

# Design of Multi-Carrier Modulation for Doubly Selective Channels Based on a Complexity-Constrained Achievable Rate Metric

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**Abstract**—Multi-carrier modulation (MCM) is widely studied as a communication technique for time- and frequency selective (hence, doubly selective) channels. MCM schemes are usually designed either to reduce decoding complexity, to exploit diversity gains, or to enhance spectral efficiency. However, no known scheme accounts for all three concerns. In this paper, we propose a design metric based on complexity-constrained achievable rate that accounts for all three of these concerns. We then use this metric to characterize a trade-off between achievable rate and implementation complexity, assuming frequency-domain processing at the receiver. Finally, we use this metric to compare several MCM schemes under the same level of receiver complexity.<sup>1</sup>

## I. INTRODUCTION

Multi-carrier modulation (MCM) has been extensively studied as a practical method for communication over channels which exhibit both time-selective and frequency-selective fading, i.e., doubly selective (DS) fading. The principle challenge faced when using MCM over these channels is effectively combating a rich and quickly varying inter-symbol interference (ISI) plus inter-carrier interference (ICI) response. For reasons of complexity, practical MCM reception strategies decode data by taking into account only the *significant* ISI/ICI coefficients. This is more important in single-input multiple-output (SIMO) systems, where the number of channel coefficients scales with the number of receive antennas. For example, consider that, in a 128 subcarrier system with 2 receive antennas, there will be  $2 \times 128 \times 128 = 32768$  ICI coefficients alone, each of which can be expected to change substantially from one MCM symbol to the next. If we consider ISI from adjacent symbols, then the total number of channel coefficients becomes  $3 \times 32768 = 98304$ . Since the processing of so many coefficients is generally impractical, it is common to design the MCM system so that, say, all but adjacent subcarrier ICI is suppressed. For such a system, there would be only  $2 \times 3 \times 128 = 768$  significant coefficients to consider. We consider the use of only a few significant ISI/ICI coefficients as a form of *complexity constraint* at the receiver.

For a given set of channel spreading characteristics, the number of significant ISI/ICI coefficients is a function of the MCM pulse shape as well as the time-frequency spacing

between the MCM pulses. The time-frequency spacing affects the system's spectral efficiency and hence must be chosen with care. The pulse shape determines the time-frequency dispersion of the pulses, which affects the number of significant ISI/ICI coefficients. It also determines the correlation between these coefficients, which affects the diversity that can be exploited by the decoder. In short, MCM schemes should be designed with decoding complexity, spectral efficiency, as well as diversity exploitation in mind.

We are not aware of any MCM design strategies which attempt to address all three of these aspects simultaneously, however. For example, the classical MCM schemes which minimize ISI/ICI subject to (bi)orthogonal pulse constraints (e.g., [1]–[4]) admit good ISI/ICI suppression only at relatively low spectral efficiencies (as a consequence of the Balian-Low Theorem [5]) and never consider coefficient correlation. As another example, modern MCM schemes can attain much higher spectral efficiencies while maintaining only a small number of non-negligible ISI/ICI coefficients (e.g., [6]–[9]) but do so without explicit concern for ISI/ICI coefficient correlation. As yet another example, so-called “maximum diversity” pulses [10], [11] have been proposed at the price of low spectral efficiency and high implementation complexity.

In this paper, we propose a design metric that incorporates all three design metrics, i.e., implementation complexity as well as spectral efficiency and coefficient correlation. Specifically, we propose to measure the *achievable rate* of a generic SIMO-MCM scheme *under a local-ICI processing constraint*, i.e., when decoding a subcarrier, only the significant ICI from neighboring subcarriers is considered. Note that the number of considered ICI coefficients determines the complexity of the reception strategy.

We show the utility of the achievable metric in two ways. First, we use the derived metric to characterize a trade-off between achievable rate and implementation complexity for MCM schemes. Next, we compare the performance of select MCM schemes with receivers with the same level of implementation complexity.

The paper is organized as follows: Sec. II describes the system model, Sec. III presents the achievable rate metric, Sec. IV considers uses of the proposed metric and Sec. V

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concludes.

