Joint Channel Estimation and Sequence Detection over Doubly Dispersive Channels

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Problem Description:

- Uncoded block transmission over an ISI channel that varies significantly over the block.
- Data symbols and channel are both unknown. At least one known pilot symbol.
- Interested in near-optimal sequence detection with reasonable complexity.

Related Work:

- Per-survivor processing (PSP): trellis-based equalization using the surviving partial-paths as training for adaptive channel estimation.
- Joint estimation/MLSD for singly-dispersive channels.
**System Model:**

Received samples are \( \{r_n\}_{n=0}^{N-1} \), where

\[
r_n = \sum_{l=0}^{N_h-1} h_{n,l} s_{n-l} + v_n,
\]

- \( h_{n,l} \): time-\( n \) response to an impulse at time \( (n-l) \).
- \( N_h \): discrete channel spread.
- \( \{s_n\}_{n=0}^{N-1} \): symbols from finite alphabet \( Q \)
- \( \{v_n\}_{n=0}^{N-1} \): CWGN with variance \( \sigma^2 \).

We assume WSSUS fading:

\[
E\{h_{n,l}h_{n-m,l-p}^*\} = \rho_m \sigma_l^2 \delta_p
\]

Note: holds for single-carrier transmission over a time-varying ISI channel, or multicarrier transmission over a frequency-varying ICI channel.
BEM Approximation (Used by Receiver):

The receiver employs a basis expansion model (BEM)

\[ h_{n,l} \approx \sum_{p=0}^{N_b-1} b_{n,p} \theta_{p,l} \quad \text{for } n \in \{0, \ldots, N - 1\}. \]

\( \{b_{n,p}\}_{n=0}^{N-1} \) : \( p^{th} \) basis waveform

\( N_b \) : number of basis waveforms

\( \theta_{p,l} \) : coefficient for \( p^{th} \) basis waveform and \( l^{th} \) lag

BEM options include:

- oversampled complex exponential: \( b_{n,p} = e^{j \frac{2\pi}{NK} np}, K \geq 1 \)

- polynomial: \( b_{n,p} = n^p \)

- Karhunen-Loeve: \( \{b_{n,p}\}_{n=0}^{N-1} \) is the \( p^{th} \) largest eigenvector of Toeplitz correlation matrix defined from \( \{\rho_m\}_{m=0}^{N-1} \)
BEM-Approximated System Model:

\[ r_n = B_n S^n_0 \theta + v_n \]

where, by example,

\[
\begin{bmatrix} r_2 \\ r_1 \\ r_0 \end{bmatrix} = \begin{bmatrix} b_2^H \\ b_1^H \\ b_0^H \end{bmatrix} \begin{bmatrix} s_2 I_{N_b} & s_1 I_{N_b} \\ s_1 I_{N_b} & s_0 I_{N_b} \\ s_0 I_{N_b} & s_{-1} I_{N_b} \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} + \begin{bmatrix} v_2 \\ v_1 \\ v_0 \end{bmatrix}
\]

\[ b_n = [b_{n,0}, \ldots, b_{n,N_b-1}]^H : \text{time-}n \text{ basis values} \]

\[ \theta_l = [\theta_{0,l}, \ldots, \theta_{N_b,l}]^T : \text{lag-}l \text{ BEM coefficients} \]

Note:

- \( \theta \in \mathbb{C}^{N_b N_h} \) contains all time-varying channel parameters
- \( S^n_0 \) contains data symbols \( s_n = [s_n, \ldots, s_0]^T \)
Noncoherent Data Detection:

MLSD criterion:

\[
\hat{s}_n = \arg \max_{s_n} p(r_n|s_n)
\]

With prior channel pdf \( p(\theta) \),

\[
p(r_n|s_n) = \int_\theta \frac{\mathcal{C}\mathcal{N}(B_n S_0^m \theta, \sigma^2 I)}{\mathcal{C}\mathcal{N}(0, R_\theta)} p(r_n|s_n, \theta) p(\theta) \, d\theta
\]

After some algebra, we obtain a quadratic noncoherent metric

\[
\hat{s}_n = \arg \min_{s_n} \left\{ \frac{r_n^H \Phi_{s_n} r_n + \log \det(\Sigma_{s_n})}{\mu(s_n)} \right\} \approx \arg \min_{s_n} \mu(s_n)
\]

for \( \Phi_{s_n} = (B_n S_0^m R_\theta (B_n S_0^m)^H + \sigma^2 I_{n+1})^{-1} \).
Estimation/Detector Interpretation:

Can write noncoherent metric as

$$
\mu(s_n) = \underbrace{\sigma^{-2} \| r_n - B_n S_0^n \hat{\theta}_{s_n} \|^2}_{\text{"coherent" ML metric}} + \underbrace{\sigma^{-2} \hat{\theta}_{s_n}^H R_{\theta}^{-1} \hat{\theta}_{s_n}}_{\text{prior reconciliation}}
$$

where $\hat{\theta}_{s_n}$ is the MMSE estimate of $\theta$ from $r_n$ given $s_n$:

$$
\hat{\theta}_{s_n} = \mathbb{E}\{\theta r_n^H | s_n\} \mathbb{E}\{r_n r_n^H | s_n\}^{-1} r_n
$$

In other words, the noncoherent metric $\mu(s_n)$ adapts to the channel that is implicitly estimated with $s_n$ as training.

