optimal ρ for a given λ (denoted by $\rho^*(\lambda)$) is the value of ρ at which the LHS and RHS curves intersect for that λ. We observe from the figure that $\rho^*(\lambda)$ decreases with increasing λ. Since $\rho^*(\lambda) = 4.1$ when $\lambda = \infty$, the optimal ρ is greater than or equal to 4.1. Figure 4 shows the variation of $\rho^*(\lambda)$ as a function of λ. From

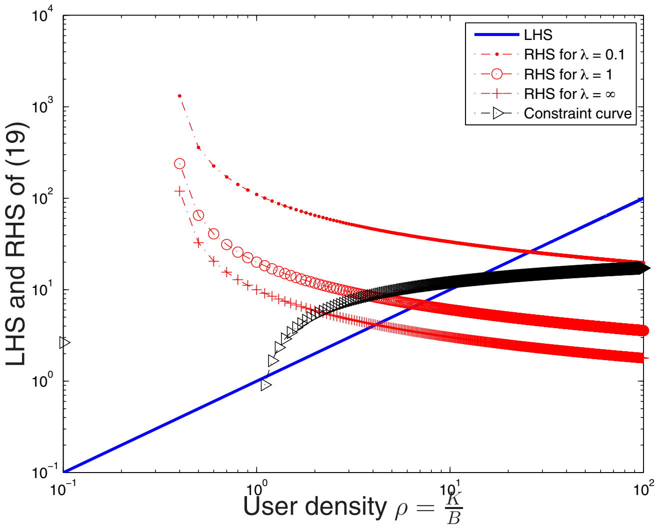

Fig. 3. LHS and RHS of (19) as a function of ρ.

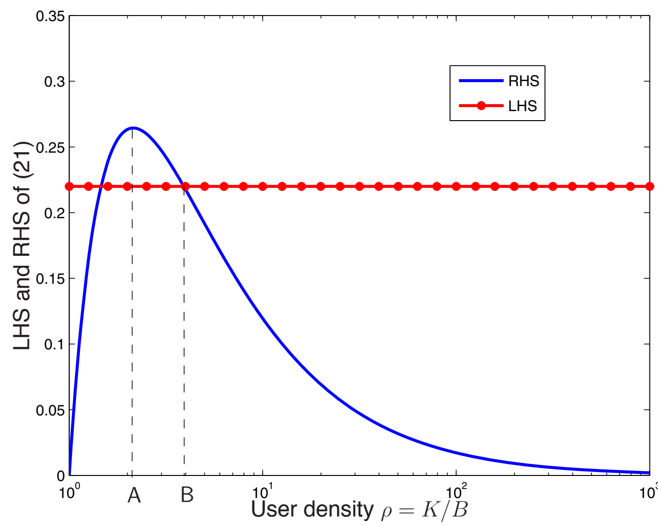

Fig. 5. LHS and RHS of (21) as a function of ρ.

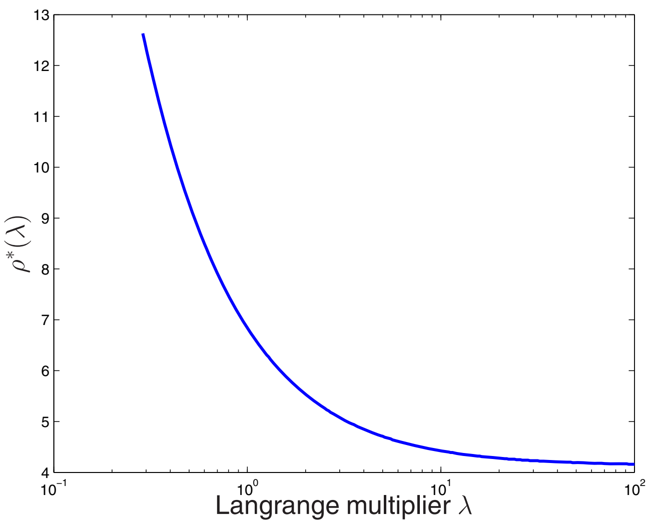

Fig. 4. Optimal user-density, i.e, $\rho^*(\lambda)$, as a function of λ.

the plot, we observe that $\rho^*(\lambda)$ exists only for $\lambda > 0.29$, and satisfies $4.1 \leq \rho^*(\lambda) \leq 12.7$ users/BS. Furthermore, the optimal user-density $\rho^*(\lambda)$ is a strictly-decreasing convex function of the cost associated with violating the per-user throughput constraint, i.e., λ.

Principle 4. *In a large extended multi-cellular network, if the users are charged a fixed amount regardless of the number of bits they download and there is a unit cost for each TX incurred by the service provider, then there is a finite range of values for $\frac{K}{B}$ in order to maximize return-on-investment of the service provider while maintaining a minimum per-user throughput.*

