Channel Estimation and Precoder Design for mmWave Communications: The Sparse Way

Phil Schniter

THE OHIO STATE UNIVERSITY

Supported in part by NSF grants CCF-1018368 and CCF-1218754.
(And thanks to Robert Heath, Jr. for lending me a few graphics!)

Asilomar — Nov’14
Potential:
- Huge amount of bandwidth available \(\rightarrow\) Huge throughput?
- Many antennas fit in a small form-factor \(\rightarrow\) Massive MIMO?

Challenges:
- Path-loss/shadowing are \(\sim 40\)dB worse than in microwave bands.
- Huge bandwidth leads to serious implementational issues.

---

Physical measurements in dense urban NLOS environments suggest that mmW channels are extremely sparse.\(^2\)

Can expect at most 3-4 clusters, with very little angle/delay-spread per cluster.

mmW uses Massive Arrays

- To counter path-loss, **massive arrays** are used at both Tx and Rx.
- The goal is **beamforming gain**, not spatial multiplexing gain. (These systems are **power-limited**, not bandwidth-limited.)
- **Narrow beams** also reduce fading, multipath, and interference.
Contributions

We propose...

- sparsity-exploiting low-complexity space-time channel estimation,
- mutual-information-maximizing beamforming & waterfilling,
- aperture shaping to ensure that physical sparsity manifests as MIMO channel sparsity.
System Model

$N \times N_r$ matrix of received samples at block $t$:

\[
Y_t = \sum_{d=0}^{N_d-1} J_d X_t H_d + W_t
\]

where

$J_d$ : cyclic $d$-delay matrix

$X_t$ : $N \times N_t$ transmitted signal at block index $t$

$H_d$ : $N_t \times N_r$ MIMO channel at delay $d$

$W_t$ : AWGN of variance $\nu_w$

and

$N_d$ : channel delay spread

$N$ : length of block transmission (plus $N_d$-length cyclic prefix)

$N_t$ : number of transmit antennas

$N_r$ : number of receive antennas.
MIMO Channel Models

Per-path channel parameters:

\[(\beta_l, \tau_l, \theta_{t,l}, \theta_{r,l}) = (\text{gain}_l, \text{delay}_l, \text{transmit-angle}_l, \text{receive-angle}_l)\]

MIMO channel matrix at delay \(d\):

\[H_d = \sum_{l=1}^{L} \beta_l \ p_{\text{srrc}}(d \ T_c - \tau_l) \ f_{N_t}(\theta_{t,l}) \ f_{N_r}(\theta_{r,l})^H\]

note: \(\tau_l, \theta_{t,l}, \theta_{r,l}\) are not discrete!

Virtual\(^3\) MIMO channel matrix at delay \(d\):

\[G_d = F_{N_t}^H \ H_d F_{N_r}\]

\(F_{N_t}, F_{N_r}\) : unitary DFT matrices.

---

Sparsity of the Virtual MIMO Channel

The elements of $G_d$ are the complex channel gains at discrete transmit and receive angles and discrete delay $d$.

Example of virtual MIMO coefficients due to a single path ($N_t = 11 = N_r$):

Sparse physical scattering does not yield sparse virtual channel coefs!
We can restore angle-domain sparsity via aperture shaping,\(^4\) i.e., windowing of the transmit and receive antenna gains:

\[
\overline{G}_d = F_{N_t}^H \text{diag}(w_t) H_d \text{diag}(w_r^*) F_{N_r}
\]

where max-SINR windows \(w_t, w_r\) are solved via a generalized-eigenvector problem.

To facilitate low-complexity channel estimation, we propose to

1. construct the space-time training signal as $X_t = F_N^H S_t F_{N_t}$, where $S_t$ has i.i.d entries in $\{1, j, -1, -j\}$,

2. FFT-process the observations, giving the observation structure

$$F_N Y_t F_{N_r} = \text{AWGN}(\nu_w) + \sqrt{N} \left[ \text{diag}(s_{t,1}) F_{N_d} \cdots \text{diag}(s_{t,N_t}) F_{N_d} \right] [g_1 \cdots g_{N_r}]$$

where $g_j \in \mathbb{C}^{N_d N_t \times 1}$ contains virtual chan coefs for $j$th Rx antenna,

3. and, if needed, stack measurements across $T$ blocks, giving a total of $NT$ scalar measurements per $N_d N_t$ scalar unknowns.

