Pulse-Shaped FDM for Doubly-Dispersive Channels

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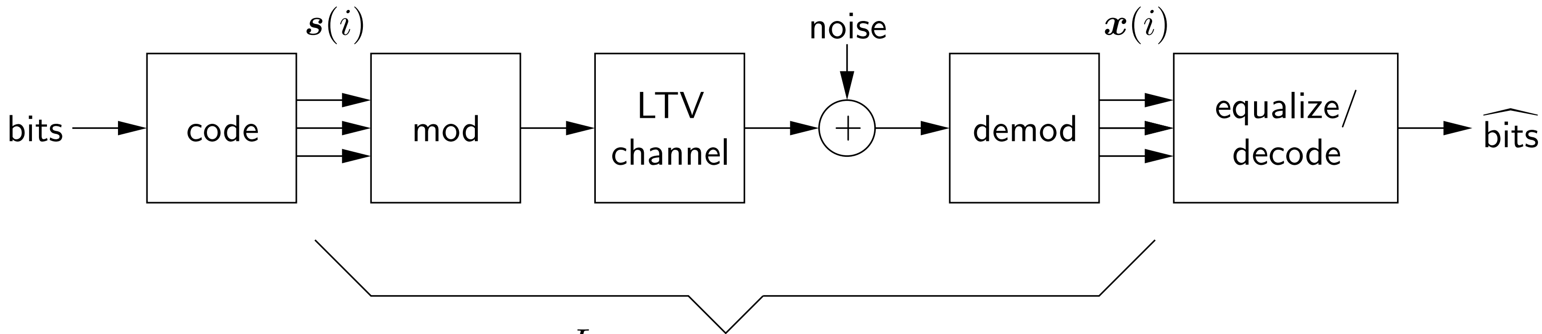
Nov. 8, 2004

Multicarrier Modulation:



$$\boldsymbol{x}(i) = \sum_{j=-L_{\text{pre}}}^{L_{\text{pst}}} \boldsymbol{\mathcal{H}}(i,j)\boldsymbol{s}(i-j) + \boldsymbol{w}(i)$$

“**LTV MIMO channel**”

- Modulator: multicarrier symbols $\{\boldsymbol{s}(i)\} \rightarrow$ waveforms,
- Demodulator: waveforms $\rightarrow$ multicarrier observations $\{\boldsymbol{x}(i)\}$.

How should we design modulator & demodulator?

How should we design equalizer/decoder?

The Doubly-Dispersive Channel:

- We focus on time-frequency (i.e., doubly) dispersive channels.
- No fixed eigenbasis for these channels, so ISI/ICI is unavoidable in the absence of transmitter channel knowledge.
- Without dispersion, Nyquist theory specifies a maximum of 1 symbol/sec/Hz for interference-free modulation/demodulation.
- Roughly, as symbol/sub-carrier spacings are increased,
  - ISI/ICI decreases (good!), but
  - modulation efficiency decreases (bad!).

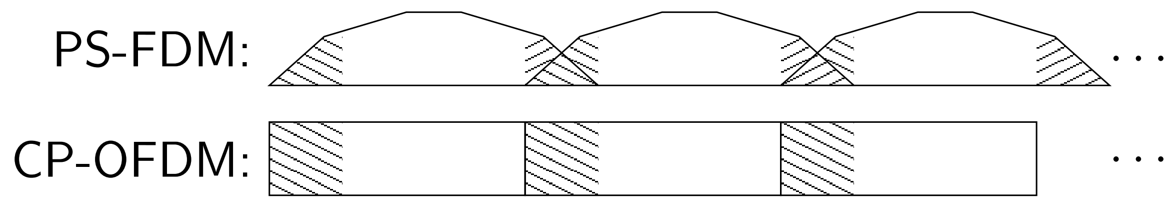
$\rightsquigarrow$ *Inherent tradeoff between modulation efficiency and ISI/ICI.*

Modulator Design Philosophy:

- Traditionally, (bi)orthogonal pulse families:
  - Zero ISI/ICI in non-dispersive (i.e., trivial) channels.
  - Low ISI/ICI in non-trivial channels $\Leftrightarrow$ low modulation efficiency.
- Our approach: non-(bi)orthogonal pulse families:
  - We don't expect trivial channels, so why design for them?
  - We do expect to have an equalizer, so why not leverage it?
- Main ideas:
  - Shape, rather than suppress, ISI/ICI.
  - Design waveforms to yield a target ISI/ICI response that
    - is attainable (i.e., suited to the typical channel),
    - allows low-complexity equalization/decoding.
  - An outage-capacity analysis suggests that shaping has advantages over suppression. (More later. . .)

Pulse-Shaped FDM:

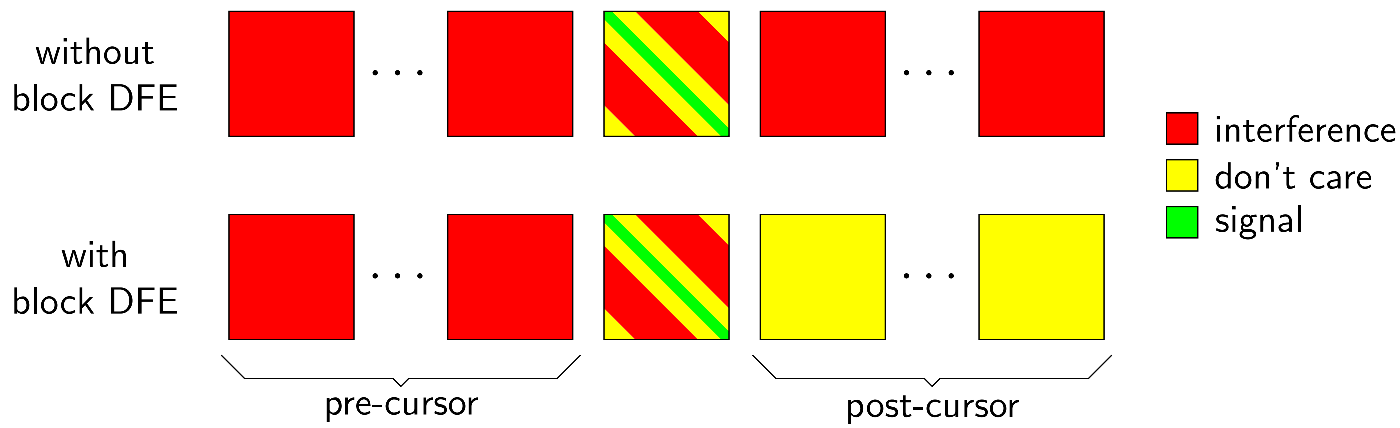
- Like CP-OFDM but with smooth overlapping mod/demod pulses.



- Complexity on par with CP-OFDM.
- Since ISI/ICI always present, no explicit need for a guard interval.
  - Higher modulation efficiency than OFDM.
  - Possible to overload the signal space (i.e., >1 symbol/sec/Hz), though equalization/decoding becomes more challenging.

Pulse Design:

Target MIMO channel $\{\boldsymbol{\mathcal{H}}(i, -L_{\text{pre}}), \ldots, \boldsymbol{\mathcal{H}}(i, L_{\text{pst}})\}$:



Joint SINR-maximizing pulses $\{\boldsymbol{a}, \boldsymbol{b}\}$:

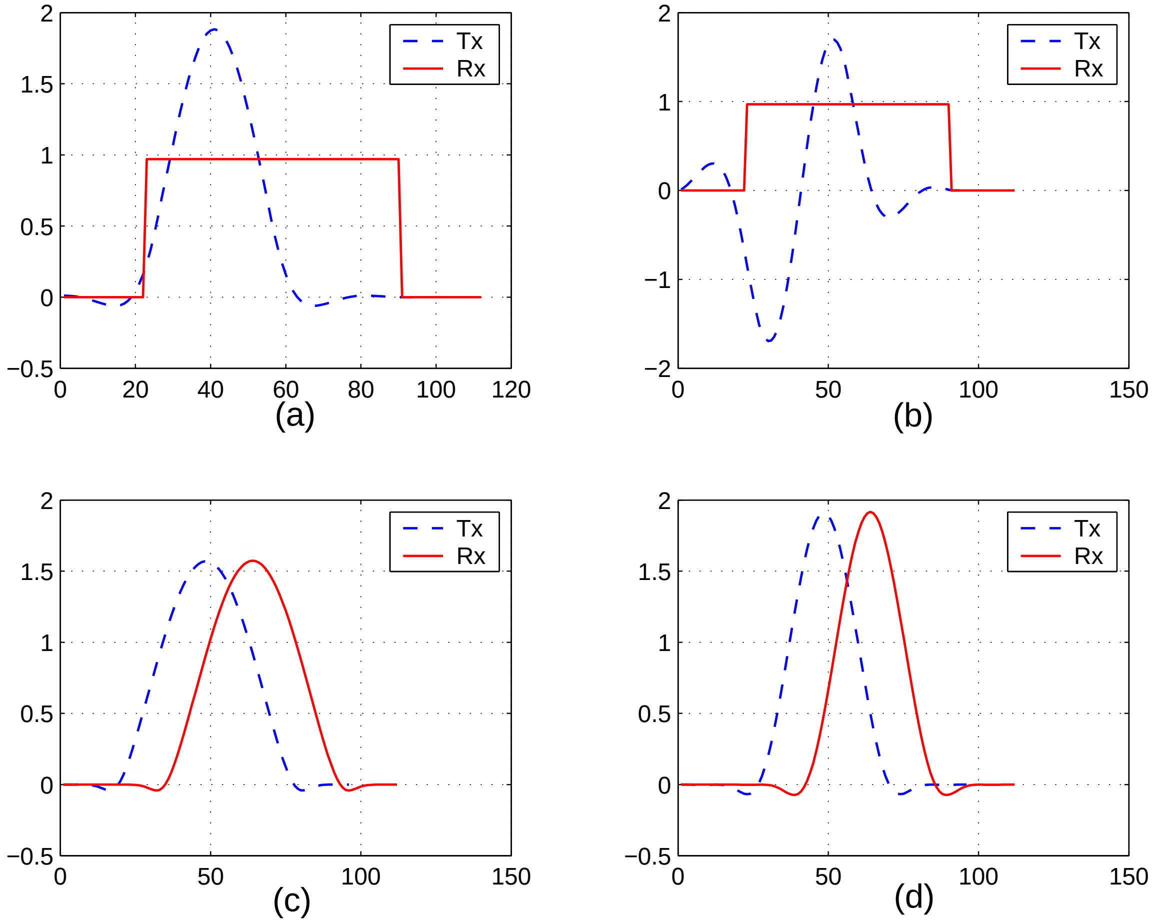
$$\boldsymbol{a}^{(i)} = \underset{\|\boldsymbol{a}\|^2 = N_s}{\arg\max} \frac{\boldsymbol{a}^H \boldsymbol{P}_{\text{s}}(\boldsymbol{b}^{(i)})\boldsymbol{a}}{\boldsymbol{a}^H \boldsymbol{P}_{\text{ni}}(\boldsymbol{b}^{(i)})\boldsymbol{a}} \quad \text{Tx pulse}$$

$$\boldsymbol{b}^{(i+1)} = \underset{\|\boldsymbol{b}\|^2 = N_s}{\arg\max} \frac{\boldsymbol{b}^H \boldsymbol{Q}_{\text{s}}(\boldsymbol{a}^{(i)})\boldsymbol{b}}{\boldsymbol{b}^H \boldsymbol{Q}_{\text{ni}}(\boldsymbol{a}^{(i)})\boldsymbol{b}} \quad \text{Rx pulse}$$

$\leadsto$ alternate between two generalized eigenvalue problems.