Note: Brute-force search evaluates $|Q|^N$ metrics!!
Fast Adaptive Sequential Decoding:

1. Suboptimal breadth-first tree search via the M-algorithm:
   - Say $\mathcal{S}_n$ contains the $M$ “best” estimates of $s_n$. For each extension $s_{n+1} = [s_n^s]$, where $s_n \in \mathcal{S}_n$ and $s \in Q$, calculate the noncoherent metric $\mu(s_{n+1})$. Then keep $M$ best in $\mathcal{S}_{n+1}$.
   - In total, evaluates $M|Q|N$ noncoherent metrics.
   - Performance almost indistinguishable from brute-force.

2. Fast metric computation:
   - Updating $\hat{\theta}_{s_n}$ requires only about $nN_bN_h + 4N_b^2N_h^2$ operations.

Assuming $N > N_bN_h$ (i.e., an underspread channel), the total complexity of calculating $\hat{s}_{N-1}$ is $\mathcal{O}(M|Q|N^2N_bN_h)$. 
Fast Metric Computation:

Can write MMSE estimate as

\[
\hat{\theta}_s \ = \ \Sigma_s^{-1} A_n^H r_n \\
\Sigma_s \ = \ A_n^H A_n + \sigma^2 R_{\theta}^{-1} \\
A_n \ = \ B_n S_0^n \in \mathbb{C}^{(n+1)\times N_bN_h}
\]

Noticing a rank-one update:

\[
\Sigma_{s_{n+1}} \ = \ \Sigma_s + a_{n+1} a_{n+1}^H \\
a_{n+1}^H \ = \ b_{n+1}^H S_{n+1}^{n+1} \in \mathbb{C}^{N_bN_h} \\
\Sigma_{s_{n+1}}^{-1} \ = \ \Sigma_s^{-1} - \frac{(\Sigma_s^{-1} a_{n+1})(\Sigma_s^{-1} a_{n+1})^H}{1 + a_{n+1}^H \Sigma_s^{-1} a_{n+1}},
\]

so complexity of calculating \( \hat{\theta}_s \) is \( \mathcal{O}(nN_bN_h) \) when \( n > N_bN_h \).

Given \( \hat{\theta}_{s_{n+1}} \), the complexity of calculating \( \mu(s_{n+1}) \) is also \( \mathcal{O}(nN_bN_h) \) when \( n > N_bN_h \).
Construction of the Transmission Frame:

Pilots:

- One pilot symbol needed to resolve channel/data phase ambiguity.
- $N_h$ leading pilots useful for “initializing” the metric, allowing for good M-alg performance with small $M$.
- $N_hN_b$ pilots required for pilot-aided estimation-then-detection.

Diversity:

- $N_h - 1$ trailing zeros needed to make full delay-diversity accessible.
- Doppler diversity not accessible without coding/precoding. (This issue will be treated in future work.)

We insert $N_hN_b$ leading pilots to facilitate a fair comparison with estimation-then-detection schemes, and we insert $N_h - 1$ trailing zeros to make delay-diversity accessible.
Numerical Experiments:

- BPSK symbols, $N = 25$
- WSSUS Jakes channel with delay spread $N_h = 2$ and single-sided Doppler spread $f_d T_s \in \{0.002, 0.005\}$.
- Receiver BEMs ($N_b = 2$):
  1. Karhunen-Loeve (KL)
  2. Oversampled Complex Exponential (OCE)
- Reference Algorithms
  1. ML with perfect $\{h_{n,l}\}$ (genie-aided)
  2. ML with MMSE-$\hat{\theta}$ from pilots+data (genie-aided)
  3. ML with MMSE-$\hat{\theta}$ from pilots
  4. PSP with RLS-$\{\hat{h}_{n,l}\}$
Effect of Metric Approximation and Choice of $M$:

$f_d T_s = 0.005$
Performance with KL-BEM:

\[ f_d T_s = 0.002 \]

\[ f_d T_s = 0.005 \]

- Kalman–PSP–LVA
- M–alg + KL–BEM
- ML with genie–estimated BEM
- ML with perfect CSI
Performance with OCE-BEM:

\[ f_d T_s = 0.002 \]

\[ f_d T_s = 0.005 \]
Conclusions:

- Joint channel/symbol estimation for quickly varying ISI channels.
- Leveraged BEM channel approximation, M-algorithm, fast MMSE-channel estimation.
- Less than 1 dB from optimal performance at complexity $O(M|Q|N^2N_bN_h)$.
- Significantly outperforms decoupled channel/symbol estimation.
- Outperforms PSP-RLS, especially at high Doppler spreads.