Design of Multi-carrier Modulation for Doubly Selective Channels based on a Complexity Constrained Achievable Rate Metric

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**The Doubly Selective Channel**

- Channel modeled as linear time-varying system:

\[
    r_n = \sum_{l=0}^{N_h-1} h_{n,l} t_{n-l} + v_n
\]

  - Large delay spread \( \Rightarrow N_h \gg 1 \)
  - Rapid time variations modeled by WSSUS Rayleigh fading

\[
    E[h_{n_1,l_1} h_{n_2,l_2}^*] = \sigma_{l_1}^2 J_0 \left(2\pi f_d (n_1 - n_2)\right) \delta_{l_1-l_2}
\]

- Popular to use multi-carrier modulation (MCM) on such channels
MIMO – MCM

N sub-carriers

Beam-forming

MIMO Mod.

MIMO Mod.

N_t Transmit Antennas

S

N x 1

MIMO

DS

Wireless

Channel

Data

Estimates

N x 1

Receiver

Processing

MCM

Demod.

MCM

Demod.

N r Receive Antennas

N x 1

N x 1

N x 1
MIMO - MCM

\[ S_{N \times 1} \]

Beamforming

\[ \begin{array}{c}
N \times 1 \\
\vdots \\
N \times 1 \\
\end{array} \]

MIMO Mod.

\[ N_t \text{ Transmit Antennas} \]

MIMO DS Wireless Channel

Effective MIMO Freq. Domain Channel

\[ r_{N \times 1} \]

Receiver Processing

\[ \begin{array}{c}
N \times 1 \\
\vdots \\
N \times 1 \\
\end{array} \]

MCM Demod.

\[ N_r \text{ Receive Antennas} \]

\[ N_{sub-carriers} \]

Data Estimates

\[ N_{x1} \]
The Challenge

- Delay spread $\Rightarrow$ inter-symbol interference (ISI)

- Doppler spread $\Rightarrow$ inter-carrier interference (ICI)

- Optimal receiver has very high complexity
  - When $N=512$, $N_t = N_r = 2$
  - More than $1 \times 10^6$ complex ICI coefficients !!

Can the structure of the MIMO Frequency Domain Channel be exploited?
MIMO - Sub-carrier Coupling Matrix

- Use guards / pulse shaping to suppress ISI
- ICI Profile
- $D_h$ = Significant ICI radius

ICI Power Profile (dB) of 2x2 MIMO-OFDM system with $N = 256$ sub-carriers
– Consider a local-ICI processing receiver which:

\[
(2D_r + 1)N_r = (2(D_r + D_i) + 1)N_r
\]

• Local-ICI processing performance/complexity controlled by “\(D_r\)”
Consider a local-ICI processing receiver which:

- Cancels ICI using previous symbol estimates $\Rightarrow$ Partial-SIC (P-SIC)
Local-ICI Processing

- Consider a local-ICI processing receiver which:
  - Cancels ICI using previous symbol estimates ⇒ Partial-SIC (P-SIC)
  - Performs local linear combining after P-SIC
  - Maximizes rate on each sub-carrier
Measuring Performance

– Local-ICI processing applicable to many MCM schemes…
– Choice of MCM scheme affects:
  • Decoding Complexity
  • Spectral Efficiency
  • Diversity Exploitation

– “Practical” performance metric should incorporate effects of all the above

– We measure the achievable ergodic rate when local-ICI processing is used on a generic MIMO-MCM system
Achievable Rate Metric (ARM)

- Assumptions:
  - Coding & Decoding over large blocks of MCM symbols
  - Each sub-carrier coded independently
  - Use of i.i.d. (complex) Gaussian codebooks

- ARM Definition:
  \[ R = E_H \left[ \sum_{k=0}^{N-1} \log(1 + \gamma_k) \right] \]

  - \( \gamma_k \): SINR for \( k^{th} \) sub-carrier using partial SIC and local linear combining.
  - SINR is function of beamforming vectors, local linear combiner and ICI coefficients.