*Notation:* In the paper,  $(\cdot)^T$  to denote transpose, and  $(\cdot)^H$  the conjugate transpose. The  $K \times K$  identity matrix is denoted by  $\mathbf{I}_K$  and its  $l^{\text{th}}$  column by  $\mathbf{e}_K(l)$ . Additionally,  $[\mathbf{B}]_{m,n}$  denotes the element in the  $m^{\text{th}}$  row and  $n^{\text{th}}$  column of  $\mathbf{B}$ , where row/column indices begin with zero. Kronecker product is denoted by  $\otimes$ . Also,  $\delta_l$  denotes the Kronecker delta with argument  $l$ ,  $\langle \cdot \rangle_N$  the modulo- $N$  operation, and  $\mathbb{C}$  the set of all complex numbers. Expectation is denoted by  $E(\cdot)$  and autocovariance by  $\Sigma_{\mathbf{b}} := E(\mathbf{b}\mathbf{b}^H) - E(\mathbf{b})E(\mathbf{b}^H)$ . Finally, the mutual information (MI) between two random entities  $X$  and  $Y$  is denoted by  $I(X; Y)$ . When conditioned on a realization of the random entity  $Z$ , MI is denoted by  $I(X; Y | Z)$ .

## II. SYSTEM MODEL AND RECEPTION STRATEGY

In this paper, we focus on a single-input multiple-output (SIMO) MCM system with  $N_r$  receive antennas and  $N$  subcarriers, and refer to it as a  $(N_r, N)$  SIMO-MCM system. In the considered system, coding is done over large blocks of (say,  $N_b$ ) MCM symbols. Further, we assume that coding is done independently for each subcarrier using i.i.d. (complex) Gaussian codebooks, and that an average power constraint is enforced. Thus, in the considered  $(N_r, N)$  SIMO-MCM system, the  $i^{\text{th}}$  MCM symbol is  $\mathbf{s}(i) = [s_0(i), s_1(i), \dots, s_{N-1}(i)]^T$ , where  $s_k(i)$  is the  $i^{\text{th}}$  symbol of the codeword on the  $k^{\text{th}}$  subcarrier. The average power constraint and the use of i.i.d. Gaussian codebooks implies that each MCM symbol is zero mean i.i.d. complex Gaussian distributed with covariance matrix  $\Sigma_{\mathbf{s}} = \mathbf{I}_N$ . The components of  $\mathbf{s}(i)$  are then modulated onto separate subcarriers using a (possibly non-rectangular) MCM pulse shape.

The transmitted signal encounters a wireless channel en route to each receive antenna. Each channel is Rayleigh faded with a delay spread of  $N_h$  chips, a uniform delay-power profile, a (chip normalized) single-sided maximum Doppler spread of  $f_d T_c$ , and satisfies the wide sense stationary uncorrelated scattering (WSSUS) property [12]. In this paper, we assume that these channels are statistically independent.

At each receive antenna, the received signal is demodulated using another MCM pulse shape, and the resulting frequency domain observation corresponding to transmitted MCM symbol  $\mathbf{s}(i)$  is  $\mathbf{r}(i) := [\mathbf{r}_0(i), \mathbf{r}_1(i), \dots, \mathbf{r}_{N-1}(i)]^T$ , where the entries of  $\mathbf{r}_k(i) \in \mathbb{C}^{N_r}$  correspond to the  $k^{\text{th}}$  subcarrier at the  $N_r$  receive antennas. It is assumed that ISI is negligible, i.e., that  $\mathbf{s}(i)$  only influences  $\mathbf{r}(i)$  and has a negligible effect on  $\mathbf{r}(i_1)$ , when  $i_1 \neq i$ . This can be ensured by using guards to suppress inter block interference (IBI) perfectly at the expense of spectral efficiency, as in traditional cyclic prefix orthogonal frequency division multiplexing (CP-OFDM) [13], [10], [14]. Alternatively, specially designed pulse shaped MCM (PS-MCM) can be used to adequately (but not perfectly) suppress IBI without sacrificing spectral efficiency as in [6], [8], [15]. Then, the observation  $\mathbf{r}(i)$  can be expressed as

$$\mathbf{r}(i) = \mathbf{H}(i)\mathbf{s}(i) + \mathbf{w}(i), \quad (1)$$

where  $\mathbf{w}(i) \in \mathbb{C}^{N_r N}$  are samples of zero mean additive complex Gaussian noise with covariance  $\Sigma_{\mathbf{w}}$  and  $\mathbf{H}(i) \in \mathbb{C}^{N_r N \times N}$  is the SIMO frequency domain channel matrix (SIMO-FDCM). The SIMO-FDCM can be obtained by

$$\mathbf{H} = \sum_{k_r=1}^{N_r} \mathcal{H}_{k_r}(i) \otimes \mathbf{e}_{N_r}(k_r), \quad (2)$$