Consider a regular extended network with fixed N, similar to that assumed in Principle 3. Here, we assume a revenue model for the service provider wherein the service provider charges each user a fixed amount regardless of the number of bits the user downloads. Then, the return-on-investment of the service provider is proportional to the user-density $\rho = \frac{K}{B}$. In large systems (large K), the associated optimization problem is:

$$\max_{K,B} s\frac{K}{B} \quad \text{s.t.} \quad \frac{cB\log(1 + P_{\mathsf{con}}\beta^2 r_0^{-2\alpha}\log\frac{Kr_0^2}{BR^2})}{K} \geq \bar{s} \quad (20)$$

for some constants $c, s, \bar{s} > 0$. Here, s depends on the amount users are charged by the service provider, and c can be bounded according to (14)-(15). For simplicity of analysis, let $\beta = r_0 = R = P_{\mathsf{con}} = 1$ (in respective SI units). The above problem becomes convex in $\rho \triangleq \frac{K}{B}$. Let the optimal solution be denoted by ρ^*. Now, the constraint in terms of ρ is

$$\frac{\bar{s}}{c} \leq \frac{\log(1 + \log\rho)}{\rho}., \quad (21)$$

The plot of LHS and RHS of (21) as a function of ρ (for $\rho \geq 1$) is plotted in Fig. 5. Examining (21) and Fig. 5, we note that the per-user throughput constraint is satisfied only if $\frac{\bar{s}}{c} \in [0, 0.26]$. Moreover, for a given value of $\frac{\bar{s}}{c}$, the set of feasible ρ lies in a closed set (for which the RHS curve remains above the LHS curve). The maximum value of ρ in this closed set, i.e., the value of ρ at point B in Fig. 5, is the one that maximizes the objective in (20), i.e., sK/B. Hence, it is the optimal ρ for the given value of $\bar{s}/c$. Let us denote it by $\rho^*(\bar{s}/c)$. Note that $\rho^*(\bar{s}/c) \geq 2.14$ (since point B lies to the right of point A in Fig. 5).

If $\bar{s}/c$ is known exactly, then the optimal user-density $\rho^* = \rho^*(\bar{s}/c)$. If not, we can write from (14)-(15) that $c_{\mathsf{lb}} \leq c \leq c_{\mathsf{ub}}$, for some positive constants $c_{\mathsf{lb}}, c_{\mathsf{ub}}$. Then, $\rho^* \in [\rho^*(\bar{s}/c_{\mathsf{lb}}), \rho^*(\bar{s}/c_{\mathsf{ub}})]$. Moreover, since $\rho^*(\bar{s}/c) \geq 2.14$ for all $\bar{s}/c \in [0, 0.26]$, we have $\rho^*(\bar{s}/c_{\mathsf{ub}}) \geq \rho^*(\bar{s}/c_{\mathsf{lb}}) \geq 2.14$.

IV. Maximum Sum-Rate Achievability Scheme

In the previous section, we derived general performance bounds and proposed design principles based on them for two specific types of networks - dense and regular-extended. In this section, we propose a distributed scheme for achievability of max-sum-rate under the above two types of networks. To this end, we construct a tight approximation of $\mathcal{C}^*$ and find a distributed resource allocation scheme that achieves the same sum-rate scaling law as that achieved by $\mathcal{C}^*$ for a large set of network parameters. Let us define an approximation of $\mathcal{C}^*$ as follows:

$$\mathcal{C}^*_{\mathsf{LB}} \triangleq \max_{\boldsymbol{p}\in\mathcal{P}} \mathrm{E}\left\{ \max_{\boldsymbol{u}\in\mathcal{U}} \sum_{i=1}^{B}\sum_{n=1}^{N} \log\left(1+\frac{\gamma_{i,u_{i,n},n}\, p_{i,n}}{1+\sum_{j\neq i}\gamma_{j,u_{i,n},n}\, p_{j,n}}\right)\right\}. \quad (22)$$

Note that $\mathcal{C}^*_{\mathsf{LB}} \le \mathcal{C}^*$. To analyze $\mathcal{C}^*_{\mathsf{LB}}$, we first give the following theorem.

Theorem 2. *Let $\{X_1,\dots,X_T\}$ be i.i.d. random variables with cumulative distribution function (cdf) $F_X(\cdot)$. Then, for any monotonically non-decreasing function $V(\cdot)$, we have*

$$\left(1-e^{-S_1}\right)V\left(l_{T/S_1}\right) \le \mathrm{E}\left\{V\left(\max_{1\le t\le T} X_t\right)\right\}. \quad (23)$$

Here, $S_1 \in (0,T]$ and $F_X\left(l_{T/S_1}\right) = 1-\frac{S_1}{T}$. Additionally, if $V(\cdot)$ is concave, then we have

$$\left(1-e^{-S_1}\right)V\left(l_{T/S_1}\right) \le \mathrm{E}\left\{V\left(\max_{1\le t\le T} X_t\right)\right\} \le V\left(\mathrm{E}\left\{\max_{1\le t\le T} X_t\right\}\right).$$

Proof: See Appendix D. ■

Theorem 2 can be used to bound $\mathcal{C}^*_{\mathsf{LB}}$ for finite K. In particular, for a given power allocation $\{p_{i,n}\}$, the achievable expected sum-rate can be bounded by bounding the contribution of each (i,n) towards sum-rate by appropriately selecting X_t and $V(\cdot)$ via[3] Theorem 2 and then taking the summation over all (i,n). Thereafter, by maximizing the bounds over all feasible power allocations that lie in $\mathcal{P}$, non-asymptotic bounds on $\mathcal{C}^*_{\mathsf{LB}}$ can be obtained. In the sequel, however, we will use Theorem 2 under asymptotic regime to propose a class of deterministic optimization problems that bound $\mathcal{C}^*_{\mathsf{LB}}$ for dense/extended networks and Rayleigh-fading channels[4].

