Adaptive Damping and Mean Removal for the Generalized Approximate Message Passing Algorithm

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Introduction

Goal: Make convergence of generalized approximate message passing (GAMP) [1] robust to the matrix A.

GAMP is a computationally efficient approach to MAP or approximate MMSE inference of $x \in \mathbb{R}^N$ that exploits:

- known separable signal prior $p_x(x) = \prod_i p_{x_i}(x_i)$;
- known separable $p_y(y | x) = \prod_m p_{y_m}(y_m | x_m)$, where $z_m = a_m x$.
- sufficiently large, dense, and random $A = \{a_1, \ldots, a_N\}$.  

GAMP admits rigorous analysis in the large-system limit for i.i.d zero-mean sub-Gaussian $A$.

For some $A$, GAMP can diverge.

- "Swept" GAMP (SwGAMP) [2] improves convergence by estimating $(z_m)_{m=1}^M$ and $(\bar{z}_m)_{m=1}^M$ sequentially, so it cannot exploit BLAS or fast matrix operations, and thus can be slow.

Adaptively Damped GAMP

To make convergence robust to the operator $A$, we damp the updates of GAMP.

- Not enough damping can result in GAMP divergence, while too much damping can slow GAMP convergence.

Thus, we damp adaptively by monitoring the cost function.

When GAMP converges, it minimizes the cost $[3,4]$:

MAP:

$$J_{\text{MAP}}(\hat{x}) \triangleq -\ln p_x(y | A \hat{x}) - \ln p_x(\hat{x})$$

MMSE:

$$J_{\text{MMSE}}(b_k, b_{k+1}) \triangleq D(b_k | p_x) + D(b_{k+1} | p_x z \approx z^{-1}) + H_N(b_k)$$ s.t. $E[|z|] = A E[|x|]$.

To evaluate $J_{b_k}(b_k, b_{k+1})$ before convergence, we need to evaluate certain fixed-point quantities, and for this we use a Newton-based method.

Mean Removal

To mitigate problems with non-zero mean $A$, we augment $z = A x$ via

$$\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_M \\ \delta z_m \end{bmatrix} = \begin{bmatrix} A \\ b_1 \gamma \\ b_2 \gamma \\ \vdots \\ x \end{bmatrix} \begin{bmatrix} m+1 \\ m+2 \\ \vdots \\ N+1 \\ \delta x \end{bmatrix}$$

- $\gamma$ are row averages of $A$.
- $\gamma$ are shifted column averages of $A$.
- $b_k$ equalize row/column norms.
- $\delta z$ has zero row and column averages.

with trivial GAMP signal priors and strict likelihoods on the augmented entries, i.e.,

$$p_{z_m}(z_m | x_m) \equiv \delta(z_m) \text{ for } m \in \{M+1, M+2\}.$$  

Selected References


Numerical Results

Goal: Recover $N = 1000$ Bernoulli-Gaussian $x$ under various $p_x(y | a x)$ and “difficult” $A$:

(a) Non-zero mean: $x_\text{tt} \sim \mathcal{N}(\mu, \Sigma)$ for a specified $\mu \neq 0$.
(b) Low-rank product: $A = UV$ with $U \in \mathbb{R}^{N \times R}, V \in \mathbb{R}^{M \times R}$, and $x \sim \mathcal{N}(0, 1)$, for a specified $R$.
(c) Column-correlated: $A = \mathbb{U} \mathbb{V}$ where $\mathbb{U}$ and $\mathbb{V}$ are the left and right singular vector matrices of an i.i.d $\mathcal{N}(0, 1)$ matrix and $(\mathbb{U}^H \mathbb{V})/M \sim (\kappa)_{i \sim [MN]}$, with a specified condition number $\kappa > 1$.

Report average NMSE $\tilde{x} = \mathbb{E}[\|x - \tilde{x}\|^2_2]$, runtime, and iterations over 100 realizations.

<table>
<thead>
<tr>
<th>Method</th>
<th>AL</th>
<th>AWGN</th>
<th>$\mu$</th>
<th>$\rho$</th>
<th>$\tilde{\Sigma}$</th>
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<td>0.99</td>
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<td>277.23</td>
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<td>6.72</td>
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</tbody>
</table>

Table 1: Average runtime (in seconds) and $\#$ iterations for various problem types and matrix types.

- The original GAMP algorithm diverges for mildly non-iid $A$.
- The proposed Mean-removed adaptive damping GAMP (MAD-GAMP) is much more robust to non-iid $A$.
- SwGAMP is also robust to some types of non-iid $A$, but less so for non-zero-mean matrices.
- SwGAMP takes fewer iterations to converge, but (M)AD-GAMP has faster average runtime.

Acknowledgments

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