- ARM Advantages:
  - Characterizes trade-off between complexity \( (D_r) \) and performance \( (R) \)
  - Compares performance of local-ICI processing on various MCM schemes at equal complexity
Rate Complexity Trade-off

- $N = 256$, $N_t = 1$, $N_r = 2$, 
  $f_d T_c = 0.016$, $SNR = 20$ dB

- Achievable rate saturates when $D_r > D_h$

- “Sweet spot” at $D_r = D_h$

- When $D_r = D_h$,
  - Achievable rate is more than 90% of maximum achievable rate
  - Complexity is less than 0.5% of optimal processing
Performance at $D_r = D_h = 1$

$N = 128$, $N_t = 1$, $N_r = 2$, $f_d T_c = 0.008$
Beamforming

– Wish to design beamforming vectors for sub-carriers
  • Assume CSI at transmitter
  • Desire good performance in the presence of ICI
  • Assume local-ICI processing receiver

– Consider a MIMO-OFDM system
  • MIMO-OFDM $\Rightarrow$ White Noise
  • MIMO-OFDM $\Rightarrow$ No ISI
  • Equal power across sub-carriers
Beamforming Approaches

- Traditional Max-SNR...
  - e.g., [Bolcskei et al, T.Comm., Feb ’02]
  - Not designed for ICI

- “Doubly selective” Max-SNR...
  - Maximize total received energy per sub-carrier

- Approximate Max-ARM
Traditional Max-SNR Beamforming

- Maximize SNR for each sub-carrier **independently**
- “signal” defined as $z_k^H H_k^o x_k s_k$
- Receiver performs antenna combining, but not sub-carrier combining ($D_r=0$)

Beamforming vector for $k^{th}$ sub-carrier: $x_k = v^* (H_k^o H_k^o)$
**“Doubly-Selective” Max-SNR Beamforming**

- Maximize SNR for each sub-carrier *independently*
- Receiver considers both antenna and sub-carrier combining ($D_r = D_h$)
- “signal” defined as $z_k^H H_{k,k} x_k s_k$

---

Beamforming vector for $k^{th}$ sub-carrier:

$$x_k = \nu^* (H_{k,k}^H H_{k,k})$$
Max-ARM Beamforming

– Motivation
  • SNR-maximization does not penalize ICI effects on other sub-carriers!
  • Can an ICI penalty be incorporated into beamformer design?

– Ideal solution: Maximize ARM
  • Leads to a difficult optimization problem!

\[
R = E_H \left[ \sum_{k=0}^{N-1} \log(1 + \gamma_k) \right]
\]

\[
\gamma_k = \frac{\mathcal{E}_{\text{sig}}(x_k, z_k, H)}{\mathcal{E}_{\text{ici}}(\{x_k\} k' \in \mathcal{K}_+(k), z_k, H) + \mathcal{E}_{\text{noise}}} \]
Approximate Max-ARM Beamforming

\[ R = E_H \log \left( 1 + \frac{\mathcal{E}_{\text{sig}} (x_k, z_k, H)}{\mathcal{E}_{\text{ici}} \left( \{x'_{k'} \}_{k' \in \mathcal{K}(k)} , z_k, H \right) + \mathcal{E}_{\text{noise}}} \right) \]

– Iterative optimization:
  • Given beamformers \( \{x_k\} \), choose combiners
    \[ z_k = \max_{\gamma_k} \gamma_k \text{, for } k = 0, ..., N-1 \]
    – Easy: generalized eigenvalue problem!

  • Given combiners \( \{z_k\} \), how to choose beamforming vectors \( \{x_k\} \)?
    – Note: Each \( x_k \) affects several \( \gamma_k \)

  • Use intuition from ARM for low-SNR and high-SNR regimes…
**Intuition from ARM: High-SNR Regime**

\[ R = E_H \left[ \sum_k \log(1 + \frac{\mathcal{E}_{\text{sig}}(x_k, z_k, H)}{\mathcal{E}_{\text{ici}}(\{x_{k'}\}_{k' \in \mathcal{K}_+(k)}, z_k, H) + \mathcal{E}_{\text{noise}}} \right) \]