Theorem 3. *Let a class of deterministic optimization problems be defined as follows:*

$$\mathsf{OP}\left(c,h(K)\right) \quad (24)$$

$$\triangleq \max_{\boldsymbol{p}\in\mathcal{P}} \sum_{i=1}^{B}\sum_{n=1}^{N} \log(1+p_{i,n}x_{i,n}) \quad (25)$$

$$\text{s.t. } \frac{r_0^2\, h(K)}{p^2} = e^{\frac{x_{i,n}}{\beta^2 r_0^{-2\alpha}}} \prod_{j\neq i}\left(1+\frac{p_{j,n}x_{i,n}}{c^{2\alpha}r_0^{-2\alpha}}\right) \quad (26)$$

where $h(\cdot)$ is an increasing function and c is a positive constant. Then, for large K and Rayleigh-fading channels, i.e., $|\nu_{i,k,n}| \sim \mathcal{CN}(0,1)$, we have

$$\left(1-e^{-S_1}\right)\mathsf{OP}\left(r_0, K/S_1\right) \le \mathcal{C}^*_{\mathsf{LB}} \le \left(1+\frac{\beta^2 r_0^{-2\alpha}u}{\bar{l}(2p,K)}\right)\mathsf{OP}\left(2p,K\right) \quad (27)$$

where $S_1 \in (0,K]$, and $\bar{l}(\cdot,K)$ is a large number that increases with increasing K. In particular, if $l=\hat{l}(\eta_1,\eta_2)$ is the solution to $\frac{r_0^2\eta_2}{p^2} = e^{\frac{l}{\beta^2 r_0^{-2\alpha}}}\left(1+\frac{lP_{\mathsf{con}}}{\eta_1^{2\alpha}r_0^{-2\alpha}}\right)^{B-1}$ for any η_1,η_2, then $\hat{l}(2p,K)\approx \bar{l}(2p,K)$. Further, $\mathsf{OP}(\cdot,\cdot)$ satisfies

$$1 \le \frac{\mathsf{OP}\left(c_2,h(K)\right)}{\mathsf{OP}\left(c_1,h(K)\right)} \le \left(\frac{c_2}{c_1}\right)^{2\alpha} \quad (28)$$

for positive constants c_1 and c_2 $(0<c_1\le c_2)$.

Proof: Proof sketch given in Appendix E. For complete proof, see [18]. ■

The above theorem leads to following two corollaries for dense and regular-extended networks.

Corollary 1. *For dense networks with large K and Rayleigh-fading channels, we have*

$$\left(1-\frac{1}{\log K}\right)\mathsf{OP}\left(r_0, K/\log\log K\right) \le \mathcal{C}^*_{\mathsf{LB}} \le \left(1+\frac{\beta^2 r_0^{-2\alpha}u}{\bar{l}(2p,K)}\right)\mathsf{OP}(2p,K), \text{ and} \quad (29)$$

$$0.63\,\mathsf{OP}(r_0,K) \le \mathcal{C}^*_{\mathsf{LB}} \le \left(\frac{2p}{r_0}\right)^{2\alpha}\left(1+\frac{\beta^2 r_0^{-2\alpha}u}{\bar{l}(2p,K)}\right)\mathsf{OP}(r_0,K). \quad (30)$$

where $\bar{l}(2p,K)=\Theta(\log K)$. In other words,

$$0.63 \le \frac{\mathcal{C}^*_{\mathsf{LB}}}{\mathsf{OP}\left(r_0,K\right)} \le \left(\frac{2p}{r_0}\right)^{2\alpha} + O\left(\frac{1}{\log K}\right). \quad (31)$$

Proof: Put $S_1=\log\log K$ in Theorem 3 to prove (29). Put $S_1=1$ in (27) and use (28) to prove (30). ■

Corollary 2. *For regular extended networks and Rayleigh-fading channels, if $\rho \triangleq K/B$ users are distributed uniformly in each cell and each TX schedules users only within its cell, then*

$$\left(1-\frac{1}{\log\rho}\right)\mathsf{OP}\left(r_0,\frac{\rho}{\log\log\rho}\right) \le \mathcal{C}^*_{\mathsf{LB}} \le \left(1+\frac{\beta^2 r_0^{-2\alpha}u}{\bar{l}(R\sqrt{3}/2,\rho)}\right)\mathsf{OP}\left(\frac{R\sqrt{3}}{2},\rho\right) \quad (32)$$

for large ρ. Moreover,

$$0.63\,\mathsf{OP}(r_0,\rho) \le \mathcal{C}^*_{\mathsf{LB}} \le \left(1+\frac{\beta^2 r_0^{-2\alpha}u}{\bar{l}(R\sqrt{3}/2,\rho)}\right)\left(\frac{R\sqrt{3}}{2r_0}\right)^{2\alpha}\mathsf{OP}(r_0,\rho). \quad (33)$$

Proof: Note that $p=\Theta(\sqrt{B})$ in this case. Therefore we use, instead of $h(K)$, $h(\rho)$ in Theorem 3 to obtain the above result, where $\rho=\frac{K}{B}$. Also note that $2p$ is replaced by $\frac{R\sqrt{3}}{2}$ since the maximum distance between a user and its serving TX is $\frac{R\sqrt{3}}{2}$. ■

The above two corollaries highlight the idea behind the proposed achievability strategy. In particular, we use the lower bounds in (30) and (33) to give a distributed resource allocation scheme[5]. The steps of the proposed achievability scheme are summarized below.

