Achieving the noncoherent sparse-channel's capacity prelog factor: A graphical-model approach

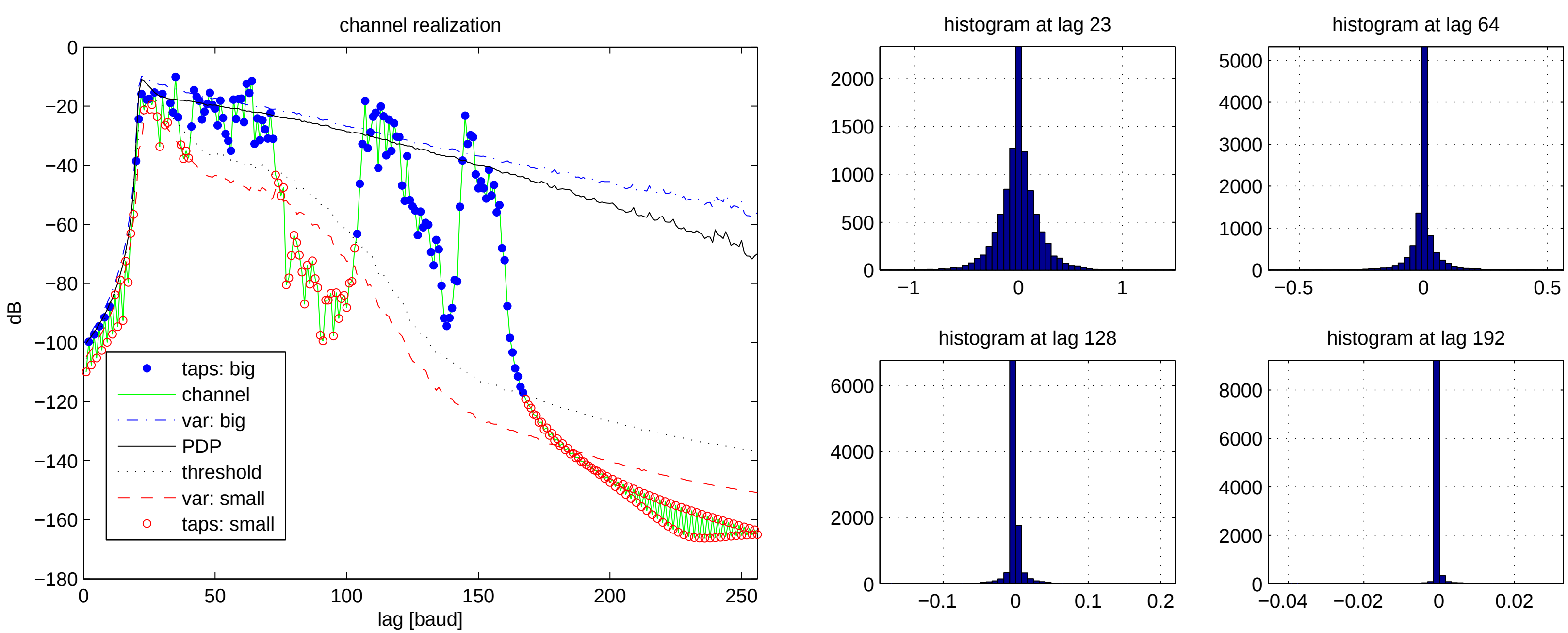
Phil Schniter

(With support from NSF grant CCF-1018368 and DARPA/ONR grant N66001-10-1-4090)

ITA @ UCSD, Feb. 2011

Motivation:

- At large communication bandwidths, communication channels are not only frequency selective but *sparse*.
- For example, consider channel taps $\boldsymbol{x} = [x_0, \ldots, x_{L-1}]$, where
  - $x_n = x(nT)$ for bandwidth $T^{-1} = 256$ MHz,
  - $x(t) = h(t) * p_{\text{RC}}(t)$, and
  - $h(t)$ is generated randomly according to the 802.15.4a method.



