OFDMA Downlink Resource Allocation via ARQ Feedback

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Setup:

- Single-antenna downlink with $K$ users
- OFDMA with $N$ subchannels
- Channels are Markov time-varying with $L$ taps
- ACK/NAK feedback from previously scheduled users
The Basic Resource Allocation Problem:

• At each time $t$, we want to schedule the "best" users (multiuser diversity) to their "best" subchannels (frequency diversity).
• We also want to optimize the powers and data-rates of assigned users.
• To make informed choices, we need channel state information (CSI).
• Feedback of each user’s CSI about each subchannel is very costly!

*Is it possible to do near-optimal resource allocation using only ACK/NAK feedback from previously scheduled users?*

*Can we learn enough about the CSI from such limited feedback?*
Detailed Objective:

At each time $t$ and subchannel $n$, choose each user $k$’s next...

- rate $r_{n,k,t+1} \in \{1, \ldots, M\}$,
- power $p_{n,k,t+1} \geq 0$,

based on ACK/NAK feedback $F_t^1$, to maximize the total future utility

$$G_{t+1}^T = \sum_{\tau=t+1}^{T} \sum_{n=1}^{N} U\left(\left(1 - \epsilon_{r_{n,k,\tau}}(\gamma_{n,k,\tau}, p_{n,k,\tau})\right) r_{n,k,\tau}\right)$$

subject to the power constraint $\sum_{n,k} p_{n,k,\tau} \leq P_{\text{max}}, \forall \tau$,

and subject to a one-user-per-subchannel constraint.

Here, $\epsilon_r(\gamma, p)$ is packet error rate and $U(\cdot)$ is a concave utility function.
Optimal ACK/NAK-based Resource Allocation:

- Notice that the current resource allocation affects not only the immediate utility, but also the subsequent ACK/NAK feedback, and hence the future utilities.

- Intuitions:
  - if we assign transmission params that are very likely to yield ACKs, we will learn very little about the changing CSI! (\(\leadsto\) “exploitation”)
  - if we assign transmission params to best inform us of CSI, the expected utility will be low. (\(\leadsto\) “exploration”)

Classic tradeoff: exploration vs exploitation.

- The optimal allocator is a partially observable Markov decision process (POMDP), at least in the simpler case of a finite set of powers. POMDP complexity is impractically high, however, forcing us to consider a suboptimal approach.
**Greedy Resource Allocation:**

- For ACK/NAK-based rate adaptation in the single-user single-channel case, we previously found that *greedy adaptation* is nearly optimal (at practical fading rates):
  

- Thus, we propose to use *greedy resource allocation* for our multi-user multi-channel problem.
The Greedy Resource-Allocation Problem:

Using the indicator $I_{n,m,k,t} \in \{0, 1\}$ to denote time-$t$ assignment of subchannel $n$ to user $k$ at MCS index $m$, the time-$t$ problem becomes

$$\max \sum_k \mathbb{E} \left\{ \sum_{n,m} U \left( I_{n,k,m,t+1} (1 - a_m e^{-b_m p_{n,k,m,t+1} \gamma_{n,k,t+1}}) r_m \right) \right\}\left| F_{1}^{t}\right\}

\text{subject to } \sum_{n,k,m} I_{n,k,m,t} p_{n,k,m,t} \leq P_{\text{max}}, \forall t,

\text{and } \sum_{k,m} I_{n,k,m,t} \leq 1, \forall n, \forall t,

where

- $\gamma_{n,k,t}$ is SNR of user $k$ at subchannel $n$ at time $t$,
- $(a_m, b_m, r_m)$ determine data rate and error rate for MCS index $m$,
- $F_{1}^{t}$ collects all ACK/NAK feedbacks collected from times 1 to $t$. 
Greedy Allocation — Practical Approximation:

Say that we relax the binary indicators to \( \tilde{I}_{n,m,k,t} \in [0, 1] \).