Note the near isometry & fast implementation of the training operator.
To Compress or Not To Compress?

Sub-Nyquist regime ($NT < N_t N_d$):
- Low training overhead.
- Requires a sparse reconstruction algorithm.

Super-Nyquist regime ($NT \geq N_t N_d$):
- Higher training overhead.
- Allows classical linear (e.g., LS, LMMSE) estimation.
- Sparse reconstruction can improve performance at very low SNR.

For example, 802.11ad (60GHz) standard uses $N = 512$ and $N_d = 128$. So $N_t = 64$ Tx antennas $\Rightarrow T \geq 16$ blocks for Nyquist sampling.
Numerical Examples

System parameters:

\[ N = 512 \quad \text{block length} \]
\[ N_d = 128 \quad \text{channel delay spread} \]
\[ N_t = 64 \quad \text{transmit antennas} \]
\[ N_r = 64 \quad \text{receive antennas (⇒ SNR gain = 18dB)} \]
\[ \text{SNR} \sim -8\text{dB} \quad (⇒ \text{subcarrier SNR} \sim 10\text{dB}) \]
\[ L = 4 \quad \text{i.i.d Rayleigh paths with uniform delay, Tx angle, Rx angle} \]

Two training lengths considered:

\[ T \in \begin{cases} 1 \text{ block} & \Rightarrow \frac{1}{16} \text{ Nyquist rate} \\ 16 \text{ blocks} & \Rightarrow \text{Nyquist rate} \end{cases} \]
Sparse reconstruction (via LASSO) shows significant gain over LMMSE at both sub-Nyquist ($T = 1$) and Nyquist ($T = 16$) sampling rates.
Numerical Example: Channel Estimation (with shaping)

Aperture shaping yields 2-5dB reduction in LASSO’s channel estimation error.
Construct the data matrix as

$$X_t = F_N^H \text{diag}(\sqrt{p}) s_t b^H F_{N_t}$$

with power allocation $p \in \mathbb{C}^{N \times 1}$, QAM $s_t$, and beamformer $b \in \mathbb{C}^{N_t \times 1}$.

The observations decouple! A sufficient statistic to estimate $s_{t,n}$ is

$$[F_N Y_t F_{N_r}]_{n,:} = s_{t,n} \sqrt{p_n} b^H G_n + \text{AWGN}(\nu_w)$$

where $G_n \in \mathbb{C}^{N_t \times N_r}$ is the MIMO channel at frequency bin $n$.

Can solve for mutual-information maximal beamformer/powers via

$$\arg\max_{p,b} \sum_{n=1}^{N} \log_2 \left( 1 + p_n \frac{b^H G_n G_n^H b}{\nu_w} \right) \quad \text{s.t.} \quad \sum_{n=0}^{N-1} p_n = N, \quad p_n \geq 0, \quad \|b\|_2 = 1.$$ 

In practice, use estimated channel $\hat{G}_n$ for Rx combining and $(p, b)$ design.
Beamforming & waterfilling is near-optimal at low SNR.

LASSO performs nearly as well as perfect CSI, even under compressed pilots ($T=1$).

LMMSE is significantly suboptimal except at high SNR and Nyquist-rate pilots ($T=16$).
Numerical Example: Spectral Efficiency (with shaping)

Aperture shaping yields 0.5dB SNR gain in the $T=1$ case, closing the gap between LASSO and perfect-CSI.
Summary

- Considered mmW systems, which operate at very low SNR using massive antenna arrays at transmitter and receiver.

- Proposed an aperture shaping scheme that promotes sparsity in the virtual MIMO channel coefficients.

- Proposed a low-complexity space-time channel estimation scheme that exploits the extreme sparsity of mmW channels.

- Proposed a beamforming + waterfilling scheme that is near-optimal at low SNR.

- Numerical experiments suggest that LASSO channel estimates yield near-optimal spectral efficiency over a wide SNR range, even under significant pilot compression.