Fast Near-Optimal Noncoherent Decoding for Block Transmission over Doubly Dispersive Channels

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Problem Description:

• Coded block transmission over a quickly time-varying frequency selective channel.

• Channel realizations unknown, but channel statistics known.

• Goal: near-optimal decoding with very low complexity and very few pilots.

Approach:

• Turbo reception (soft noncoherent equalization \implies soft decoding).

• Soft decoder: off-the-shelf.

• Soft noncoherent equalizer: a novel design leveraging...
  – tree-search based on M-algorithm,
  – basis expansion model (BEM) for channel variation,
  – fast metric update (linear complexity).
Channel Model:

Received samples are \( \{r_n\}_{n=0}^{N-1} \), where

\[
r_n = \sum_{l=0}^{N_h-1} h_{n,l} s_{n-l} + v_n
\]

\( h_{n,l} \): time-\( n \) response to an impulse at time \( (n - l) \)

\( N_h \): discrete delay spread

\( \{s_n\}_{n=0}^{N-1} \): symbols mapped from coded bits \( \{x_k\}_{k=0}^{QN-1} \) into the \( 2^Q \)-ary symbol alphabet \( S \)

\( \{v_n\}_{n=0}^{N-1} \): CWGN with variance \( \sigma^2 \).

We assume WSSUS Rayleigh fading:

\[
E\{h_{n,l}h_{n-m,l-p}^*\} = \rho_m \sigma_l^2 \delta_p,
\]

where the autocorrelation \( \{\rho_m\} \) and delay-power profile \( \{\sigma_l^2\} \) are known.
Turbo Reception:

\[
L_e(x_k|r) = \ln \frac{\sum_{x: x_k=1} \exp \mu(x)}{\sum_{x: x_k=0} \exp \mu(x)} - L_a(x_k)
\]

"extrinsic LLR"

\[
\mu(x) = \ln p(r|s(x)) + \sum_{i: x_i=1} L_a(x_i)
\]

"MAP metric"

Need \(O(2^{QN})\) evaluations of \(\mu(x) \sim \text{Computationally infeasible}!!\)
**Simplified LLR Evaluation:**

The “max-log” approximation:

\[ L_e(x_k|r) \approx \max_{x \in \mathcal{L} \cap \{x:x_k=1\}} \mu(x) - \max_{x \in \mathcal{L} \cap \{x:x_k=0\}} \mu(x) - L_a(x_k) \]

\[ \mathcal{L} : \text{ set containing the } M \text{ most probable } x, \]

requires only a few evaluations of \( \mu(x) \). But how complex is this?

Say \( r = Sh + v \) where \( h \sim \mathcal{CN}(0, R_h) \) and \( v \sim \mathcal{CN}(0, \sigma^2 I) \). Then

\[ r|s \sim \mathcal{CN}(0, \underbrace{SR_hS^H + \sigma^2 I}_{\Phi(s)}) \]

\[ \Rightarrow \ln p(r|s) = -r^H \Phi^{-1} r - \ln \det \Phi - N \ln \pi \]

\[ \Rightarrow \mu(x) = -r^H \Phi^{-1} r - \ln \det \Phi - N \ln \pi + \sum_{i:x_i=1} L_a(x_i) \]

\[ \Rightarrow \text{direct evaluation of } \mu(x) \text{ is } \mathcal{O}(N^3). \text{ Still quite expensive!} \]
Fast Soft Noncoherent Equalization:

We propose a novel equalization algorithm, based on

- efficient tree search to find best $M$ bit sequences $\mathcal{L}$, and
- fast recursive update of MAP metric $\mu(x)$ using
  - a basis expansion model (BEM) for the channel’s time-variation,
  - recursive update of the implicit MMSE channel estimate $\hat{\theta}$.

The result is near-MAP performance with complexity that is
- linear in the block length, and
- quadratic in the channel length.
**BEM Approximation:**

\[ h_{n,l} \approx \sum_{p=0}^{N_b-1} b_{n,p} \theta_{p,l} \quad \text{for } n \in \{0, \ldots, N-1\}. \]

\[ \{b_{n,p}\}_{n=0}^{N-1} : p^{th} \text{ basis waveform} \]

\[ N_b : \text{number of basis waveforms} \]

\[ \theta_{p,l} : \text{coefficient for } p^{th} \text{ basis waveform and } l^{th} \text{ lag.} \]

Basis choices include:

- **complex exponential basis:** \( b_{n,p} = e^{j \frac{2 \pi}{N} (p \frac{N_b-1}{2}) n} \),

- **polynomial basis:** \( b_{n,p} = n^p \)

- **Karhunen-Loeve basis:** \( \{b_{n,p}\}_{n=0}^{N-1} \) is the \( p^{th} \) largest eigenvector of the Toeplitz matrix defined from the autocorrelation \( \{\rho_m\}_{m=0}^{N-1} \).