Then the KKT conditions become (suppressing the time-\( t \) notation):

\[
\forall n, k, m, \quad \mu = a_m b_m r_m \mathbb{E}\{\gamma_{n,k} e^{-b_m p_{n,k,m} \gamma_{n,k}} \mid F\}
\]

\[
\forall n, k, m, \quad \lambda_n = r_m \mathbb{E}\{1 - a_m e^{-b_m p_{n,k,m} \gamma_{n,k}} \mid F\} - \mu p_{n,k,m}
\]

where \( \{\lambda_n\}_{n=1}^N \) and \( \mu \) are Lagrange multipliers. A practical alg is then:

1. Initialize \( \mu \) at a small value.

2. For each subchannel \( n \),
   - For each \((k, m)\)...
     - calculate \( p_{n,k,m} \) from (1) with \( \tilde{I}_{n,k,m} = 1 \), forcing \( p_{n,k,m} \geq 0 \).
     - plug \( p_{n,k,m} \) into (2) and calculate the corresponding \( \lambda_n(k, m) \).
   - Find \( (k^*, m^*) = \arg \max_{(k,m)} \lambda_n(k, m) \).
   - Set \( I_{n,k^*,m^*} = 1 \) and \( I_{n,k,m} \mid (k,m) \neq (k^*,m^*) = 0 \).

3. If \( \sum_n p_{n,k^*,m^*} > P_{\text{max}} \), increase \( \mu \) and repeat, else stop.
**Example Performance of Greedy Approximation:**

<table>
<thead>
<tr>
<th>$N$</th>
<th>$K$</th>
<th>$M$</th>
<th>greedy goodput</th>
<th>approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>9</td>
<td>5.9884</td>
<td>5.988</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>9</td>
<td>6.3501</td>
<td>6.3499</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>9</td>
<td>10.3251</td>
<td>10.3249</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>9</td>
<td>10.9778</td>
<td>10.9774</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>9</td>
<td>14.0573</td>
<td>14.0571</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>9</td>
<td>14.9653</td>
<td>14.9651</td>
</tr>
</tbody>
</table>

*The practical approximation yields 99.99% of the goodput attained by the true greedy scheme.*
Tracking the SNR distribution:

The greedy allocator tracks the SNR by updating the SNR distributions

\[ p(\gamma_{n,k,t+1} \mid F_t^t), \quad \forall \text{ users } k \text{ and subchannels } n. \]

The SNR evolves as follows:

- **Markov evolution of time-domain channel taps:**
  \[ h_{l,k,t+1} = (1 - \alpha)h_{l,k,t} + \alpha w_{l,k,t}, \quad w_{l,k,t} \sim \mathcal{CN}(0, 1), \]

- **subchannel gains as a function of time-domain channel taps:**
  \[ H_{n,k,t} = \sum_{l=0}^{L-1} h_{l,k,t} e^{-j\frac{2\pi}{N}nk}, \]

- **subchannel SNRs as a function of subchannel gains:**
  \[ \gamma_{n,k,t} = K|H_{n,k,t}|^2. \]
Tracking the SNR distribution (cont.):

SNR tracking can be done as follows:

\[
p(\gamma_{n,k,t+1} \mid F^t_1) = \int_{h_{k,t+1}} p(\gamma_{n,k,t+1} \mid h_{k,t+1}) p(h_{k,t+1} \mid F^t_1) \quad (\text{approx of) Dirac delta}) \quad (3)
\]

\[
p(h_{k,t+1} \mid F^t_1) = \int_{h_{k,t}} \underbrace{p(h_{k,t+1} \mid h_{k,t})}_{\text{Markov prediction}} p(h_{k,t} \mid F^t_1) \quad (4)
\]

\[
p(h_{k,t} \mid F^t_1) = \frac{p(f_{k,t} \mid h_{k,t}) p(h_{k,t} \mid F^{t-1}_1)}{\int_{h_{k,t}'} p(f_{k,t} \mid h_{k,t}') p(h_{k,t}' \mid F^{t-1}_1)} \quad \text{(Bayes rule)} \quad (5)
\]

\[
p(f_{k,t} \mid h_{k,t}) = \prod_{n=1}^{N} p(f_{n,k,t} \mid \gamma_{n,k,t}(h_{k,t})) \quad (6)
\]

\[
p(f_{n,k,t} = f \mid \gamma_{n,k,t}) = \begin{cases} 
\sum_m I_{n,k,m,t} a_m e^{-b_m p_{n,k,m,t} \gamma_{n,k,t}} & f = 0 \\
\sum_m I_{n,k,m,t} (1 - a_m e^{-b_m p_{n,k,m,t} \gamma_{n,k,t}}) & f = 1 \\
1 - \sum_m I_{n,k,m,t} & f = \emptyset
\end{cases} \quad (7)
\]
Tracking the SNR distribution (cont.):

Thus, for each user $k$,

1. measure feedbacks $f_{k,t}$ across all subchannels,
2. compute $p(f_{n,k} \mid \gamma_{n,k,t}(h_{k,t}))$ on $h$-lattice using error-rate rules (6)-(7),
3. compute $p(h_{k,t} \mid F_{1}^{t})$ on $h$-lattice by updating previous posterior via (5),
4. compute $p(h_{k,t+1} \mid F_{1}^{t})$ on $h$-lattice via Markov-prediction step (4),
5. compute $p(\gamma_{k,t+1} \mid F_{1}^{t})$ on $\gamma$-lattice via $h$-to-$\gamma$ conversion step (3).