Typical Max-SINR Pulse Shapes:

$$N = 64 \text{ carriers}, \quad T_{\mathsf{ISI}} = \frac{T_s}{2}, \quad \eta_o = 1 \text{ sym/sec/Hz}.$$



Outage Capacity:

- Definition of outage capacity C_o via probability P_o:

$$P_o \;:=\; \Pr\big\{\mathcal{I}^{(j)} < C_o\big\}$$

- Example setup with $M = 2, L_{\text{pre}} = 1, L_{\text{pst}} = 1$:

$$\begin{bmatrix} \boldsymbol{x}(1) \\ \boldsymbol{x}(0) \end{bmatrix} = \left[\begin{array}{c|cc|c} \boldsymbol{\mathcal{H}}(1,-1) & \boldsymbol{\mathcal{H}}(1,0) & \boldsymbol{\mathcal{H}}(1,1) & \\ & \boldsymbol{\mathcal{H}}(0,-1) & \boldsymbol{\mathcal{H}}(0,0) & \boldsymbol{\mathcal{H}}(0,1) \end{array}\right] \left[\begin{array}{c} \boldsymbol{s}(2) \\ \hline \boldsymbol{s}(1) \\ \boldsymbol{s}(0) \\ \hline \boldsymbol{s}(-1) \end{array}\right] + \begin{bmatrix} \boldsymbol{w}(1) \\ \boldsymbol{w}(0) \end{bmatrix}$$