In (2),  $\mathcal{H}_{k_r}(i) \in \mathbb{C}^{N \times N}$  is the subcarrier coupling matrix (SCM) for the  $k_r^{\text{th}}$  receive antenna where  $[\mathcal{H}_{k_r, k_t}(i)]_{m_1, m_2}$  depicts the influence of  $s_{m_2}(i)$  on the observation for the  $m_1^{\text{th}}$  subcarrier at the  $k_r^{\text{th}}$  receive antenna. Thus, off-diagonal entries of the SCM cause ICI. (See [6]–[8], [15] for an expression relating  $\mathcal{H}_{k_r}$  to the channel impulse response.) The typical low-pass nature of Doppler spreading for the WSSUS Rayleigh faded channel implies that the ICI due to a subcarrier is significant only in observations for a few neighboring subcarriers and is restricted to rather low levels outside this neighborhood. The radius (in number of subcarriers) within which a subcarrier causes significant ICI is called the ICI spread,  $D_h$ . The ICI spread is much smaller than the number of subcarriers for most MCM schemes. As a result, significant entries of the SIMO-FDCM  $\mathbf{H}(i)$  are located in a quasi-block-banded region depicted in Fig. 1. Next, we describe a low complexity local-ICI processing receiver that exploits the structure of the SIMO-FDCM.

As a result of the coding strategy used, blocks of  $N_b$  observation vectors are processed together and subcarriers are decoded individually at the receiver. Inspired by minimum mean squared error equalization and sequential interference cancellation receivers for i.i.d. Rayleigh faded MIMO channels [16], we design receiver processing along the same lines. That is to say, in the reception strategy considered, local linear combining is performed for the first subcarrier ( $k = 0$ ) for each of the  $N_b$  blocks and the obtained symbol estimates are used to decode information on the first subcarrier. Assuming a judicious rate-allocation and consequent error free decoding, the interference from the symbols on the first subcarrier can be regenerated and removed from observations for neighboring subcarriers on which it has significant influence. These steps are then repeated for the second ( $k = 1$ ) subcarrier, and so on.

Recall that each subcarrier influences observations for a few neighboring subcarriers. We, thus, consider a receiver where only observations for subcarriers within a radius of  $D_r$  subcarriers around that subcarrier,  $\mathbf{r}_k(i)$ , are used in the reception process, i.e.  $\mathbf{r}_k(i) := [\mathbf{r}_{\langle k-D_r, N \rangle}^T(i), \dots, \mathbf{r}_{\langle k+D_r, N \rangle}^T(i)]^T$ . Then  $\mathbf{r}_k(i) \in \mathbb{C}^{(2D_r+1)N_r}$  can be written as

$$\mathbf{r}_k(i) = \sum_{k'=0}^{N-1} \mathbf{h}_{k, k'}(i) s_{k'}(i) + \mathbf{w}_k(i), \quad (3)$$

where  $\mathbf{h}_{k, k'} \in \mathbb{C}^{N_r(2D_r+1)}$  is a vector with entries from the SIMO-FDCM that captures the influence of  $s_{k'}(i)$  on  $\mathbf{r}_k(i)$  and  $\mathbf{w}_k(i)$  is a vector of noise samples with covariance  $\Sigma_{\mathbf{w}_k}$  that affect  $\mathbf{r}_k(i)$ . Notice from Fig. 1 that only  $2(D_r + D_h) + 1$  subcarriers have significant influence on  $\mathbf{r}_k(i)$ . In the proposed

receiver, only these  $(2D+1)N_r \times (2(D_r+D_h)+1)$  channel coefficients are used in processing for a given subcarrier (see Fig. 1). Thus, we only cancel interference from neighboring subcarriers that have already been decoded. This *partial sequential interference cancellation* (P-SIC) process can be expressed as

$$\mathbf{y}_k(i) = \mathbf{r}_k(i) - \sum_{k' \in \mathcal{K}(k)} \mathbf{h}_{k,k'}(i) s_{k'}(i), \quad (4)$$

where the index set  $\mathcal{K}(k)$  is defined as

$$\mathcal{K}(k) := \{ \langle k \pm l \rangle_N : 1 \leq l \leq D_h + D_r \} \cap \{ l : l < k \}, \quad (5)$$

Next, the observations in  $\mathbf{y}_k(i)$  are linearly combined. Since only the observations corresponding to subcarriers in the neighborhood of any given subcarrier are combined, we call this step *local combining* (LC). LC can be represented by

$$\phi_k(i) = \mathbf{z}_k^H(i) \mathbf{y}_k(i), \quad (6)$$

where  $\mathbf{z}_k(i)$  is referred to as the *local linear combiner* (LLC). The final expression for  $\mathbf{z}_k(i)$  will be derived in Sec. III.