This costs $O(KNQ_{h}^{L} + KLQ_{h}^{L+1} + KNQ_{\gamma}Q_{h}^{L})$, where

$Q_{h} = \text{number of grid points used per dimension of } h$-lattice,
$Q_{\gamma} = \text{number of grid points used per dimension of } \gamma$-lattice.
## Numerical Experiments:

### Setup:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>2</td>
</tr>
<tr>
<td>$N$</td>
<td>2</td>
</tr>
<tr>
<td>$L$</td>
<td>2</td>
</tr>
<tr>
<td>$E{\gamma_{n,k,t}}$</td>
<td>25dB = 330</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>${0.01, 0.001, 0.0001}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Plots show (versus packet index $t$):

- **goodput of**
  - approximate-greedy with genie-aided CSI
  - approximate-greedy with tracked CSI
  - approximate-greedy with prior CSI (and round robin)
- **power/rate/user of** approximate-greedy with tracked CSI
Goodput for $\alpha = 0.0001$: 

2 users, 2 subcarrier, $\alpha = 1e^{-4}$, 200 packets 

- genie-CSI avg = 11.494 
- tracked-CSI avg = 11.3354 
- prior-CSI avg = 6.6481
Allocations for $\alpha = 0.0001$: 

- 2 users, 2 subcarriers, $\alpha = 1e-4$, 200 packets
- User 1 after ACK
- User 2 after ACK
- User 1 after NACK
- User 2 after NACK
- Rate change up or down

Graphs showing power and SNR estimates for subcarriers 1 and 2.
Goodput for $\alpha = 0.001$:

2 users, 2 subcarrier, $\alpha = 1e^{-3}$, 200 packets

- genie-aided CSI avg = 11.0324
- tracked CSI avg = 10.574
- prior CSI avg = 7.3398
Allocations for $\alpha = 0.001$:

- 2 users, 2 subcarriers, $\alpha = 1e^{-3}$, 200 Packets

- Power Subcarrier 1
  - User 1 after ACK
  - User 2 after ACK
  - User 1 after NACK
  - User 2 after NACK
  - Rate change up or down

- SNR estimate Subcarrier 1
  - User 1
  - User 2
  - Actual SNR for corresponding users

- Power Subcarrier 2
  - User 1
  - User 2

- SNR estimate Subcarrier 2
  - User 1
  - User 2

- Packet Number
  - SNR estimate Subcarrier 1
  - SNR estimate Subcarrier 2
  - Power Subcarrier 2
  - Power Subcarrier 1
**Goodput for** $\alpha = 0.01$:

2 users, 2 subcarrier, $\alpha = 1e−2$, 200 packets

<table>
<thead>
<tr>
<th>Method</th>
<th>Avg Goodput</th>
</tr>
</thead>
<tbody>
<tr>
<td>Genie-CSI</td>
<td>11.0086</td>
</tr>
<tr>
<td>Tracked CSI</td>
<td>9.8795</td>
</tr>
<tr>
<td>Prior CSI</td>
<td>8.1891</td>
</tr>
</tbody>
</table>

Packet Number

Total goodput in all subcarriers
Allocations for $\alpha = 0.01$:

2 users, 2 subcarriers, $\alpha = 1e-2$, 200 Packets

- User 1 after ACK
- User 2 after ACK
- User 1 after NACK
- User 2 after NACK
- Rate change up or down

Power Subcarrier 1

SNR estimate Subcarrier 1

Power Subcarrier 2

SNR estimate Subcarrier 2

Packet Number
Summary:

- **Goal**: Allocation of \{user schedule, powers, rate\} to maximize finite-horizon expected goodput under an instantaneous total-power constraint and a one-user-per-subcarrier constraint.
- The optimal resource allocator is a POMDP, which is computationally impractical.
- We settle for greedy resource allocation, thought to be near-optimal for practical fading rates.
- The greedy allocator itself is computationally impractical, and so we settle for a practical approximation (99.99% exact).
- To maintain CSI, we track the SNR distribution of each user at each subcarrier (conditioned on past ACK/NAK feedback).
- Preliminary experiments for 2 users and 2 subchannels indicates that our practical algorithm does a decent job of SNR tracking and goodput maximization.