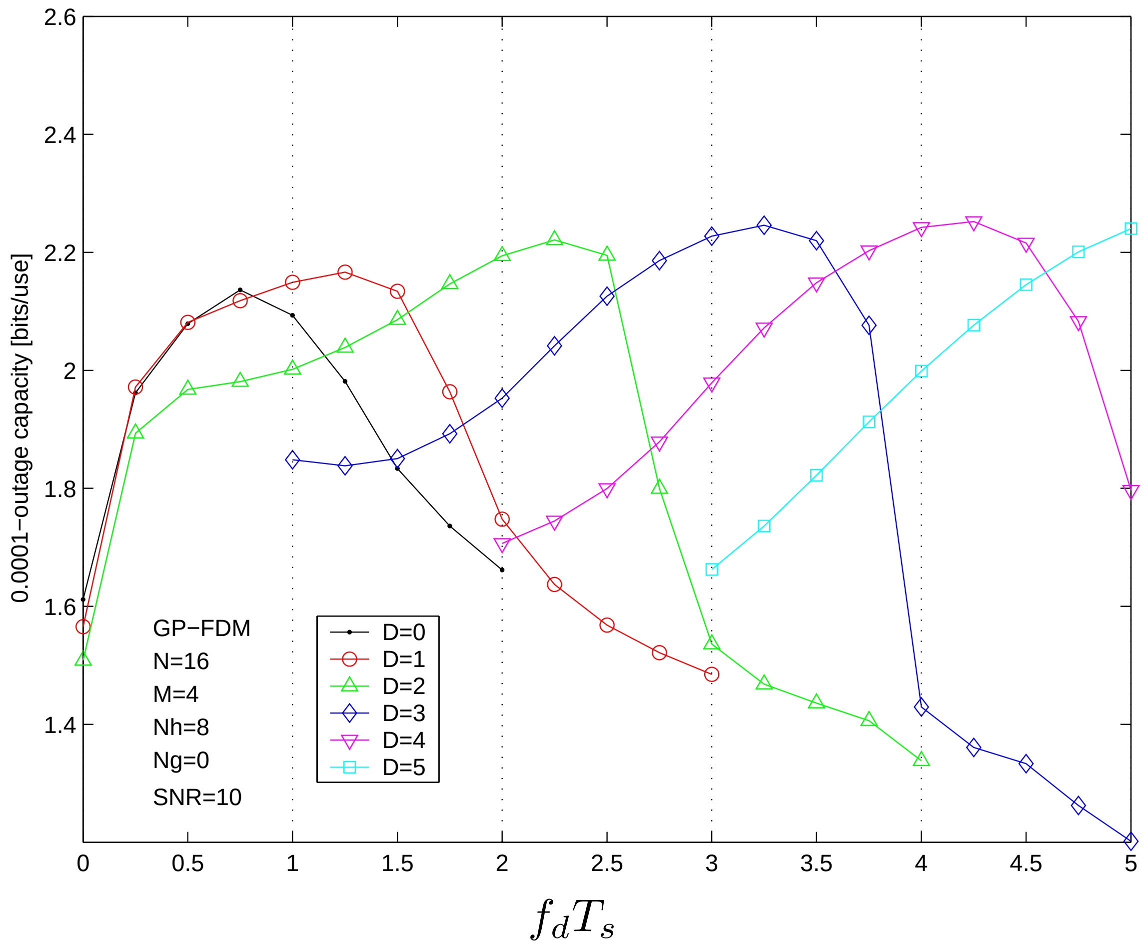
$$\underbrace{\begin{bmatrix} \boldsymbol{x}(1) \\ \boldsymbol{x}(0) \end{bmatrix}}_{\boldsymbol{x}^{(0)}} = \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{H}}(1,0) & \boldsymbol{\mathcal{H}}(1,1) \\ \boldsymbol{\mathcal{H}}(0,-1) & \boldsymbol{\mathcal{H}}(0,0) \end{bmatrix}}_{\boldsymbol{\mathcal{H}}^{(0)}} \underbrace{\begin{bmatrix} \boldsymbol{s}(1) \\ \boldsymbol{s}(0) \end{bmatrix}}_{\boldsymbol{s}^{(0)}} + \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{H}}(1,-1) & \\ & \boldsymbol{\mathcal{H}}(0,1) \end{bmatrix} \begin{bmatrix} \boldsymbol{s}(2) \\ \boldsymbol{s}(-1) \end{bmatrix} + \begin{bmatrix} \boldsymbol{w}(1) \\ \boldsymbol{w}(0) \end{bmatrix}}_{\boldsymbol{v}^{(0)}}$$

- Mutual info (bits/sec/Hz) between Gaussian $\boldsymbol{s}^{(j)}$ and $\boldsymbol{x}^{(j)}$

$$\mathcal{I}^{(j)} \;=\; \frac{1}{MN_s} \log_2 \det\big(\boldsymbol{I}_{MN} + \boldsymbol{\mathcal{H}}^{(j)H} \boldsymbol{R}_v^{-1} \boldsymbol{\mathcal{H}}^{(j)}\big)$$

where $N_s = BT_s$ and M is # of m.c. symbols in a code block.

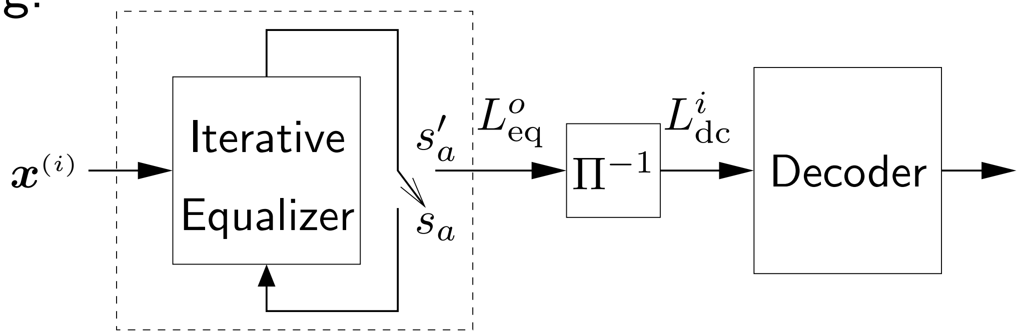
Outage Capacity vs $f_{\mathrm{d}}T_s$ for various D:



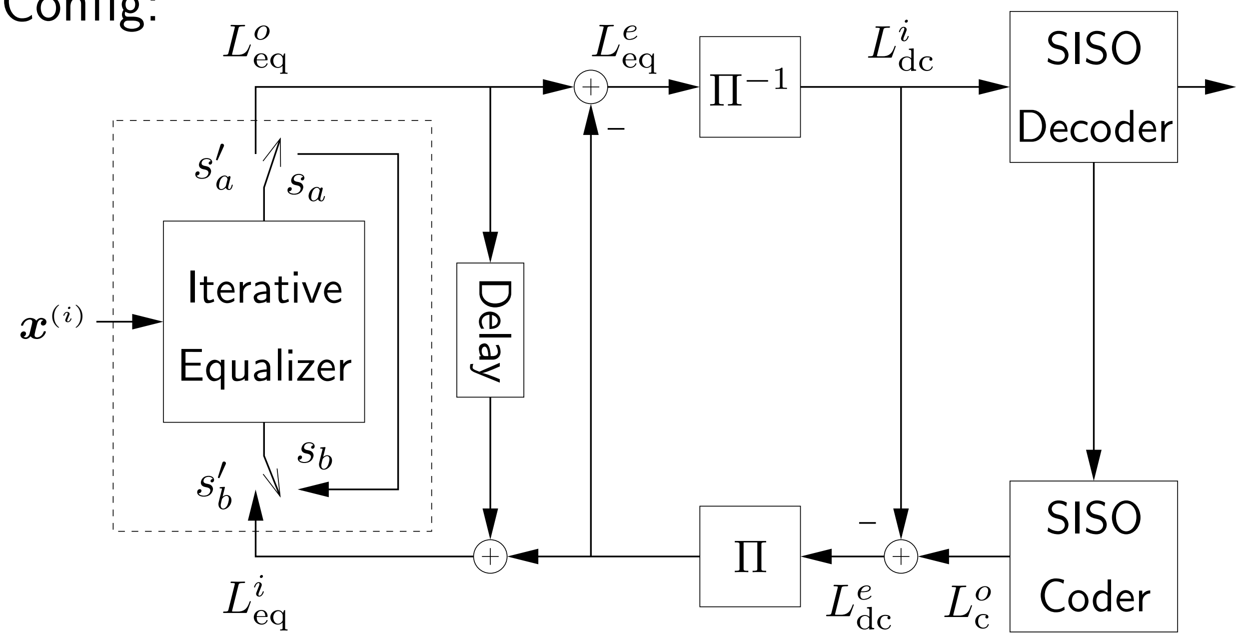
$\Rightarrow$ Max-SINR pulse designs based on an ICI radius of $\approx f_{\mathrm{d}}T_s$ have a capacity advantage at higher Dopplers!

Equalizer/Decoder Structure:

Serial Config:



Turbo Config:



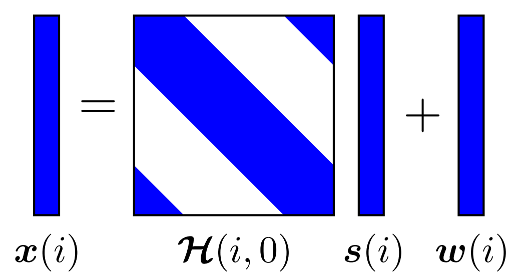
Iterative Equalization:

- System model:

$$\boxed{\boldsymbol{x}(i) = \boldsymbol{\mathcal{H}}(i,0)\boldsymbol{s}(i) + \boldsymbol{w}(i) + \boldsymbol{\varepsilon}(i),}$$