Simplified Channel Model:

Consider a discrete-time channel that is

- *block-fading* with block size N,
- *frequency-selective* with L taps (where $L < N$),
- *sparse* with S non-zero complex-Gaussian taps (where $0 < S \leq L$),

where *both the channel coefficients and support are unknown* to the receiver.

Important questions:

1. What is the capacity of this channel?
2. How can we build a *practical* comm system that operates near this capacity?

Noncoherent Capacity of the Sparse Channel:

For the unknown N-block-fading, L-length, S-sparse channel described earlier, Pachai-Kannu & Schniter [1] established that

1. In the high-SNR regime, the ergodic capacity obeys

$$C_{\text{sparse}}(\text{SNR}) = \frac{N-S}{N}\log(\text{SNR}) + \mathcal{O}(1).$$

2. To achieve the prelog factor $R_{\text{sparse}} = \frac{N-S}{N}$, it suffices to use
   - pilot-aided OFDM (with N subcarriers, of which S are pilots)
   - with (necessarily) *joint* channel estimation and data decoding.

Key points:

- The effect of *unknown channel support* manifests only in the $\mathcal{O}(1)$ offset.
- While [1] uses constructive proofs, the scheme proposed there is impractical.

[1] A. Pachai-Kannu and P. Schniter, "On communication over unknown sparse frequency selective block-fading channels," arXiv 1006.1548, June 2010.

Practical Communication over the unknown Sparse Channel:

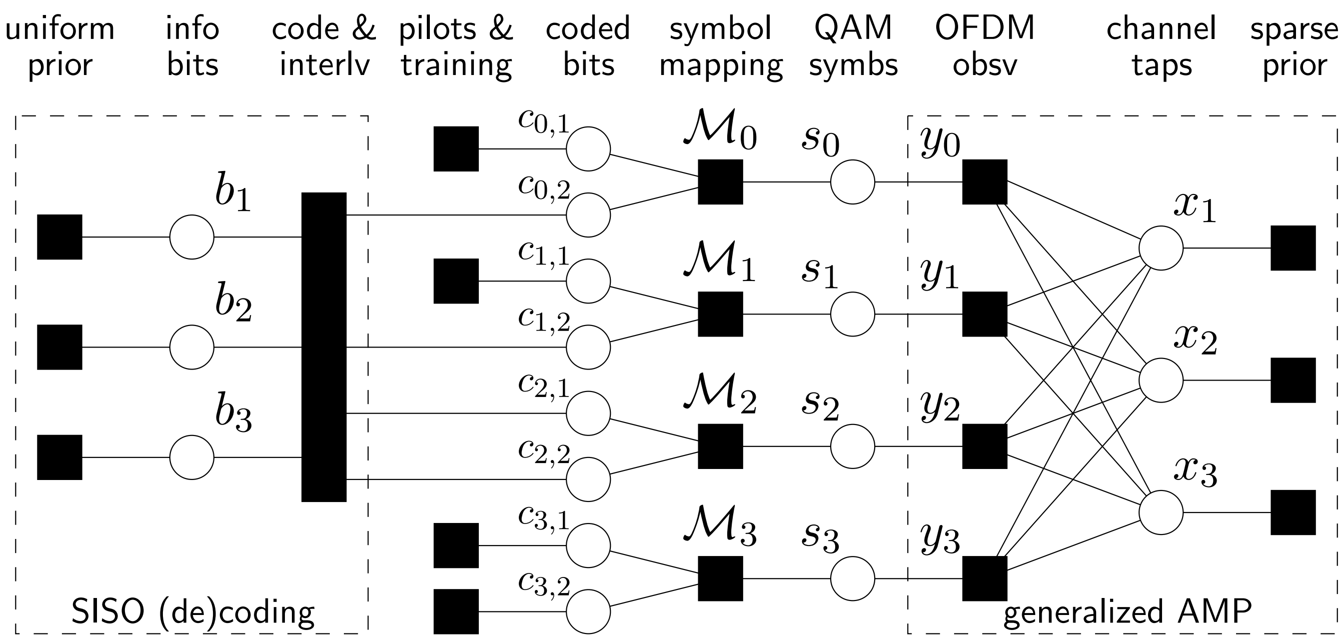
We now propose a communication scheme that. . .