[3]For example, by setting $T=K$, $X_t=\frac{\gamma_{i,t,n}\,p_{i,n}}{1+\sum_{j\neq t}\gamma_{i,t,n}\,p_{i,n}}$ and $V(x)=\log(1+x)$.

[4]Theorem 3 can be easily extended for Nakagami-m, Weibull, and Log-Normal fading channels.

[5]One could also use the lower bounds in (29) and (32) to obtain an alternate distributed resource allocation scheme.

1) Find the best power allocation (denoted by $\{P_{i,n}\}$) by solving the LHS of (30) for dense networks, or LHS of (33) for regular-extended networks. This can be computed offline.
2) For each TX i and resource-block n, schedule the user $k(i,n)$ that satisfies:

$$k(i,n) = \operatorname*{argmax}_k \frac{P_{i,n}\gamma_{i,k,n}}{1+\sum_{j\neq i} P_{j,n}\gamma_{j,k,n}}. \quad (34)$$

We propose that each user k calculates $\frac{P_{i,n}\gamma_{i,k,n}}{1+\sum_{j\neq i} P_{j,n}\gamma_{j,k,n}}$ for each (i,n) combination and feeds back the value to TX i, thus making the algorithm distributed.

We will now compare low-powered peer-to-peer networks and high-powered single TX systems to give a design principle using the bounds in Corollary 1.

Principle 5. *The sum-rate of a peer-to-peer network with B transmit nodes (geographically distributed antennas), each transmitting at a fixed power $\bar{P}$ across every resource-block, increases linearly with B only if $B = O\left(\frac{\log K}{\log\log K}\right)$. If $B = \Omega\left(\frac{\log K}{\log\log K}\right)$, then there is no gain with increasing B. Further, the gain obtained by implementing a peer-to-peer network over a high-powered single-TX system (with power $B\bar{P}$ across each resource-block) is*

$$\begin{cases} \Theta(B) & \text{if } B = O\left(\frac{\log K}{\log\log K}\right), \\ \Theta\left(\frac{\log K}{\log\log K}\right) & \text{if } B = \Omega\left(\frac{\log K}{\log\log K}\right) \text{ and } B = O(\log K), \\ \Theta\left(\frac{\log K}{\log B}\right) & \text{if } B = \Omega(\log K). \end{cases} \quad (35)$$

In this case, we consider a peer-to-peer networks with B nodes randomly distributed in a circular area of fixed radius p. Assuming fixed power allocation, we have $P_{i,n} = \bar{P}$ for all i,n. Therefore, from (26), we get

$$x_{i,n} \approx \Theta\left(\min\left\{ \beta^2 r_0^{-2\alpha} \log \frac{r_0^2 h(K)}{p^2}, \frac{1}{\bar{P}}\left(\frac{c}{r_0}\right)^{2\alpha} \sqrt[B-1]{\frac{r_0^2}{p^2} h(K)} \right\} \right), \quad (36)$$

and $\mathsf{OP}(c,h(K)) = \sum_{i,n} \log(1+\bar{P}x_{i,n}) = \Theta\big(\min\{BN\log\log h(K), N\log h(K)\}\big)$. Note that for a fixed power allocation scheme, $\mathcal{C}^*_{\mathsf{LB}} = \Theta\big(\mathsf{OP}(c,K)\big)$ also denotes the expected maximum achievable sum-rate. Therefore, using Corollary 1 with $h(K) = K$, the max-sum-rate under fixed power-allocation scales as:

$$\mathcal{C}^*_{\mathsf{LB}} = \Theta\big(\min\{BN\log\log K, N\log K\}\big). \quad (37)$$

In other words, if $B = O\left(\frac{\log K}{\log\log K}\right)$, then $\mathcal{C}^*_{\mathsf{LB}} = \Theta(BN\log\log K)$, i.e., we get a linear scaling in max-sum-rate w.r.t. B. Note that this is also the scaling of the upper bound on max-sum-rate given in Lemma 1. However, if $B = \Omega\left(\frac{\log K}{\log\log K}\right)$, then $\mathcal{C}^*_{\mathsf{LB}} = \Theta(N\log K)$.

One can also view the above scenario as a multi-antenna system with a single base-station in which all B transmitters are treated as co-located antennas (i.e., $B = M$). Then, comparing our results to those in [2], we note that our results extend the results in [2]. In particular, [2] showed that linear scaling of sum-rate $\mathcal{C}^*_{\mathsf{LB}}$ w.r.t. number of antennas M holds when $M = \Theta(\log K)$ and does not hold when $M = \Omega(\log K)$. We establish that even if M scales slower than $\log K$, the achievable sum-rate scaling is not linear in M unless $M = O\left(\frac{\log K}{\log\log K}\right)$. Only in the special case of $M = \Theta(\log K)$ is $\mathcal{C}^*_{\mathsf{LB}} = \Theta(N\log K) = \Theta(NM)$. Another way to state the above result is that for a given number of users K (K is large), the achievable sum-rate increases with increasing M only until $M = O\left(\frac{\log K}{\log\log K}\right)$, beyond which it stabilizes.