Finally, notice that the integer parameter  $D_r$  controls both the performance (increasing  $D_r$  increases the number of observations used, enhancing performance) and the complexity of local processing (since the complexity is dominated by the  $\mathcal{O}((2D_r+1)^3 N_r^3)$  design of the LLC for each subcarrier). In the sequel, we derive a metric to measure the performance of a  $(N_r, N)$  SIMO-MCM system with local processing at the receiver.

### III. THE ACHIEVABLE RATE METRIC

In this section, we focus on deriving the achievable rate of the proposed local processing receiver. As each MCM symbol is processed in identical fashion, we consider one MCM symbol *w.l.o.g* and drop the MCM symbol indices for brevity.

The mutual information (MI) between the observation and the MCM symbol for such a system<sup>2</sup> can be written using the chain rule [17] as

$$I(\mathbf{r}; \mathbf{s}) = I(\mathbf{r}; s_0) + \sum_{k=1}^{N-1} I(\mathbf{r}; s_k | \{s_{k'}\}_{k'=0}^{k-1}). \quad (7)$$

Since, to save computation, only  $\mathbf{r}_k$  is used in demodulating the  $k^{\text{th}}$  subcarrier, the achievable rate of our  $(N_r, N)$  SIMO-MCM system reduces to

$$c(D_r) := I(\mathbf{r}_0; s_0) + \sum_{k=1}^{N-1} I(\mathbf{r}_k; s_k | \{s_{k'}\}_{k'=0}^{k-1}). \quad (8)$$

An application of the data processing inequality [17] shows that  $c(D_r)$  is a lower bound for  $I(\mathbf{r}; \mathbf{s})$ . An achievable lower bound for  $c(D_r)$  is calculated here by bounding each term on the *r.h.s.* of (8) from below. This lower bound emulates the local processing strategy from Sec. II.

<sup>2</sup>We consider coherent communication. The mutual information calculated here is conditioned on the realization of  $\mathbf{H}$ . However, we drop the conditioning on  $\mathbf{H}$  from the notation for brevity.

The  $k^{\text{th}}$ -subcarrier observation after P-SIC can be written, from (3) and (4), as

$$\mathbf{y}_k = \mathbf{h}_{k,k} s_k + \sum_{k' \in \bar{\mathcal{K}}(k)} \mathbf{h}_{k,k'} s_{k'} + \mathbf{w}_k. \quad (9)$$

In (9), the indices of the subcarriers that contribute interference to  $\mathbf{y}_k$  are collected in  $\bar{\mathcal{K}}(k)$ . Then

$$\bar{\mathcal{K}}(k) := \{ l : 0 \leq l < N \} \setminus (\mathcal{K}(k) \cup \{k\}). \quad (10)$$

Realize that P-SIC reduces interference, while leaving the signal component unchanged, as seen in (9). Thus

$$I(\mathbf{y}_k, s_k | \{s_{k'}\}_{k' \in \mathcal{K}(k)}) < I(\mathbf{r}_k; s_k | \{s_{k'}\}_{k'=0}^{k-1}) \quad (11)$$

Next, let  $\mathbf{z}_k \in \mathbb{C}^{(2D_r+1)N_r}$  be a LLC operating on  $\mathbf{y}_k$ . Then LC in (6) can be written using (9) as

$$\phi_k = \mathbf{z}_k^H \mathbf{h}_{k,k} s_k + \sum_{k' \in \bar{\mathcal{K}}(k)} \mathbf{z}_k^H \mathbf{h}_{k,k'} s_{k'} + \mathbf{z}_k^H \mathbf{w}_k. \quad (12)$$

