Reduced Complexity Tracking of Doubly-Selective Channels

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Problem:
Estimation and tracking of doubly-selective channels.

Approach:
Use current and previous pilots as well as previously decoded data.

Goal:
Near optimal performance without large matrix inversions.
• Transmission Model:

\[ i - 1 \quad \text{FRAME} \]

\[ \cdots \quad \text{DATA} \quad \cdots \]

\[ N_f \]

• Reception Model:

\[ y_{n}^{(i)} = \sum_{d=0}^{N_h-1} h_{n,d}^{(i)} t_{n-d} + v_{n}^{(i)} \]

\[ y_{d}^{(i)} = T_{d}^{(i)} h_{d}^{(i)} + v_{d}^{(i)} \]

\[ y_{p}^{(i)} = T_{p} h_{p}^{(i)} + v_{p}^{(i)} \]
Reception Model (cont.):

\[ y^{(i)} = \begin{bmatrix} y^{(i)}_d \\ y^{(i)}_p \end{bmatrix} = \begin{bmatrix} T^{(i)}_d \\ T^{(i)}_p \end{bmatrix} \begin{bmatrix} h^{(i)}_d \\ h^{(i)}_p \end{bmatrix} + \begin{bmatrix} v^{(i)}_d \\ v^{(i)}_p \end{bmatrix} \]

- Estimate \( h^{(i)}_d \) given \( \{y^{(j)}\}_{j \leq i} \) and \( \{T^{(j)}_d\}_{j \leq i-1} \) and \( T_p \).

(assume channel process is WSS)
Background:

- Pilot-aided Wiener Estimation (PW)
- Pilot-aided Decision-directed Wiener Estimation (PDW)
- Pilot-aided Decision-directed Kalman Estimation (PDK)
Pilot-aided Wiener Estimation:

- For $M$ frames:

$$
\begin{bmatrix}
  y_p^{(i)} \\
  \vdots \\
  y_p^{(i-M)} \\
\end{bmatrix}
= \begin{bmatrix}
  T_p \\
  \vdots \\
  T_p \\
\end{bmatrix}
\begin{bmatrix}
  h_p^{(i)} \\
  \vdots \\
  h_p^{(i-M)} \\
\end{bmatrix}
+ \begin{bmatrix}
  v_p^{(i)} \\
  \vdots \\
  v_p^{(i-M)} \\
\end{bmatrix}
$$

$$
\hat{h}_d^{(i)} \bigg|_{\text{pilot}} = \left( R_{h_p,h_d} T_M^H (T_M R_{h_p,h_p} T_M^H + \sigma_v^2 I)^{-1} y_p^{(i)} \right)_{\text{LTI}}.
$$

- Advantage: time-invariant estimator.

- Limitation: if $f_d > \frac{1}{2N_f}$, channel is undersampled.
Pilot-aided Decision-directed Wiener Estimation:

- For $M$ frames:
  \[
  \begin{bmatrix}
  y_p^{(i)} \\
  y^{(i-1)} \\
  \vdots \\
  y^{(i-M)}
  \end{bmatrix}
  =
  \begin{bmatrix}
  T_p \\
  T^{(i-1)} \\
  \vdots \\
  T^{(i-M)}
  \end{bmatrix}
  \begin{bmatrix}
  h_p^{(i)} \\
  h^{(i-1)} \\
  \vdots \\
  h^{(i-M)}
  \end{bmatrix}
  +
  \begin{bmatrix}
  v_p^{(i)} \\
  v^{(i-1)} \\
  \vdots \\
  v^{(i-M)}
  \end{bmatrix}
  \]

  \[
  \hat{h}_{d,(i)} \bigg|_{\text{wiener}} = R_{\hat{h}_w,h_d} T_w^{(i)H} (T_w R_{\hat{h}_w,h_w} T_w^{(i)H} + \sigma_v^2 I)^{-1} y_w^{(i)}.
  \]

- Limitation: large matrix inversion per each frame.
Pilot-aided Decision-directed Kalman Estimation:

- AR dynamic model: \( h^{(i)} = A_k h^{(i-1)} + D_k w_k^{(i-1)} \).

- Current observation: \( y_k^{(i-1)} = \begin{bmatrix} y_p^{(i)} \\ y_d^{(i-1)} \end{bmatrix} \).

- Together

\[
y_k^{(i-1)} = C_k^{(i-1)} h^{(i-1)} + v_k^{(i-1)},
\]

where \( C_k^{(i-1)} \) contains data, pilots and \( A_k \),
and where \( v_k^{(i-1)} \) contains noise and channel variation.