Now, for fair comparison with lower-powered peer-to-peer network, we assume that in case of the high-powered single-TX system, $P_{1,n} = B\bar{P}$ for all n. Then, for a high-powered single-TX system, $\mathcal{C}^*_{\mathsf{LB}} = \Theta(N\log(B\bar{P}\log K))$. Hence, the gain of peer-to-peer networks over a high-powered single-TX system is given by (35).

In the above design principle, the total power allocated by each transmitter is $N\bar{P}$. Replacing $\bar{P}$ by $\frac{P_{\mathsf{con}}}{N}$, one can calculate the scaling of achieved sum-rate when a sum-power constraint of P_{con} must be met at each transmitter in a dense network (or peer-to-peer network with B nodes). Repeating the above analysis, we obtain that the equal power allocation scheme achieves a sum-rate scaling of $\Theta(BN\log\log K)$, which is same as that of the upper bound of $\mathcal{C}^*$ in Theorem 1, as long as $B = O\left(\frac{\log K}{\log\log K}\right)$ and $N = O(\log K)$. Since the proposed distributed user and power outperforms the equal-power allocation scheme, the sum-rate scaling remains optimal for the proposed algorithm in the aforementioned range of B, N.

V. A Note on MISO vs SISO Systems

Until now, we discussed systems where either every transmitter had a single antenna or different transmitters were treated as geographically distributed antennas with independent power constraints (i.e., P_{con} at each TX). We wrap up our analysis with a discussion on multiple antennas at each TX followed by conclusions in Section VI.

We use the opportunistic random scheduling scheme proposed in [2], which achieves the max-sum-rate in the scaling sense for fixed power-allocation schemes. Assume that each TX has M antennas and each user (or, receiver) has a single antenna. Every TX constructs M orthonormal random beams ϕ_m ($M \times 1$) for $m \in \{1,\ldots,M\}$ using an isotropic distribution [21]. With some abuse of notation, let the user scheduled by TX i across resource block n using beam m be denoted by $u_{i,n,m}$. Then, the signal received by $u_{i,n,m}$ across resource block n is

$$\begin{aligned} &y_{u_{i,n,m},n} \\ &= \boldsymbol{H}_{i,u_{i,n,m},n}\Big(\boldsymbol{\phi}_m\, x_{i,u_{i,n,m},n} + \sum_{m'\neq m} \boldsymbol{\phi}_{m'}\, x_{i,u_{i,n,m'},n}\Big) \\ &\quad + \sum_{j\neq i}\sum_{\tilde{m}=1}^{M} \boldsymbol{H}_{j,u_{i,n,\tilde{m}},n}\, \boldsymbol{\phi}_{\tilde{m}}\, x_{j,u_{j,n,\tilde{m}},n} + w_{u_{i,n,m},n}, \end{aligned} \quad (38)$$

where $\boldsymbol{H}_{i,k,n} = \beta R_{i,k}^{-\alpha}\boldsymbol{\nu}_{i,k,n} \in \mathbb{C}^{1\times M}$ is the channel-gain matrix, $\boldsymbol{\nu}_{i,k,n}$ is the $1\times M$ vector containing i.i.d. complex Gaussian random variables, and $w_{k,n} \sim \mathcal{CN}(0,1)$ is AWGN that is i.i.d. for all (k,n). Abbreviating $\mathrm{E}\{|x_{i,u_{i,n,m},n}|^2\}$ by

$p_{i,n,m}$, we can write the SINR corresponding to the combination (i,k,n,m) as:

$$\mathsf{SINR}_{i,k,n,m} = \frac{p_{i,n,m}\gamma_{i,k,n,m}}{1+\sum_{m'\neq m} p_{i,n,m'}\,\gamma_{i,k,n,m}+\sum_{j\neq i}\sum_{\tilde{m}=1}^{M} p_{j,n,\tilde{m}}\,\gamma_{j,k,n,\tilde{m}}}, \quad (39)$$

where $\gamma_{i,k,n,m} \triangleq |\boldsymbol{H}_{i,k,n}\,\boldsymbol{\phi}_m|^2$ for all (i,k,n,m). Since $\boldsymbol{H}_{i,k,n}\,\boldsymbol{\phi}_m$ are i.i.d. over all (k,n,m) [2], $\gamma_{i,k,n,m}$ are i.i.d. over (k,n,m). A lower bound on max-sum-rate, similar to that in (22), under opportunistic random beamforming can be written as:

$$\mathcal{C}^*_{\mathsf{LB,MISO}} \triangleq \max_{\{p_{i,n,m}\geq 0 \text{ for all } i,n,m\}} \mathrm{E}\left\{\max_{\{u_{i,n,m}\}} \sum_{i=1}^{B}\sum_{n=1}^{N}\sum_{m=1}^{M}\log\left(1+\mathsf{SINR}_{i,u_{i,n,m},n,m}\right)\right\} \quad (40)$$

$$\text{s.t.} \sum_{n,m} p_{i,n,m} \leq P_{\mathsf{con}} \text{ for all } i. \quad (41)$$