- is practical, with complexity of only $\mathcal{O}(N \log_2 N + N|\mathbb{S}|)$ per block,
- (empirically) achieves the optimal prelog factor $R_{\text{sparse}} = \frac{N-S}{N}$,
- significantly outperforms LASSO-based "compressed channel sensing" (CCS).

Our scheme uses. . .

- a conventional transmitter: pilot-aided BICM OFDM,
- a novel receiver: based on loopy *belief propagation* (BP)
  - key enabler: "**generalized AMP**" algorithm [Rangan 10]
    building on [Guo/Wang 07, Donoho/Maleki/Montanari 09, Bayati/Montanari 10]

Factor Graph for pilot-aided BICM-OFDM:



○ = random variable ■ = posterior factor

To jointly infer all random variables, we perform loopy-BP via the sum-product algorithm, using carefully chosen message approximations in each dashed box.

[2] P. Schniter, "Belief-propagation-based joint channel estimation and decoding for spectrally efficient communication over unknown sparse channels," arXiv:1012.4519, Dec 2010.

Numerical Results — Perfectly Sparse Channel:

Transmitter:

- LDPC codewords with length ~ 10000 bits.
- 2^M-QAM with $2^M \in \{4, 16, 64, 256\}$ and multi-level Gray mapping.
- OFDM with $N = 1024$ subcarriers.
- P "*pilot subcarriers*" and T "*training MSBs*."

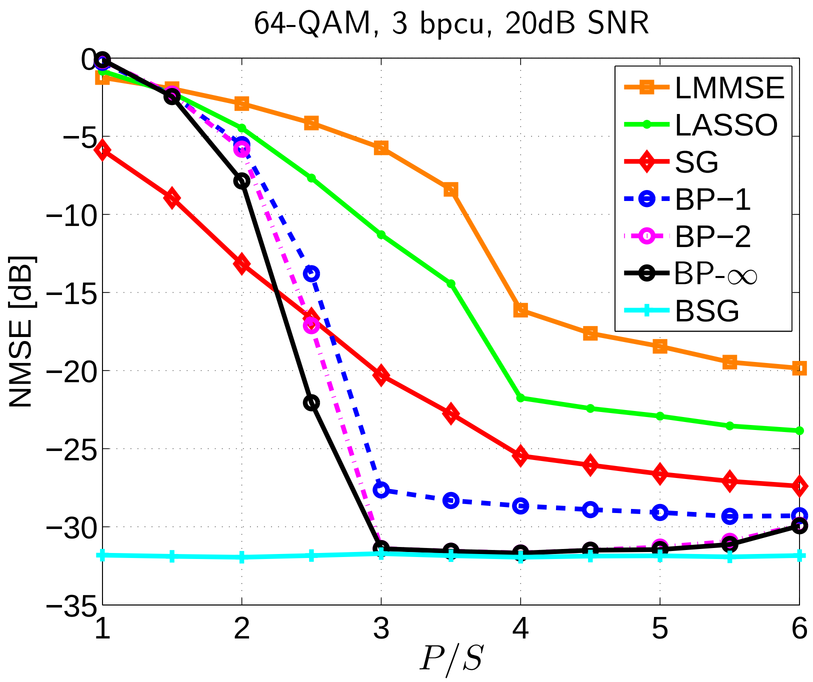
Channel:

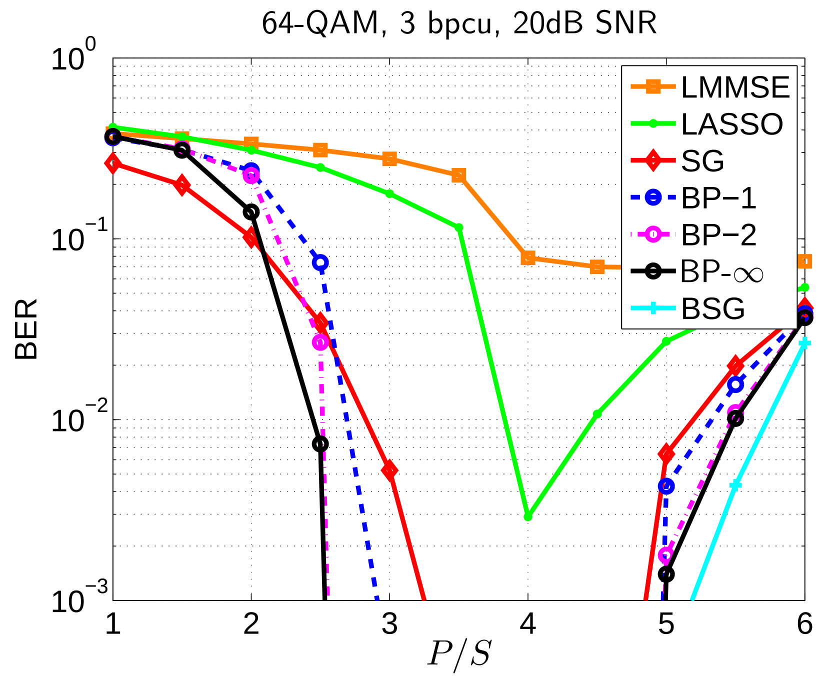
- Length $L = 256 = N/4$.
- Sparsity $S = 64 = L/4$.

Reference Schemes:

- Pilot-aided LASSO was implemented using SPGL1 with genie-aided tuning.
- Pilot-aided LMMSE, support-aware MMSE, and bit+support-aware MMSE channel estimates were also tested.

NMSE & BER versus pilot/sparsity ratio (at SNR=20dB, $T=0$):