Since the ICI will be Gaussian,

$$I(\mathbf{z}_k^H \mathbf{y}_k, s_k | \{s_{k'}\}_{k' \in \mathcal{K}(k)}) = \log(1 + \gamma_k(\mathbf{z}_k)), \quad (13)$$

where  $\gamma_k(\mathbf{z}_k)$  is the SINR of the  $k^{\text{th}}$  subcarrier after combining with  $\mathbf{z}_k$ . As a result,  $\gamma_k(\mathbf{z}_k)$  can be expressed using expectations over the joint source-noise distribution as

$$\gamma_k(\mathbf{z}_k) = \frac{\mathbb{E} [|\mathbf{z}_k^H \mathbf{h}_{k,k} s_k|^2]}{\mathbb{E} \left[ \left| \sum_{k' \in \bar{\mathcal{K}}(k)} \mathbf{z}_k^H \mathbf{h}_{k,k'} s_{k'} + \mathbf{z}_k^H \mathbf{w}_k \right|^2 \right]} \quad (14)$$

$$= \frac{\mathbf{z}_k^H \mathbf{h}_{k,k} \mathbf{h}_{k,k}^H \mathbf{z}_k}{\mathbf{z}_k^H \left( \sum_{k' \in \bar{\mathcal{K}}(k)} \mathbf{h}_{k,k'} \mathbf{h}_{k,k'}^H + \boldsymbol{\Sigma}_{\mathbf{w}_k} \right) \mathbf{z}_k}. \quad (15)$$

In (15),  $\boldsymbol{\Sigma}_{\mathbf{w}_k}$  is the covariance of additive noise (and any neglected IBI). It is well known that the MMSE combiner in (16) maximizes  $\gamma_k$  and, consequently, the MI in (13).

$$\mathbf{z}_k^* = \left( \sum_{k' \in \bar{\mathcal{K}}(k)} \mathbf{h}_{k,k'} \mathbf{h}_{k,k'}^H + \boldsymbol{\Sigma}_{\mathbf{w}_k} \right)^{-1} \mathbf{h}_{k,k}. \quad (16)$$

In fact, MMSE combining is information lossless given that  $\mathbf{w}_k$  is Gaussian distributed [16], *i.e.*

$$I(\mathbf{y}_k, s_k | \{s_{k'}\}_{k' \in \mathcal{K}(k)}) = \log(1 + \gamma_k(\mathbf{z}_k^*)). \quad (17)$$

Thus,  $\mathbf{z}_k^*$  appears to be the obvious choice for the LLC. However, notice that  $\mathbf{z}_k^*$  is a function of channel coefficients for subcarriers beyond the radius of  $D_r + D_h$  subcarriers around the  $k^{\text{th}}$  subcarrier. Thus, we use the following (suboptimal) combiner instead:

$$\tilde{\mathbf{z}}_k = \left( \sum_{k' \in \bar{\mathcal{K}}(k)} \mathbf{h}_{k,k'} \mathbf{h}_{k,k'}^H + \boldsymbol{\Sigma}_{\mathbf{w}_k} \right)^{-1} \mathbf{h}_{k,k}. \quad (18)$$

In (18), the set  $\tilde{\mathcal{K}}(k)$  is defined as

$$\tilde{\mathcal{K}}(k) := \bar{\mathcal{K}}(k) \cap \{ \langle k \pm l \rangle_N : 1 \leq l \leq D_r + D_h \}. \quad (19)$$

The intuition for using  $\tilde{z}_k$  is that, with proper choice of  $D_r$ , the ICI due to symbols beyond a radius of  $D_r + D_h$  subcarriers can be made small compared to the ICI due to symbols within this radius. Then

$$I(\tilde{\mathbf{z}}_k^H \mathbf{y}_k, s_k | \{s_{k'}\}_{k' \in \mathcal{K}(k)}) = \log(1 + \gamma_k(\tilde{z}_k)), \quad (20)$$

and it is easily seen that

$$I(\mathbf{y}_k, s_k | \{s_{k'}\}_{k'=0}^{k-1}) \geq \log(1 + \gamma_k(\tilde{z}_k)). \quad (21)$$

Using the discussion in this section, it is straightforward to show that

$$I(\mathbf{r}, \mathbf{s}) \geq c(D_r) \geq \sum_{k=0}^{N-1} \log(1 + \gamma_k(\tilde{z}_k)). \quad (22)$$

Finally, averaging over channel realizations, we arrive at

$$E_{\mathbf{H}} [I(\mathbf{r}, \mathbf{s})] \geq \underbrace{E_{\mathbf{H}} \left[ \sum_{k=0}^{N-1} \log(1 + \gamma_k(\tilde{z}_k)) \right]}_{C(D_r)}. \quad (23)$$

Henceforth, we refer to  $C(D_r)$  in (23) as the achievable rate metric (ARM).

