Full-Duplex MIMO Relaying: Achievable Rates Under Limited Dynamic Range

Brian P. Day*, Adam R. Margetts†, Daniel W. Bliss‡ and Philip Schniter*

*Department of Electrical and Computer Engineering, The Ohio State University
†Advanced Sensor Techniques Group, MIT Lincoln Laboratory
‡School of Electrical Computer and Energy Engineering, Arizona State University
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Opinions, interpretations, conclusions, and recommendations are those of the authors and are not necessarily endorsed by the United States Government.
Source communicates to Destination through decode and forward Relay

- **MIMO** at all terminals
- Relay operates in **full-duplex** mode

**Fundamental challenges:**
- high self-interference (as high as 100dB!)
- limited dynamic range due to non-ideal transmitter and receiver hardware (power amp noise, non-linearities in ADC/DAC, oscillator phase noise, AGC noise)

**Fundamental question:**
- What is the maximum achievable rate of such systems?
System Model

- \( H_{ij} \) are MIMO Rayleigh fading propagation channels, assumed to be unknown and static.
- \( n_i \) is AWGN thermal noise of unit variance.
- \( \rho_i \) represents SNR and \( \eta_i \) represents INR.
- Dynamic range limitation modeled by signal-power dependent additive interference \( c_j \) at transmitters and \( e_i \) at receivers.
  - Facilitates tractable achievable-rate analysis.
  - Recent work (e.g. Rice, Lincoln) confirms the fidelity of this model.
Each receive chain is corrupted by additive Gaussian interference with power proportional to the intended receive power; similar for each transmit chain

\[ e_i(t) \sim \mathcal{CN}(0, \beta \text{diag}(\Phi_i)), \quad e_i(t) \perp u_i(t), \quad e_i(t) \perp e_i(t') \mid_{t' \neq t} \]

\[ c_j(t) \sim \mathcal{CN}(0, \kappa \text{diag}(Q_j)), \quad c_j(t) \perp x_j(t), \quad c_j(t) \perp c_j(t') \mid_{t' \neq t} \]

where \( \Phi_i = \text{Cov}(u_i) \) and \( Q_j = \text{Cov}(x_j) \).
Transmission Protocol

- During Epoch $i$, the source communicates the $i^{th}$ packet to the relay, while the relay simultaneously communicates the $(i - 1)^{st}$ packet to the destination. $\implies$ Enables full-duplex communication.

- Before the first data epoch, we have a training epoch where we perform least-squares channel estimation.

- Data communication parameters (e.g. transmit covariance matrices) are designed to maximize the achievable rate.
We allow two distinct transmit covariance matrices per data epoch. The two periods per data epoch can differ in duration.
Partial Interference Cancellation

- We show that the relay’s received signal can be modeled as

\[ y_r(t) = \sqrt{\rho_r} \hat{H}_{sr} x_s(t) + v_r(t) \]

where \( v_r \) is the aggregate interference including transmitter/receiver dynamic-range induced self-noise, channel-estimation error, and thermal noise. Similarly, we can write \( y_d \) with interference \( v_d \).

- We write the relay’s aggregate interference as

\[ v_r(t) \triangleq \sqrt{\eta_r} \hat{H}_{rr} x_r(t) + \sqrt{\rho_r} \hat{H}_{sr} c_s(t) - D_{sr}^{\frac{1}{2}} \tilde{H}_{sr} (x_s(t) + c_s(t)) + n_r(t) \]

\[ + \sqrt{\eta_r} \hat{H}_{rr} c_r(t) - D_{rr}^{\frac{1}{2}} \tilde{H}_{rr} (x_r(t) + c_r(t)) + e_r(t) \]

where \( \sqrt{\eta_r} \hat{H}_{rr} x_r(t) \) is known by the relay and can be eliminated using interference cancellation.
Lower-Bounding the Achievable Rate

- Mutual information characterization is complicated by the fact that the aggregate interference $\nu_i$ is non-Gaussian when channel-estimation error is non-zero.

- We therefore lower-bound the mutual information by replacing $\nu_i$ with a Gaussian noise of identical covariance, i.e.,

\[
I_{sr}(Q[l]) = \log \det \left( I + \rho_r \hat{H}_{sr} Q_s[l] \hat{H}_{sr}^H \hat{\Sigma}_r^{-1} [l] \right)
\]

where $\hat{\Sigma}_r = \text{Cov}(\nu_r | \hat{H}_{sr}, \hat{H}_{rr})$ and $Q[l] \triangleq \{Q_s[l], Q_r[l]\}$. A similar expression is found for $I_{rd}(Q[l])$.

- We can also upper-bound the mutual information by ignoring the channel estimation error component.
Maximizing the Achievable-Rate Lower-Bound

For full-duplex operation, the Source → Destination rate is bottlenecked by the smallest of \( \{ I_{sr}, I_{rd} \} \).