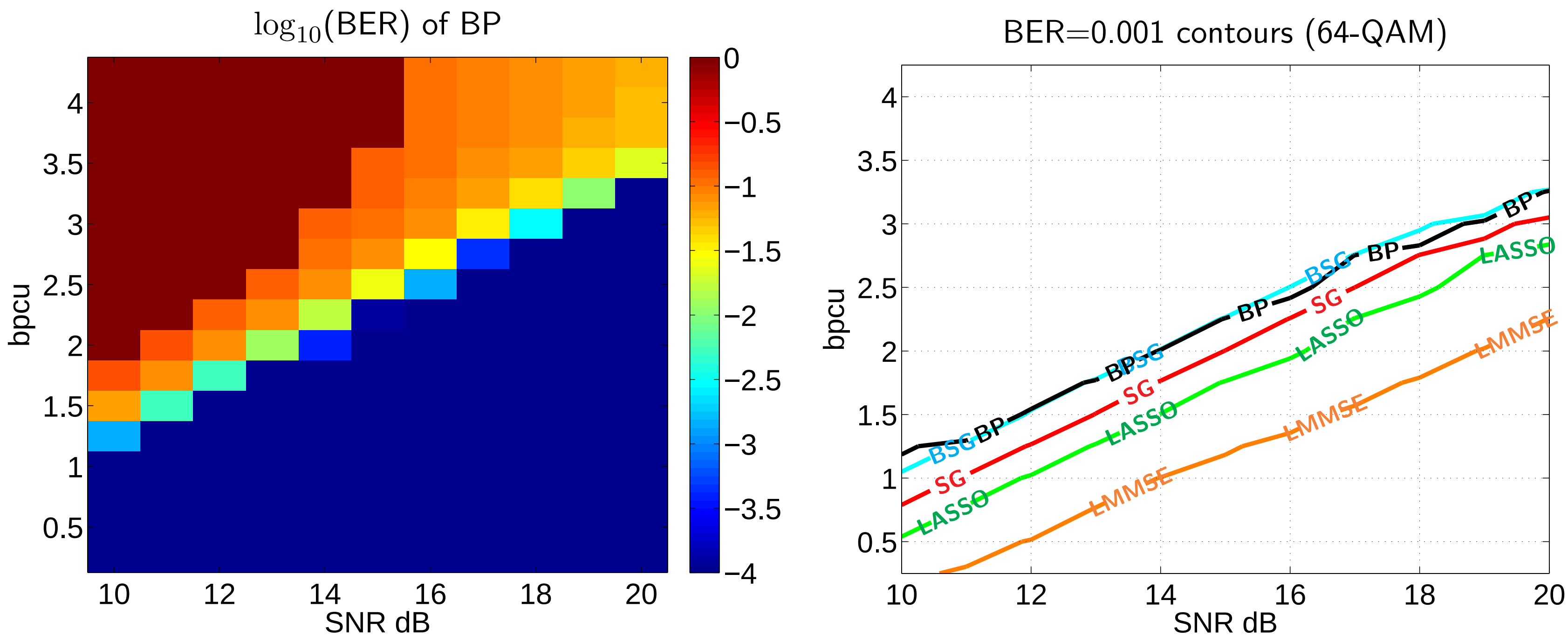


implementable schemes	reference schemes
LMMSE = LMMSE-based CCS LASSO = LASSO-based CCS BP-n = BP after n turbo iterations	SG = support-aware genie BSG = bit- and support-aware genie

Observations:

- For pilot-based methods, channel estimation MSE improves monotonically with P.
- As P grows very large, BER suffers due to decrease in code-rate (since bpcu is fixed).
- For CCS, $P=4S=L$ gives best tradeoff. (Note $P=L$ is the Nyquist pilot rate!)

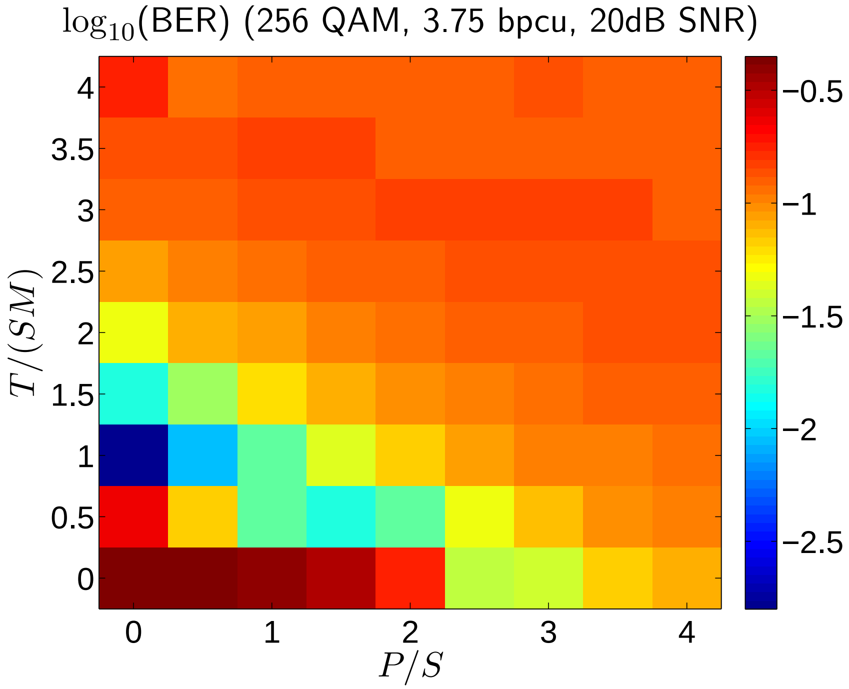
Outage Rate versus SNR (with $P\!=\!4S\!=\!L$ pilots and $T\!=\!0$ training):