The above optimization problem is similar to that in (22) with BM transmitters. Therefore, repeating the analysis in (22)-(30) under dense networks for the problem in (40)-(41), we get $\mathcal{C}^*_{\mathsf{LB,MISO}} = \Theta\big(\mathsf{OP}_{\mathsf{MISO}}(r_0,K)\big)$, where

$$\mathsf{OP}_{\mathsf{MISO}}(c,h(K)) \triangleq \max_{\{p_{i,n,m}\geq 0\}} \sum_{i=1}^{B}\sum_{n=1}^{N}\sum_{m=1}^{M}\log(1+p_{i,n,m}\,x_{i,k,n,m}) \quad (42)$$

$$\text{s.t.} \sum_{n,m} p_{i,n,m} \leq P_{\mathsf{con}} \text{ for all } i, \text{ and for all } (i,m)$$

$$\left(1+\frac{p_{i,n,m}x_{i,k,n,m}}{c^{2\alpha}r_0^{-2\alpha}}\right)\frac{r_0^2 h(K)}{p^2} = e^{\frac{x_{i,k,n,m}}{\beta^2 r_0^{-2\alpha}}}\prod_{j}\prod_{\tilde{m}=1}^{M}\left(1+\frac{p_{j,n,\tilde{m}}x_{i,k,n,\tilde{m}}}{c^{2\alpha}r_0^{-2\alpha}}\right).$$

VI. Conclusion

In this paper, we developed bounds on the downlink max-sum-rate in large OFDMA based networks and derived the associated scaling laws with respect to number of users K, transmitters B, and resource-blocks N. Our bounds hold for a general spatial distribution of transmitters, a truncated path-gain model, and a general channel-fading model. We evaluated the bounds under asymptotic situations in *dense* and *extended* networks in which the users are distributed uniformly for Rayleigh, Nakagami-m, Weibull, and LogNormal fading models. Using the derived results, we proposed four design principles for service providers and regulators to achieve QoS provisioning along with system scalability. According to the first principle, in dense-femtocell deployments, for a minimum per-user throughput requirement, we showed that the system is scalable only if BN scales as $\Omega\big(\frac{K}{\log\log K}\big)$. In the second principle, we considered the cost of bandwidth to the service provider along with the cost of the transmitters in regular extended networks and showed that under a minimum return-on-investment and a minimum per-user throughput requirement, the system is not scalable under fixed B and is scalable under fixed N only if $B=\Theta(K)$. In the third and fourth principles, we considered different pricing policies in regular extended networks and showed that the user density must be kept within a finite range of values in order to maximize the return-on-investment, while maintaining a minimum per-user rate. Thereafter, towards developing an achievability scheme, we proposed a deterministic distributed resource allocation scheme and developed an additional design principle. In particular, we showed that the max-sum-rate of a peer-to-peer network with B transmitters increases with B only when $B = O\big(\frac{\log K}{\log\log K}\big)$. Finally, we showed how our results can be extended to MISO systems.

Appendix A
Proof Sketch of Theorem 1

By ignoring the interference for each realization of $\{\gamma_{i,k,n}\}$, $\mathcal{C}_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{\nu}}(\boldsymbol{U},\boldsymbol{P})$ can be upper bounded by $\sum_{i=1}^{B}\sum_{n=1}^{N}\log\big(1 + P_{i,n}\max_k\gamma_{i,k,n}\big)$. Therefore, $\mathcal{C}_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{\nu}}(\boldsymbol{U},\boldsymbol{P}) \leq \sum_{i=1}^{B}\sum_{n=1}^{N}\log\big(1 + P_{\mathsf{con}}\max_k\gamma_{i,k,n}\big)$. Moreover, since $\frac{1}{N}\sum_n P_{i,n} \leq \frac{P_{\mathsf{con}}}{N}$, we get $\mathcal{C}_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{\nu}}(\boldsymbol{U},\boldsymbol{P}) \leq N\sum_{i=1}^{B}\log\big(1 + \frac{P_{\mathsf{con}}}{N}\max_{n,k}\gamma_{i,k,n}\big)$ using Jensen's inequality on powers. Taking expectation on both sides of above two inequalities, we get the upper bounds in (9) and (10). For lower bound, we allocate equal power to all resource-blocks at every TX giving

$$\mathcal{C}_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{\nu}}(\boldsymbol{U},\boldsymbol{P}) \geq \sum_{i=1}^{B}\sum_{n=1}^{N}\log\left(1+\frac{P_{\mathsf{con}}\,\gamma_{i,k_{i,n},n}}{N+P_{\mathsf{con}}\sum_{j\neq i}\gamma_{j,k_{i,n},n}}\right), \quad (43)$$

where $k_{i,n}$ is any choice of user allocated on resource block n by TX i. Next, evaluating $\mathcal{C}^* = \mathrm{E}\{\mathcal{C}_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{\nu}}(\boldsymbol{U},\boldsymbol{P})\}$ using the property $\log\big(1+d_1/d_2\big) \geq \frac{\log(1+d_1)}{d_2}$ for all $d_1 \geq 0, d_2 \geq 1$ and selecting $k_{i,n} = \mathrm{argmax}_k\,\gamma_{i,k,n}$ for all (i,n), we get the desired lower bound.