#### IV. APPLICATIONS OF THE ARM

The ARM can be used to study various aspects of low-complexity local-ICI processing on SIMO-MCM systems. Two such examples follow. First, we present an approach to select a suitable  $D_r$  using the ARM in Sec. IV-A. Next, we compare the performance of local processing on select MCM schemes in Sec. IV-B.

##### A. Rate-Complexity Trade-off

In this section, we present an approach to select a suitable value of the design parameter  $D_r$  by characterizing the trade-off between the achievable rate and complexity of local processing. In the sequel, we refer to this trade-off as the *rate complexity trade-off* (RCT).

The previously defined ARM can be used to measure the performance of a SIMO-MCM system as a function of  $D_r$ . Meanwhile, the complexity of local processing is  $\mathcal{O}((2D_r + 1)^3 N_r^3 N)$  per MCM-symbol, which is cubic in  $D_r$ . Thus, RCT for a given SIMO-MCM system can be described by the pair  $(D_r, C(D_r))$ . The RCTs for various  $(N_r = 2, N = 256)$  SIMO-MCM systems are plotted in Fig. 2. In Fig. 2, MSINR refers to the jointly optimized max-SINR pulses from [6], [15], while GP refers to the MCM scheme with Gaussian prototype pulses that are dilated to minimize out-of-target ICI/ISI, and OFDM refers to standard CP-OFDM. For this particular example, transmission is over channels with  $N_h = 16$  equal power taps, each with  $f_d T_c = 0.016$ , at SNR 20dB. This setup yields ICI spread  $D_h = 4$ .

Figure 2 shows similar RCT trends for each of the MCM schemes. When  $D_r \leq D_h$ , the achievable rate  $C(D_r)$  grows quickly in  $D_r$ . When  $D_r > D_h$ , however,  $C(D_r)$  grows much more slowly in  $D_r$ . Intuitively, this can be explained as

follows. When  $D_r < D_h$ , increases in  $D_r$  gather significantly more signal energy, and hence lead to significantly higher achievable rates. Increasing  $D_r$  beyond  $D_h$  gathers only traces of additional signal energy, and hence only marginal increases in achievable rate. In conclusion, the RCT “sweet spot” corresponds to  $D_r = D_h$ . For this value, the systems tested support over 90% of the maximum achievable rate. Yet, the complexity incurred is less than 0.5% of that for optimal processing.

##### B. Comparison of MCM Schemes

We compare the rates achieved by local processing on the same set of MCM schemes that were considered for the RCT in Sec. IV-A. The MCM schemes are used on a  $(2, 128)$  SIMO-MCM system, where transmission is over channels with  $N_h = 16$  chip delay spreads, uniform power profiles, and  $f_d T_c = 0.008$  (hence,  $D_h = 1$ ). Local processing at the receiver uses  $D_r = D_h = 1$ . Each data point is an average of measurements over  $10^3$  channel realizations.

In Fig. 3, it is clear that the MSINR scheme outperforms CP-OFDM, which, in turn, performs better than the Gaussian prototype pulsed (GPP) scheme. Recall that as the SNR increases, performance is limited by uncanceled out-of-band ICI and IBI. The results in Fig. 3 can directly be related to the out-of-band ICI/IBI suppression capabilities of these MCM schemes. For instance, the MSINR pulses provide the best out-of-band ICI/IBI suppression, and consequently, support the highest rate of the three. Also note that the GPP scheme suppresses ICI better than CP-OFDM. However, CP-OFDM suppresses IBI completely at the expense of spectral efficiency, whereas, the the performance of GPP is hampered by IBI. For our setup, the loss in rate due to the guards in CP-OFDM is smaller than the rate loss due to uncanceled IBI in the GPP scheme. Thus for our setup, CP-OFDM performs better than GPP MCM.

In a nutshell, the schemes compared here are designed according to different philosophies and have different spectral efficiencies. Yet, the ARM provides a *fair* means of comparing them under the assumption of low-complexity reception.

#### V. CONCLUSION

In this paper, we derived an achievable rate metric to measure the performance of local processing for SIMO-MCM systems with a complexity constraint. The utility of the metric was shown in two ways, by characterizing a trade-off between achievable rate and implementation complexity, and by making a fair comparison of local processing on various MCM schemes with a complexity constraint.