Note: \( N_b = N \) yields zero approximation error, though typically \( N_b = 2 \).
Sequential Processing based on the BEM:

\[ r_n := [r_0, r_1, \ldots, r_n]^T = B_n S_0^m \theta + v_n \]

where, by example,

\[
\begin{bmatrix}
    r_0 \\
    r_1 \\
    r_2
\end{bmatrix} = \begin{bmatrix}
    b_0^H \\
    b_1^H \\
    b_2^H
\end{bmatrix} \begin{bmatrix}
    s_0 I_{N_b} & s_{-1} I_{N_b} \\
    s_1 I_{N_b} & s_0 I_{N_b} \\
    s_2 I_{N_b} & s_1 I_{N_b}
\end{bmatrix} \begin{bmatrix}
    \theta_0 \\
    \theta_1
\end{bmatrix} + \begin{bmatrix}
    v_0 \\
    v_1 \\
    v_2
\end{bmatrix}
\]

\[ b_n = [b_{n,0}, \ldots, b_{n,N_b-1}]^H : \text{time-}n \text{ basis elements} \]
\[ \theta_l = [\theta_{0,l}, \ldots, \theta_{N_b,l}]^T : \text{lag-}l \text{ BEM coefficients} \]

Note:

- \( \theta \in \mathbb{C}^{N_b N_h} \) contains all unknown channel coefficients,
- \( S_0^m \) contains data symbols \( s_n := [s_0, s_1, \ldots, s_n]^T \).
Fast Metric Update:

Say $x_n$ contains bits from $s_n$. Can compute $\mu(x_n)$ given $\mu(x_{n-1})$ via

$$a_n = [s_n b_n^H, s_{n-1} b_n^H, \ldots, s_{n-Nh+1} b_n^H]^H \in \mathbb{C}^{NhNb}$$

$$d_n = \Sigma_{n-1}^{-1} a_n$$

$$\alpha_n = (1 + a_n^H d_n)^{-1}$$

$$\Sigma_n^{-1} = \Sigma_{n-1}^{-1} - \alpha_n d_n d_n^H$$

$$\mu(x_n) = \mu(x_{n-1}) - \frac{\alpha_n}{\sigma^2} |r_n - a_n^H \hat{\theta}_{n-1}| - \ln(\pi \alpha_n) + \sum_{i : x_i = 1, x_i \in s_n} L_a(x_i)$$

$$\hat{\theta}_n = (I_{NhNb} - \alpha_n d_n a_n^H) \hat{\theta}_{n-1} + (1 - \alpha_n d_n^H a_n) r_n d_n,$$

using only $2(NhNb)^2 + 9NhNb + 8$ multiplications!
Fast Tree Search:

Breadth-first search via the M-algorithm:

- Say $L'_n$ contains the $M$ “best” estimates of $x_n$. For each extension $x_{n+1} = [x_n]$, where $x_n \in L'_n$ and $x \in \{0, 1\}^Q$, calculate the metric $\mu(x_{n+1})$. Then collect the $M$ best extensions in the set $L'_{n+1}$.

- Doing this for $n = 0, \ldots, N - 1$ requires the evaluation of $M2^Q N$ MAP metrics, yielding $L' := L'_{N-1}$, an estimate of the $M$ most probable bit vectors.

- Performance almost indistinguishable from full search (i.e., $L' \approx L$).

In total, $2M2^Q N (N_b N_h)^2$ multiplications are required to compute the MAP metrics $\{\mu(x) : x \in L'\}$. Note that this complexity is

- *linear* in the block length $N$,
- *quadratic* in the channel length $N_b N_h$. 
Construction of the Transmission Block:

Pilots:

- One pilot symbol sufficient to resolve channel/data phase ambiguity.
- $N_p > 1$ pilots provide a good “initialization” of $\mu(x_{N_p})$, helping improve the accuracy of tree search.
- $N_p \geq N_h N_b$ needed for channel-estimation followed by coherent decoding.

Guard:

- A ZP guard interval of length $N_h - 1$ prevents inter-block interference and enables capture of diversity from delay spread.
- Note: we hope to capture Doppler diversity through iteration with soft decoder.
Numerical Experiments (Setup):

Transmitter:
- rate-$\frac{1}{2}$ LDPC coding, frame length 4096, QPSK ($Q = 2$),
- block length $N = 64$,
- $N_p \in \{3, 6, 9\}$ pilots.

Channel:
- WSSUS Rayleigh (via Jakes) with $N_h = 3$ taps at $f_D T_s = 0.002$.

Receiver:
- KL-basis with $N_b = 2$,
- search parameter $M \in \{16, 32, 64, 128\}$,
- inner (i.e., LDPC decoding) iterations $\leq 60$,
- outer (i.e., turbo) iterations $\in \{1, 2, 4, 8, 12, 16\}$.
Choice of search parameter $M$:

![Graph showing BER vs Eb/No for different M values](image_url)
Choice of # outer iterations:

![Graph showing BER vs. Eb/No (dB) for different iterations.]
Choice of # pilots $N_p$ / genie-aided comparison:

![Graph showing BER vs. Eb/No(dB) for different pilots and estimation methods.]

- Soft Kalman Estim $N_p=9$
- Kalman–LVA $N_p=9$
- NC (Proposed) $N_p=9$
- Genie–Estim CH
- Perfect CH Knowledge
Numerical Experiments (Interpretations):

Genie-aided references:

1. perfectly known channel,
2. channel estimated from 100% training.

Only about 2 dB away!

Maximum diversity order offered by channel:

- \( f_D T_s = 0.002 \Rightarrow \) coherence time = 500 symbols.
- 4096-symbol frame \( \Rightarrow \) 8 coherence intervals.
- 3 taps \( \times 8 \) coherence intervals = 24 degrees of freedom.

The BER slopes confirm that our scheme achieves maximum diversity!
Conclusions:

- We presented a novel scheme for the reception of coded transmissions over quickly varying multipath channels.
- Leveraged a BEM approximation of channel, the M-algorithm, a fast recursive update for the MAP metric, and the turbo principle.
- Achieved performance $\approx 2$ dB away from genie-aided bounds at a complexity of $\approx 2M2^Q(N_bN_h)^2$ mults per QAM symbol.
- Only need one pilot per block, though a few more help performance.