Therefore, our metric is

\[
I_\tau (Q) = \min \left\{ \sum_{l=1}^{2} \tau[l] I_{sr}(Q[l]), \sum_{l=1}^{2} \tau[l] I_{rd}(Q[l]) \right\}
\]

\[
\triangleq I_{sr,\tau}(Q) \quad \triangleq I_{rd,\tau}(Q)
\]

where \( Q \triangleq \{ Q_s[1], Q_s[2], Q_r[1], Q_r[2] \} \).

Our optimization problem becomes \( \max_Q I_\tau (Q) \) with power and positivity constraints

\[
Q \in Q_\tau \triangleq \left\{ \begin{array}{l}
\sum_{l=1}^{2} \tau[l] \text{tr} (Q_s[l]) \leq 1, \quad Q_s[l] \geq 0 \quad \forall l \in \{1, 2\} \\
\sum_{l=1}^{2} \tau[l] \text{tr} (Q_r[l]) \leq 1, \quad Q_r[l] \geq 0 \quad \forall l \in \{1, 2\}
\end{array} \right\}
\]

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We convert the maximin problem to a weighted sum-rate optimization problem

$$\max_{\zeta \in [0,1]} \max_{Q \in Q_\tau} (\zeta I_{sr,\tau}(Q) + (1 - \zeta)I_{rd,\tau}(Q))$$

where we find $\zeta$ via bisection search.

To maximize $\tau$-weighted sum-rates $I_{sr,\tau}(Q)$ and $I_{sr,\tau}(Q)$, we have developed a Gradient Projection algorithm.

The projection step is performing waterfilling over both spatial and temporal degrees of freedom.

Finally we maximize with respect to the time-share $\tau$ using a grid search.
Numerical Results

We will now show the achievable-rate bounds in the following plots:
- versus INR $\eta_r$
- versus training length

In the plots, we show our proposed scheme as well as the following schemes:
- Half-duplex with optimized covariance matrices and time-sharing parameter $\tau$
- Our proposed scheme without performing interference cancellation
- Our proposed scheme using only one period per data epoch
Achievable-Rate Lower-Bound vs Training Length $T$

![Graph showing achievable-rate lower-bound vs training length $T$. The graph includes three lines for different SNR values: $\eta_r = 0$ dB, $\eta_r = 40$ dB, and $\eta_r = 100$ dB. The $x$-axis represents the training length $T$, and the $y$-axis represents the min-rate (bpcu). Different lines represent upper and lower bounds for the min-rate at various SNR levels.]

- Upper-Bound
- Lower-Bound

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Conclusion

- We characterized the achievable-rate of MIMO decode-and-forward full-duplex relaying.

- We considered dynamic range limitations at the transmitter and receiver, as well as channel-estimation error from the training-based least-squares.

- Our solution required solving a non-convex optimization problem, for which we applied the projected gradient method.

- An analytic approximation that writes mutual information as an explicit function of the SNRs, INRs, numbers of antennas, and dynamic-range parameters $\kappa$ and $\beta$ was also derived (see paper).
Thanks!
Backup Slides
Gradient Projection Algorithm

We find the achievable-rate lower-bound via Gradient Projection:

\[ \text{for } k = 1, 2, 3, \ldots \]

\[ P_r^{(k)}[1] = Q_r^{(k)}[1] + G_r^{(k)}[1] \]

\[ P_r^{(k)}[2] = Q_r^{(k)}[2] + G_r^{(k)}[2] \]

\[ (\tilde{Q}_r^{(k)}[1], \tilde{Q}_r^{(k)}[2]) = \mathcal{P}_{Q_r \tau} (P_r^{(k)}[1], P_r^{(k)}[2]) \]

\[ Q_r^{(k+1)}[1] = Q_r^{(k)}[1] + \gamma^{(k)} (\tilde{Q}_r^{(k)}[1] - Q_r^{(k)}[1]) \]

\[ Q_r^{(k+1)}[2] = Q_r^{(k)}[2] + \gamma^{(k)} (\tilde{Q}_r^{(k)}[2] - Q_r^{(k)}[2]) \]

\[ \langle \text{Similar repeated for } Q_s[1] \text{ and } Q_s[2] \rangle \]

where \( G_r^{(k)}[l] \) is the gradient, and \( \mathcal{P}_{Q_r \tau} (\cdot) \) projects the period 1 and period 2 covariances onto the constraint set. \( \gamma^{(k)} \) is chosen via the Armijo stepsize rule.
Achievable-Rate Lower-Bound Contour over SNR and INR

![Achievable-Rate Lower-Bound Contour over SNR and INR](image.png)
The complicated nature of the optimization problem motivates us to approximate its solution.

Making simplifying assumptions, we are able to find straightforward optimal transmit covariance matrices for both full-duplex and half-duplex operation.

Our analytic approximate solution is simply the maximum of the full-duplex and half-duplex approximate solutions.
Analytic Approximation Contour over SNR and INR

Approximation

Gradient Projection Optimization

INR $\eta_r$ [dB]

SNR $\rho_r$ [dB]

INR $\eta_r$ [dB]

SNR $\rho_r$ [dB]