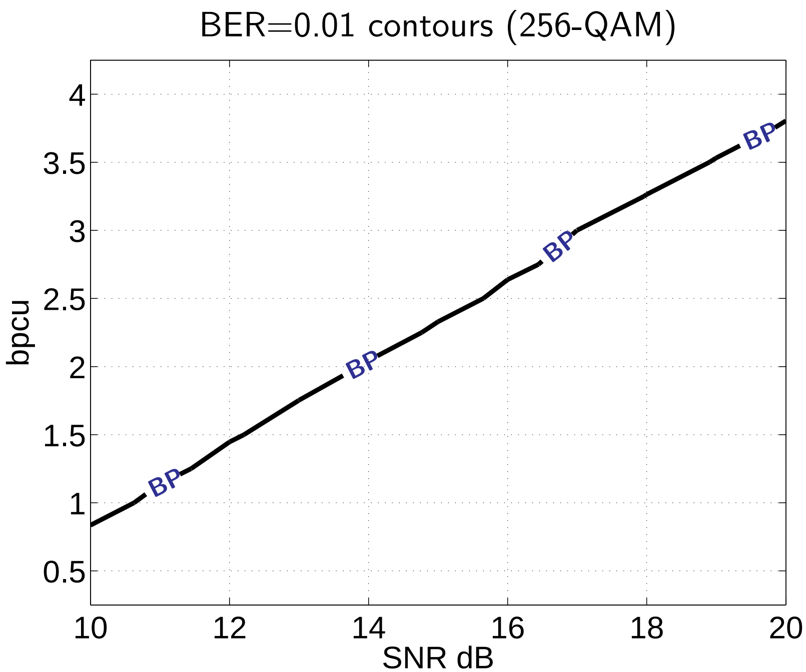


Key points:

- Belief propagation outperforms both LASSO and the support genie (SG).
- Belief propagation performs nearly as well as the bit+support-aware genie (BSG).
- With $P = L$, all approaches yield prelog factor $R = \frac{N-L}{N} = \frac{3}{4}$, which falls short of the optimal $R_{\text{sparse}} = \frac{N-S}{N} = \frac{15}{16}$.

Bit-Rate versus SNR (with $P=0$ pilots & $T=SM$ training MSBs):