Appendix B
Proof Sketch of Lemma 1 and Lemma 3

The proof consists of three steps. In the first step, we represent the lower bound of $\mathcal{C}^*$ obtained in Theorem 1 in terms of its upper bound. In particular, we show that for dense networks, $\mathcal{C}^* \geq f_{\mathsf{lo}}^{\mathsf{DN}}(r,B,N)\sum_{i,n}\mathrm{E}\big\{\log\big(1 + P_{\mathsf{con}}\max_k\gamma_{i,k,n}\big)\big\}$, where $r > 0$ is a fixed number, and $f_{\mathsf{lo}}^{\mathsf{DN}}(r,B,N) = \frac{r^2}{(1+r^2)(N+P_{\mathsf{con}}\beta^2 r_0^{-2\alpha}(1+r)B)}$. This is done by upper bounding the interference term in (43) (see denominator) using $\gamma_{j,k,n} \leq \beta^2 r_0^{-2\alpha}|\nu_{j,k,n}|^2$ for all (j,k,n) where $j \neq i$. Then, we use one-sided variant of Chebyshev's inequality (also called Cantelli's inequality) on the random variable in denominator $\sum_{j\neq i}|\nu_{j,k,n}|^2$ to obtain the aforementioned lower bound. In particular, we use the following inequality:

$$\Pr\left(\sum_{j\neq i}|\nu_{j,k,n}|^2 \leq (1+r)B\right) \geq \frac{r^2}{1+r^2}.$$

The corresponding result for regular extended-networks is $\mathcal{C}^* \geq f_{\mathsf{lo}}^{\mathsf{EN}}(r,N)\sum_{i,n}\mathrm{E}\big\{\log(1+P_{\mathsf{con}}\max_k\gamma_{i,k,n})\big\}$, where $r > 0$, $f_{\mathsf{lo}}^{\mathsf{EN}}(r,N) = \frac{(1+r^2)^{-1}r^2}{N+(1+r)c_0}$, and $c_0 = \frac{P_{\mathsf{con}}\beta^2 r_0^{2-2\alpha}}{R^2}\Big(4+\frac{\pi}{\sqrt{3}(2\alpha-2)}\Big)$.

In the second step, using fundamental coordinate geometry and probability theory, we show that under Rayleigh fading, the CDF of $\gamma_{i,k,n}$ is given by

$$F_{\gamma_{i,k,n}}(\gamma) \triangleq 1 - \frac{r_0^2}{p^2} e^{-\frac{\gamma}{\beta^2 r_0^{-2\alpha}}} - \frac{1}{\alpha\beta^2 p^2} \int_{\frac{\beta^2}{(p-d)^{2\alpha}}}^{\beta^2 r_0^{-2\alpha}} e^{-\frac{\gamma}{g}} \left(\frac{g}{\beta^2}\right)^{-1-\frac{1}{\alpha}} dg + \int_{\frac{\beta^2}{(p+d)^{2\alpha}}}^{\frac{\beta^2}{(p-d)^{2\alpha}}} e^{-\frac{\gamma}{g}} ds(g), \quad (44)$$

where $d = \sqrt{a_i^2 + b_i^2}$, and $s(g)$ is a function that satisfies $\left(1 - \frac{d}{p}\right)^2 \leq s(g) \leq 1$ for all g. We then take the limit of growth function $h(\gamma) \triangleq \frac{1-F_{\gamma_{i,k,n}}(\gamma)}{f_{\gamma_{i,k,n}}(\gamma)}$ as $\gamma \to \infty$, where $f_{\gamma_{i,k,n}}(\cdot)$ is the pdf of $\gamma_{i,k,n}$, and apply Gnedenko's theorem [22] to show that the limit (as K grows large) of an appropriately shifted version of the maximum of K i.i.d. random variables, i.e., $\max_k \gamma_{i,k,n} - \beta^2 r_0^{-2\alpha} \log \frac{K r_0^2}{p^2}$, is a random variable with a Gumbel-type cdf that is given by $\exp(-e^{-x r_0^{2\alpha}/\beta^2})$ for all $x \in (-\infty, \infty)$.

Finally, in the third step, we use the asymptotic distribution of $\max_k \gamma_{i,k,n}$ to evaluate the lower bound from first step and the upper bound in (9) and obtain the final result. Note that the upper bound in (10) is evaluated using the asymptotic distribution of $\max_{k,n} \gamma_{i,k,n}$, which is basically the maximum over KN i.i.d. random variables (instead of maximum over K i.i.d. random variables).

Appendix C
Proof Sketch of Lemma 2 and Lemma 4

The proof is similar to the proof of Lemma 1 and Lemma 3 in Appendix B. First, we show that $\max_k \gamma_{i,k,n} - l_K$ converges in distribution to a random variable with a Gumbel-type cdf, where l_K is such that $1 - F_{\gamma_{i,k,n}}(l_K) = \frac{1}{K}$. Next, using the asymptotic pdf, we evaluate the lower bound in the first step in Appendix B and the upper bound in (9) to obtain scaling laws for different fading models.

Appendix D
Proof of Theorem 2

We know that $F_{X_1}(l_{T/S_1}) = 1 - \frac{S_1}{T}$, where $S_1 \in (0, T]$. Therefore, the cdf of $\max_t X_t$ satisfies $F_{\max_t X_t}(l_{T/S_1}) = \left(1 - \frac{S_1}{T}\right)^T$. This implies, for an non-decreasing function $V(\cdot)$,

$$\begin{aligned} \mathrm{E}\left\{V\left(\max_{1\leq t\leq T} X_t\right)\right\} &\geq \Pr\left(\max_{1\leq t\leq T} X_t > l_{T/S_1}\right) V\left(l_{T/S_1}\right) \\ &= \left(1 - \left(1 - \frac{S_1}{T}\right)^T\right) V\left(l_{T/S_1}\right) \\ &\geq \left(1 - e^{-S_1}\right) V\left(l_{T/S_1}\right). \end{aligned} \quad (45)$$

Additionally, if $V(\cdot)$ is concave, then an upper bound on $\mathrm{E}\left\{V\left(\max_{1\leq t\leq T} X_t\right)\right\}$ can be obtained via Jensen's inequality. In particular, we have $\mathrm{E}\left\{V\left(\max_{1\leq t\leq T} X_t\right)\right\} \leq V\left(\mathrm{E}\left\{\max_{1\leq t\leq T} X_t\right\}\right)$.

