Adaptive Compressive Noncoherent Change Detection

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Outline:

1. Noncoherent Change Detection

2. Compressive Noncoherent Change Detection

3. Adaptive Sensing
Change Detection:

• Given a **reference signal** (or image) \( r \in \mathbb{C}^N \) and a **signal-under-test** \( x \in \mathbb{C}^N \), how do we detect the pixels that are changed?

• Set up a model:

\[
\forall n: \quad x_n = s_n c_n + (1 - s_n)(r_n + d_n) \quad \text{with unknown}\ldots
\]

\[
\begin{cases}
    s_n \in \{0, 1\} & \text{change indicators} \\
    c_n & \text{values of changed pixels} \\
    d_n & \text{small variations at “unchanged” pixels,}
\end{cases}
\]

where \( c_n \sim \text{i.i.d } p_C(\cdot) \), \( d_n \sim \text{i.i.d } p_D(\cdot) \), \( s_n \sim \text{i.i.d } p_S(\cdot) \)

• Various optimal detectors can be formulated as a **likelihood ratio test**:

\[
LR(x_n, r_n) = \frac{p(x_n, r_n | s_n = 1)}{p(x_n, r_n | s_n = 0)} = \frac{p_C(x_n)}{p_D(x_n - r_n)}
\]

• Intuition: look for outliers in **difference signal** \( x_n - r_n \).
Noncoherent Change Detection:

- Now suppose that $r$ and $x$ are phase incoherent.

- One application is radar image change detection in foliage, where pixel phases can vary significantly across looks due to wind-induced motion.

- A possible model is:

\[
\forall n : x_n = s_n c_n + (1 - s_n)(r_n e^{j\theta_n} + d_n)
\]

where $\theta_n \sim i.i.d \mathcal{U}[0, 2\pi)$ implies complete phase uncertainty.

- Change detection is still a textbook problem,

\[
\text{GLR}(x_n, r_n) : \frac{p_C(x_n)}{\min_{\theta_n} p_D(x_n - r_n e^{j\theta_n})} = \frac{p_C(|x_n|)}{p_D(|x_n| - |r_n|)} \quad \text{for circular $C$ & $D$}.
\]

Intuition: look for outliers in magnitude difference $|x_n| - |r_n|$. 
Compressive Noncoherent Change Detection:

Now consider noisy compressive linear observations $y \in \mathbb{C}^M$ with $M < N$:

$$y = Ax + w, \quad w \sim \mathcal{CN}(0, \nu^w I)$$

Challenges:

- The signal $x$ is not directly observed:
  - $\Rightarrow$ Cannot implement standard noncoherent detection without $|x_n|$.

- The signal $x$ is generally non-sparse/compressible:
  - $\Rightarrow$ Cannot use standard sparse-reconstruction to recover $x$ from $y$.

Opportunities:

- With sparse changes, we know most magnitudes $|x_n|$, approximately, and thus have a strong prior on $x$.

- In practice, the change-pattern $s$ is not i.i.d, but spatially clustered.
Proposed Approach:

We assume the generative mixture model

\[ x_n = s_n c_n + (1 - s_n)(r_n e^{j\theta_n} + d_n) \quad \text{and} \quad y = Ax + w \]

\[
\begin{align*}
    s_n &\sim \{0, 1\} \text{ Markov} \\
    c_n &\sim \mathcal{CN}(0, \nu^r) \text{ i.i.d} \\
    d_n &\sim \mathcal{CN}(0, \nu^d) \text{ i.i.d, } \nu^d \ll \nu^r \\
    \theta_n &\sim \mathcal{U}[0, 2\pi) \text{ i.i.d}
\end{align*}
\]

leading to the factor graph

and then perform inference via “turbo” approximate-message-passing.
Numerical Example:

- We compare two schemes:
  
  ![Flowchart](image)

- Simulation parameters:
  - signal length $N = 200$,
  - changes: 1D Markov chain with rate 0.1 and avg cluster length $= 11$.
  - reference-to-disturbance ratio $\frac{\nu_r}{\nu_d} = 30$ dB,
  - signal-to-noise ratio $= 15$ dB,
  - sensing matrix: $\{A_{mn}\} \sim \text{i.i.d } \mathcal{N}(0, M^{-1})$
Numerical Example:

- **AMP-based joint reconstruction-and-change-detection** outperforms the conventional method in both NSER and NMSE, even when the conventional detector can exploit clustered changes.
Adaptive Sensing:

- Now consider the **multi-step** observation model
  \[ y_t = A_t x + w_t, \quad t = 1 \ldots T \]
  and the **adaptation** of \( A_t \) (s.t. \( \|A_t\|_F^2 \leq \mathcal{E} \))
  using knowledge gained from previous measurements \( y_{t-1} \triangleq \{ y_{\tau} \}_{\tau=1}^{t-1} \).

- To infer \( x \), the approach known as **Bayesian experimental design** chooses \( A_t \) to maximize the **mutual information** \( I(X; Y_t) \) between random vectors \( X \sim p(x|y_{t-1}) \) and \( Y_t \sim p(y_t|y_{t-1}; A_t) \).

- For **Gaussian signal and noise**, we previously established that the design of MI-maximizing \( A_t \) is a **waterfilling** problem [Schniter CAMSAP 11].

- Since **turbo-AMP** produces an accurate **Gaussian posterior approximation**, it partners well with waterfilling-based adaptation. For **structured-sparse signal recovery**, this combination has been shown to yield recovery-MSE near **oracle bounds** [Schniter CAMSAP 11].
NMSE versus cumulative # measurements [Schniter CAMSAP 11]:

- Note gains from structured sparsity, adaptivity, and the combination.
- Adaptive turbo-AMP performs 1.5 dB from the support-oracle bound!
Waterfilling-based Adaptation for Noncoherent Change Detection:

- We now add **waterfilling-based adaptive sensing** to our **noncoherent change detection** scheme.

### Conventional

\[
\begin{align*}
  & y 
  \xrightarrow{\text{MMSE reconstruction}} \hat{x} 
  \xrightarrow{\text{noncoherent detection}} \hat{s} \\
  & r
\end{align*}
\]

### Proposed

\[
\begin{align*}
  & y_t \\
  & r
  \xrightarrow{\text{adaptation of } A_t} \\
  & y_t 
  \xrightarrow{\text{joint noncoherent reconstruction} \& \text{change detection (turbo-AMP)}} \hat{x}_t \\
  & r 
  \xrightarrow{\hat{s}_t}
\end{align*}
\]

- To minimize **signal-recovery normalized MSE** (NMSE), we perform waterfilling based on a Gaussian approximation of \( p(x|y_{t-1}) \).
- To minimize the **normalized change-support error rate** (NSER), we perform waterfilling based on a Gaussian approximation of \( p(s|y_{t-1}) \).
Numerical Example:

Notice that:

- the matrices designed to improve the recovery of the change pattern $s$ do significantly improve the NSER (left), and
- those designed to improve the recovery of signal $x$ do improve NMSE (right),
- but not vice versa!