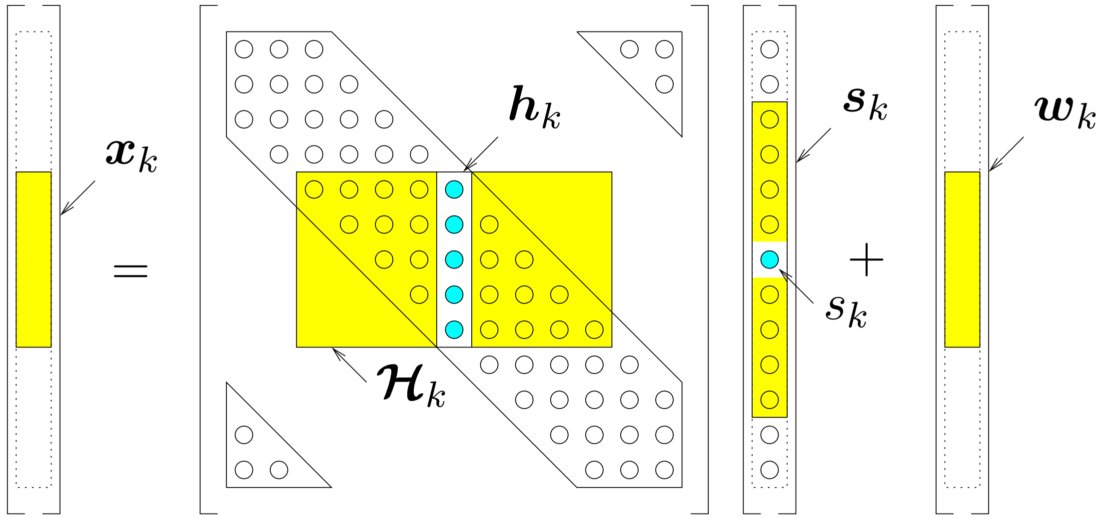
  where $\boldsymbol{\varepsilon}(i)$ represents ISI.

- With successful pulse designs. . .
  1. ISI energy $\ll$ noise energy, so $\boldsymbol{\varepsilon}(i)$ can be ignored.
  2. $\boldsymbol{\mathcal{H}}(i,0)$ has a banded structure.

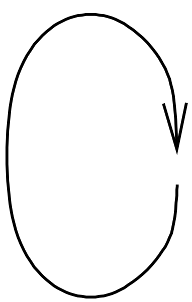


- Exploit <u>banded</u> channel and <u>finite alphabet</u> symbols.
  – Iterative MMSE Equalization (IMSE)
  – Iterative ML Equalization (IMLE)

Iterative MMSE Equalization (IMSE):

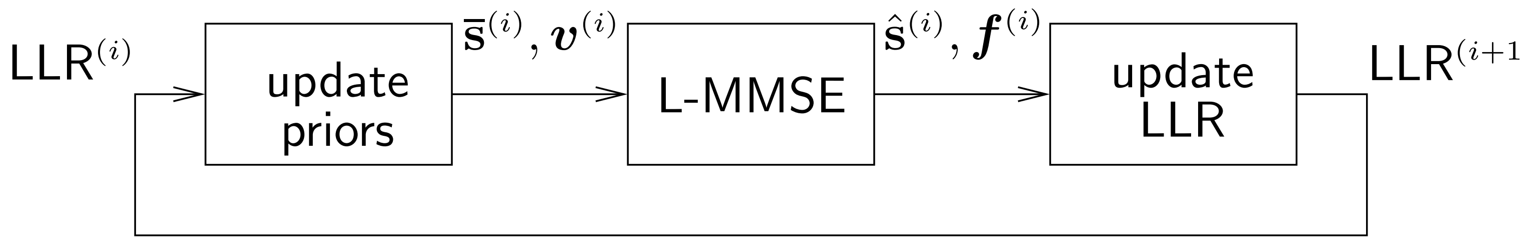


$$\boxed{\boldsymbol{x}_k = \boldsymbol{\mathcal{H}}_k \boldsymbol{s}_k + \boldsymbol{w}_k}$$



- L-MMSE estimate s_k from $\boldsymbol{x}_k$ using mean & covariance of $\boldsymbol{s}_k$.
- Assuming Gaussian error, compute LLRs(s_k).
- Using LLRs(s_k), update mean/covariance of s_k.
- $k \longrightarrow \langle k+1 \rangle_N$.