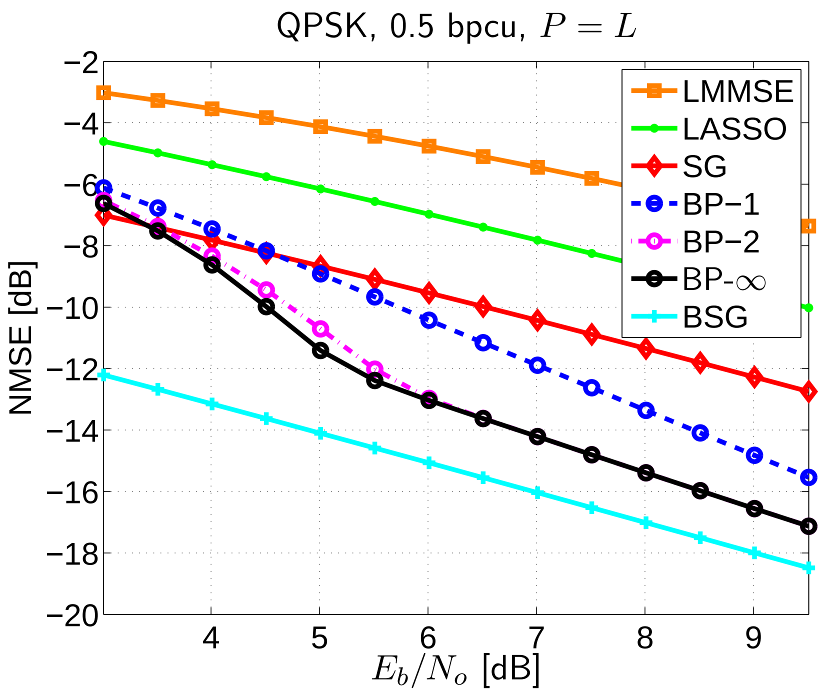


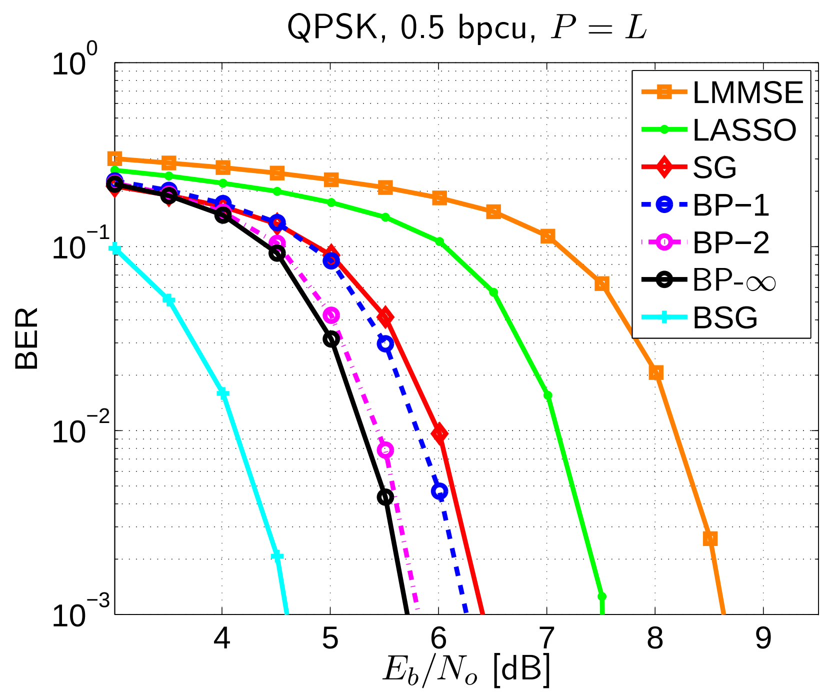


Key points:

- BP favors $P=0$ pilot subcarriers and $T=SM$ training MSBs.
- BP achieves the sparse-channel's capacity-prelog factor $R_{\text{sparse}} = \frac{N-S}{N}$.

BER versus SNR (with $P=4S=L$ pilots and $T=0$ training):





implementable schemes	reference schemes
LMMSE = LMMSE-based CCS LASSO = LASSO-based CCS BP-n = BP after n turbo iterations	SG = support-aware genie BSG = bit- and support-aware genie

Key points:

- Sparsity can be exploited even at very low SNR.
- BP is only about 1dB worse than oracle bound, and 2dB better than LASSO.

Conclusions:

- Channel sparsity manifests at larger communication bandwidths, as seen by 802.15.4a channel models.
- This motivates analysis of the (idealized) N-block, L-length, S-sparse channel, for which we derived the high-SNR ergodic noncoherent capacity

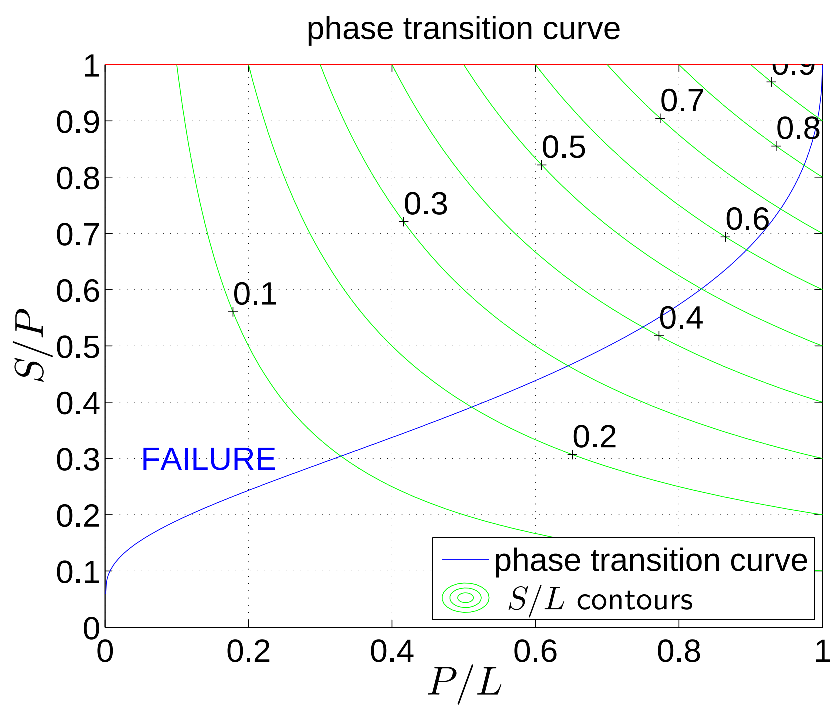
$$C_{\text{sparse}}(\text{SNR}) = \frac{N-S}{N}\log(\text{SNR}) + \mathcal{O}(1).$$