- When designing \( x_k \):
  - Residual ICI dominates noise
  - Due to P-SIC, need not consider ICI caused to “future” sub-carriers \( \mathcal{K}_+(k) \)
  - Maximize signal, minimize ICI caused to “past” sub-carriers \( \mathcal{K}_-(k) \)
Intuition from ARM: Low-SNR Regime

\[ R = E_H \left[ \sum_k \log(1 + \frac{\mathcal{E}_{\text{sig}}(x_k, z_k, H)}{\mathcal{E}_{\text{ici}}(\{x_{k'}\}_{k' \in K(k)}), z_k, H) + \mathcal{E}_{\text{noise}}} \right) \]

- When designing \( x_k \):
  - Noise dominates residual ICI
  - Need to maximize signal energy

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Approximate Max-ARM Beamforming

With low-SNR and high-SNR intuitions, we propose:

\[
x_k^* = \arg \max_{\|x_k\|=1} \frac{x_k^H (H_{k,k}^H z_k z_k^H H_{k,k}^H) x_k}{x_k^H \left( \sum_{k' \in \mathcal{K}_{(k)}} H_{k',k}^H z_k z_k^H H_{k',k}^H + \sigma^2 I \right) x_k}, \quad k = 0, \ldots, N - 1
\]

Note that:

- At high SNR, prevent causing ICI to “past” sub-carriers
- At low SNR, maximize signal energy
## Complexity

<table>
<thead>
<tr>
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<th>Traditional Max-SNR</th>
<th>“Doubly Selective” Max-SNR</th>
<th>Approximate Max-ARM</th>
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<tr>
<td><strong>Beamformer Design</strong> (per OFDM symbol)</td>
<td>$O(N N_t^3)$</td>
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<td>$O(N_i N N_t^3)$</td>
</tr>
<tr>
<td><strong>Combiner Design</strong> (per OFDM symbol)</td>
<td>$O(N N_r^3)$</td>
<td>$O(N (2 D_r + 1)^3 N_r^3)$</td>
<td>$O(N_i N (2 D_r + 1)^3 N_r^3)$</td>
</tr>
<tr>
<td><strong>Aggregate</strong> (per OFDM symbol)</td>
<td>$O(N (N_t^3 + N_r^3))$</td>
<td>$O(N ((2 D_r + 1)^3 N_r^3 + N_t^3))$</td>
<td>$O(N_i N ((2 D_r + 1)^3 N_r^3 + N_t^3))$</td>
</tr>
</tbody>
</table>
Simulation Setup

- System:
  - \((N_t, N_r = 2, N = 128)\) MIMO-OFDM system
  - Bandwidth = 1.5 MHz, Carrier Frequency = 60 GHz
  - Delay spread 10.8 \(\mu\)s, or equivalently 16 chips

- Compare with two benchmarks:
  - Traditional Max-SNR beamforming
  - Upper Bound:
    - “Doubly Selective” Max-SNR beamforming
    - Genie-aided perfect “past” and “future” ICI-cancellation at receiver
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Results - I

\[ N_t = 8 \]

\[ f_d T_c = 0.008 \]

\[ N_t = 6 \]

\[ N_t = 4 \]
Despite $v = 3 \times 138 \text{ km/hr}$, $f_d T_c = 0.016$, $D_h = 2$

$v = 3 \times 69 \text{ km/hr}$, $f_d T_c = 0.008$, $D_h = 1$
acsi at Transmitter

- Practically, perfect transmitter CSI is impossible

- Consider Time Division Duplex Operation

  - Easy to obtain CSI when in receive mode
    - pilots / decision directed channel estimation

  - This (outdated) CSI can be forward-predicted when in transmit mode

  - For our setup, we use MMSE prediction
    - Use channel correlation from WSSUS Jakes’ model
Results - III

$v = 3 \times 69 \text{ km/hr} \quad f_d T_c = 0.008 \quad D_h = 1$

$v = 3 \times 138 \text{ km/hr} \quad f_d T_c = 0.016 \quad D_h = 2$
Conclusion

– For MCM over DS channels:
  
  • Considered low-complexity receiver processing

  • Derived an ARM

  • Showed the versatility of our ARM
    – Characterized Rate / Complexity Trade-off
    – Complexity-constrained performance evaluation