- Note: slightly non-standard due to \( S_k = E \{ w_k^{(i-1)} v_k^{(i)H} \} \neq 0 \).
Kalman Estimation (cont.):

- MMSE optimal estimate of $h^{(i)}$ using $\{y_k^{(i-1)}, ..., y_k^{(0)}\}$ is

$$
\hat{h}^{(i)} \mid_{\text{kalman}} = A_k \hat{h}^{(i-1)} \mid_{\text{kalman}} + L_k^{(i-1)} \{y_k^{(i-1)} - C_k^{(i-1)} \hat{h}^{(i-1)} \mid_{\text{kalman}} \}
$$

$$
L_k^{(i-1)} = (A_k P_k^{(i-1)} C_k^{(i-1)H} + D_k S_k)
$$

$$
\times (C_k^{(i-1)} P_k^{(i-1)} C_k^{(i-1)H} + R_k)^{-1}
$$

$$
P_k^{(i-1)} = \sigma_w^2 D_k D_k^H + A_k P_k^{(i-2)} A_k^H
$$

$$
- L_k^{(i-2)} (C_k^{(i-2)} P_k^{(i-2)} A_k^H + S_k^H D_k^H)
$$

where $R_k = E\{v_k^{(i-1)} v_k^{(i-1)H}\}$

with the initializations $P_k^{(0)} = E\{h^{(0)} h^{(0)H}\}$ and $\hat{h}^{(0)} \mid_{\text{kalman}} = 0$.

- Kalman estimation requires $N_f \times N_f$ matrix inversion per frame.
Low Complexity Predictor (LCP):

1. Compute Kalman smoothed channel in previous frame.

2. Wiener predict current channel using $M$ past smoothed frames and current pilots.
Low Complexity Predictor (cont.):

Complexity of each stage (per frame):

1. Kalman Smoothing: \(\frac{N_f}{L}\) matrix inversions of size \(L \times L\).
   
   Note: \(L\) limits AR model size.

2. With some approximations, Wiener predictor is LTI.
   
   \(\Rightarrow\) no matrix inversion in prediction stage)
Approximations giving LTI Wiener Prediction:

\[
\begin{bmatrix}
y_p(i) \\
h_p(i-1) \\
\vdots \\
h_p(i-M) \\
y_l(i)
\end{bmatrix}
= 
\begin{bmatrix}
T_p & I & \cdots & I \\
& & & B \\
& & & \vdots
\end{bmatrix}
\begin{bmatrix}
h_p(i) \\
h_p(i-1) \\
\vdots \\
h_p(i-M) \\
h_l(i)
\end{bmatrix}
+ 
\begin{bmatrix}
v_p(i) \\
e^{(i-1)}
\end{bmatrix}
\]

Assumptions on smoothing error \(e^{(j)}\):

1. White smoothing error (across time and lag)

\[
E\{e^{(j)}e^{(k)H}\} = \sigma_v^2 I \delta(j - k).
\]

2. Smoothing errors uncorrelated with channel (across time and lag)

\[
E\{h^{(j)}e^{(k)H}\} = 0.
\]

\[\Rightarrow\] Prediction: \(\hat{h}_d^{(i)}|_{lcp} = \left[R_{h_l,h_d}^H(BR_{h_l,h_d}B^H + \sigma_v^2 I)^{-1}y_{l|^i}\right]_{\text{LTI}}\).
Modifications of LCP:

1. LCP with Kalman prediction (LCKP):
   - LTI $M$-frame Wiener Prediction replaced by steady-state Kalman prediction.
   - Uses only the previous smoothed frame.

2. LCP with Doppler-lag coefficients (LCPD):
   - Transform previous time-lag estimates to Doppler-lag representation.
   - Keep only significant Doppler indices for prediction of the current time-lag channel.
   - Factor of (almost) $\frac{1}{2f_d}$ savings !!

\implies Reduce memory and computation.
Simulation Results:

- Simulation parameters $N_f = 80$, $N_h = 8$.
- Relative algorithm complexity:

<table>
<thead>
<tr>
<th>Technique</th>
<th>Size of matrix inversion per frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>PW</td>
<td>n.a.</td>
</tr>
<tr>
<td>PDW</td>
<td>$(MN_f + N_h) \times (MN_f + N_h)$</td>
</tr>
<tr>
<td>PDK</td>
<td>$N_f \times N_f$</td>
</tr>
<tr>
<td>LCP, LCPD and LCKP</td>
<td>$L \times L$</td>
</tr>
</tbody>
</table>

- Typical values are $L = 10$ and $M = 2$.
- Benchmark: IIR Wiener prediction with persistent Kronecker-delta pilot transmission (PTP).
Effect of SNR @ $f_d = 0.01$

Effect of $f_d$ @ SNR=15 dB
Effect of $M @ f_d = 0.01$

Effect of $L @ f_d = 0.01$
Performance of LCPD

Performance of LCKP

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**Performance of LCPD**

- LCPD M=2, L=5
- LCP M=2, L=5
- LCPD M=2, L=10
- LCP M=2, L=10

**Performance of LCKP**

- LCP M=2, L=10
- LCKP L=10
- PDK
Conclusions:

- Performance of pilot-only estimation is limited by training interval.
- Decision-directed Wiener and Kalman approaches require large matrix inversion.
- Proposed a low-complexity two-stage estimator:
  1. Computes smoothed past channel estimates.
  2. LTI Wiener prediction of the current channel (or steady-state Kalman prediction).

Requires only small matrix inversion.

- Numerical results show that LCP performance is close to that of Kalman and Wiener estimators.