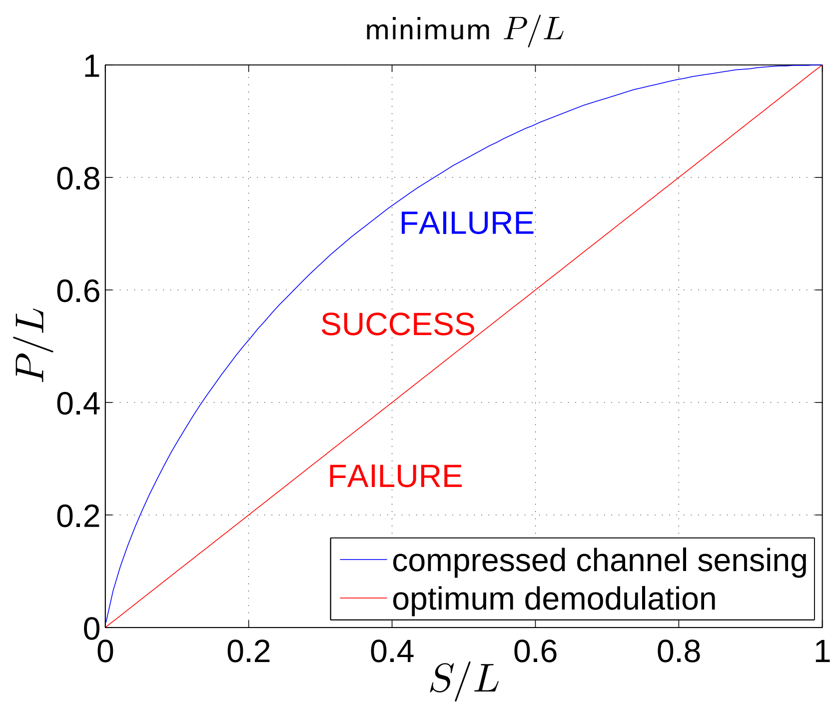
- We then proposed a factor-graph receiver with
  - outage rate matching the capacity prelog factor $\frac{N-S}{N}$,
  - BER performance within 1dB of oracle bound and 2dB beyond LASSO,
  - very low complexity: $\mathcal{O}(N\log_2 N + N2^M)$ per N-block.
- Our approach leverages recent work on "generalized approximate message passing" proposed in the context of compressed sensing.
- With 802.15.4a channels (not shown here), BER performance remains within 1dB of perfect-CSI bound and 4dB better than LASSO.

Thanks!

Performance limits of LASSO-based compressed channel sensing:

In the large system limit (i.e., $L, S, P \to \infty$) with i.i.d $\boldsymbol{F}_{\text{p}}$, the Donoho/Tanner *phase transition curve* (PTC) predicts where noiseless LASSO will fail:





The PTC translates directly to a minimum required P/L for CCS (as SNR$\to\infty$).

[5] D. L. Donoho and J. Tanner, "Observed universality of phase transitions in high-dimensional geometry, with implications for modern data analysis and signal processing," *Phil. Trans. Royal Soc. A*, 2009.