Appendix E
Proof Sketch of Theorem 3

The proof of (25)-(27) in Theorem 3 consists of three steps. First, we bound $\mathcal{C}^*_{\mathsf{LB}}$ using the fact that the path-gain in a dense network between a TX and user lies in $[(2p)^{-\alpha}\beta, r_0^{-\alpha}\beta]$. In particular, we have

$$\begin{aligned} \sum_{i=1}^{B}\sum_{n=1}^{N} \mathrm{E}\left\{\log\left(1 + \max_k \mathbb{X}_{i,k,n}(r_0)\right)\right\} &\leq \mathcal{C}^*_{\mathsf{LB}} \\ \leq \sum_{i=1}^{B}\sum_{n=1}^{N} \mathrm{E}\left\{\log\left(1 + \max_k \mathbb{X}_{i,k,n}(2p)\right)\right\} & \end{aligned} \quad (46)$$

where $\mathbb{X}_{i,k,n}(c) \triangleq \frac{\beta^2 R_{i,k}^{-2\alpha} |\nu_{i,k,n}|^2}{1+\beta^2 c^{-2\alpha} \sum_{j\neq i} P_{j,n} |\nu_{j,k,n}|^2}$ and where $c \in \{r_0, 2p\}$.

Second, we compute the distribution of $\mathbb{X}_{i,k,n}(c)$ using mathematical tools from coordinate geometry and probability theory. We show that the CDF of $\mathbb{X}_{i,k,n}(c)$ can be approximated as $F_{\mathbb{X}_{i,k,n}(c)}(x) \approx 1 - \frac{r_0^2}{p^2} e^{-\frac{x}{\beta^2 r_0^{-2\alpha}}} \prod_{j\neq i} \left(1 + \frac{p_{j,n} x}{c^{2\alpha} r_0^{-2\alpha}}\right)^{-1}$ for large x, and the growth function converges to a constant, i.e., $\lim_{x\to\infty} \frac{1-F_{\mathbb{X}_{i,k,n}(c)}(x)}{f_{\mathbb{X}_{i,k,n}(c)}(x)} = \beta^2 r_0^{-2\alpha}$. Therefore, applying Gnedenko's theorem [22], we obtain that the limiting distribution of an appropriately shifted version of $\max_k \mathbb{X}_{i,k,n}(c)$ is of Gumbel-type.

Finally, using the distribution of $\mathbb{X}_{i,k,n}(c)$ and the limiting distribution of $\max_k \mathbb{X}_{i,k,n}(c)$, we obtain new lower and upper bounds. In particular, the upper bound is obtained by applying Jensen's inequality on $\sum_{i=1}^{B}\sum_{n=1}^{N} \mathrm{E}\left\{\log\left(1 + \max_k \mathbb{X}_{i,k,n}(2p)\right)\right\}$ (see upper bound in (46)), which gives that $\mathcal{C}^*_{\mathsf{LB}} \leq \sum_{i=1}^{B}\sum_{n=1}^{N} \log\left(1 + \mathrm{E}\left\{\max_k \mathbb{X}_{i,k,n}(2p)\right\}\right)$. Now, $\mathrm{E}\left\{\max_k \mathbb{X}_{i,k,n}(2p)\right\}$ can be bounded using the limiting distribution of $\max_k \mathbb{X}_{i,k,n}(c)$. To obtain the lower bound, we apply Theorem 2 on the lower-bound in (46) with $V(\mathbb{X}_{i,k,n}(r_0)) = \log(1 + p_{i,n}\mathbb{X}_{i,k,n}(r_0))$. Combining the bounds in to a single mathematical expression, we get the desired optimization problem $\mathsf{OP}(\cdot,\cdot)$ and the bounds in (27).

To prove (28) in Theorem 3, we analyze (26) using simple algebraic methods and show that for a given $\boldsymbol{p} = \{p_{i,n} \text{ for all } i,n\}$, if $x_{i,n}(c)$ solves (26) for any given c, then we have

$$1 \leq \frac{x_{i,n}(c_2)}{x_{i,n}(c_1)} \leq \left(\frac{c_2}{c_1}\right)^{2\alpha} \text{ for all } (i,n), \quad (47)$$

where $0 < c_1 \leq c_2$. Now, since $\log(1 + ax) \leq a\log(1 + x)$ for all $x \geq 0$ and $a \geq 1$, we have $1 \leq \frac{\log(1+P_{i,n}\, x_{i,n}(c_2))}{\log(1+P_{i,n}\, x_{i,n}(c_1))} \leq \left(\frac{c_2}{c_1}\right)^{2\alpha}$. Substituting this in the objective of $\mathsf{OP}(\cdot,\cdot)$, we get (28).

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