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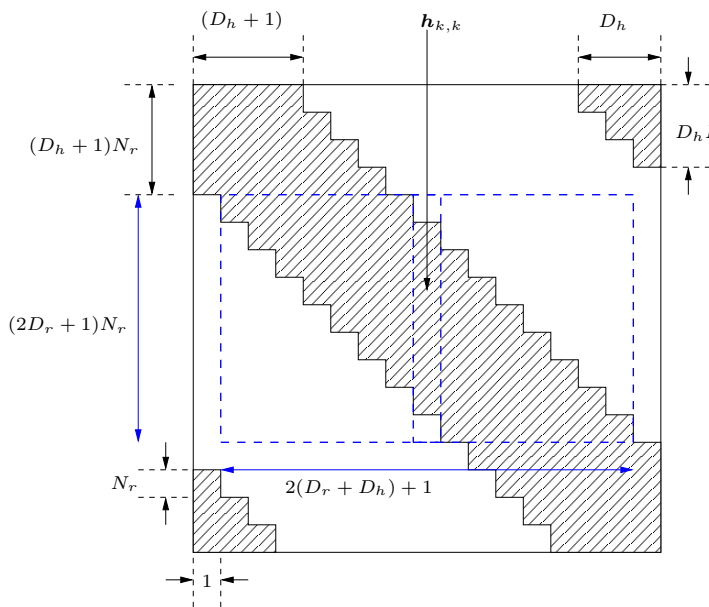


Fig. 1. Approximate structure of SIMO-FDCM. Rectangle (in dotted lines) indicates the channel coefficients used for local processing for the  $k^{\text{th}}$  subcarrier.

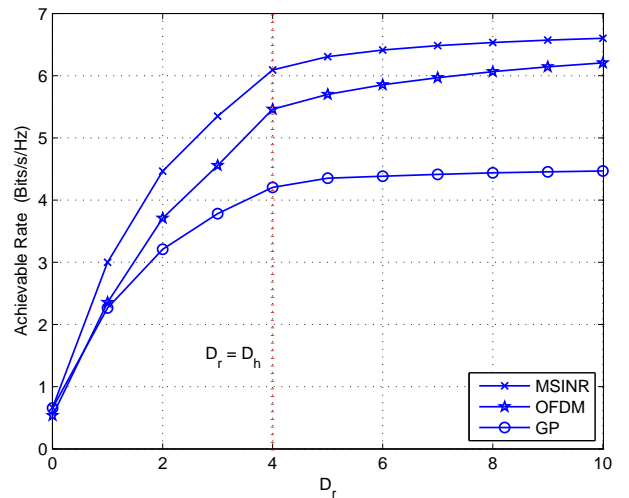


Fig. 2. Achievable rate vs. equalization complexity ( $D_r$ ) for cyclic prefix OFDM, jointly optimized max-SINR MCM scheme, and Gaussian pulsed MCM scheme for a (2, 256) SIMO-MCM system.

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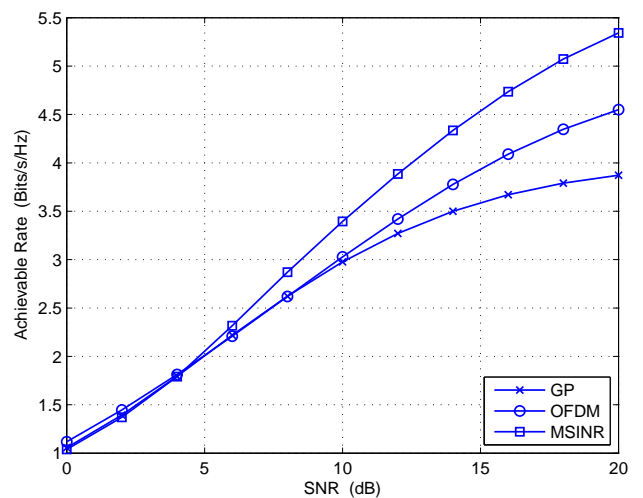


Fig. 3. Achievable rate for cyclic prefix OFDM, jointly optimized max-SINR MCM scheme, and Gaussian pulsed MCM scheme for a (2, 128) SIMO-MCM system with local processing reception using  $D_r = D_h = 1$ .