Iterative MMSE Equalization (BPSK example):



$$\bar{s}_k^{(i)} := \widehat{\mathrm{E}\{s_k|\hat{s}_k\}} = \tanh(\mathsf{LLR}_k^{(i)}/2)$$

$$v_k^{(i)} := \widehat{\mathrm{var}(s_k|\hat{s}_k)} = 1 - (\bar{s}_k^{(i)})^2$$

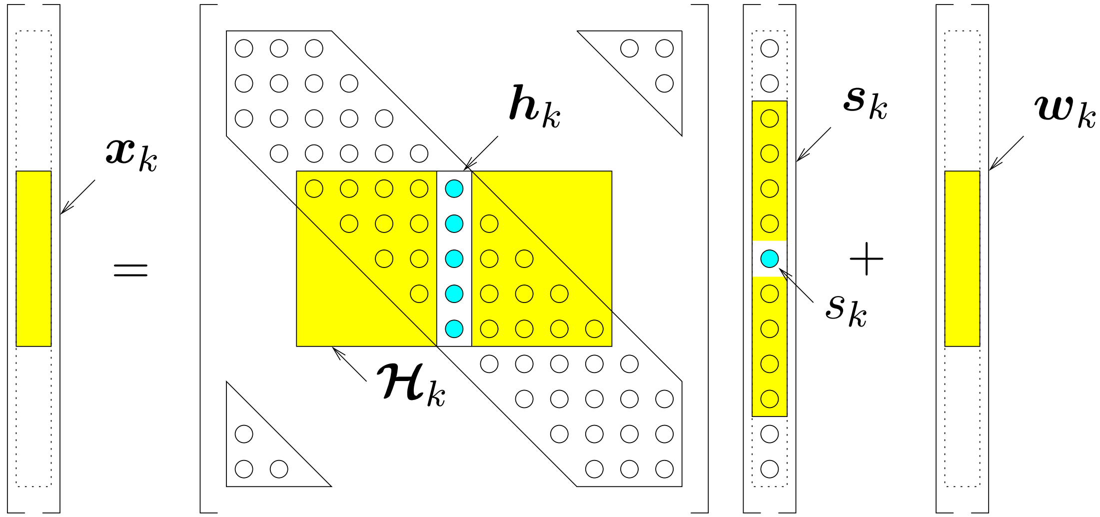
$$\boldsymbol{f}_k^{(i)} = \left(\boldsymbol{R}_w + \boldsymbol{\mathcal{H}}_k\, \mathcal{D}(\boldsymbol{v}_k^{(i)})\boldsymbol{\mathcal{H}}_k^H + \boldsymbol{h}_k \boldsymbol{h}_k^H\right)^{-1} \boldsymbol{h}_k$$

$$\hat{s}_k^{(i)} = \boldsymbol{f}_k^{(i)H}\left(\boldsymbol{x}_k - \boldsymbol{\mathcal{H}}_k \bar{\boldsymbol{s}}_k^{(i)}\right)$$

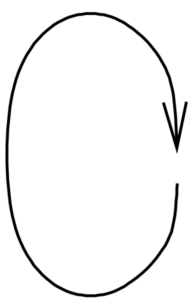
$$\mathsf{LLR}_k^{(i+1)} = \mathsf{LLR}_k^{(i)} + 4\,\mathrm{Re}(\hat{s}_k^{(i)})/(1 - \boldsymbol{h}_k^H \boldsymbol{f}_k^{(i)})$$

Complexity: $\underset{\text{iters}}{M} \times \underset{\text{syms}}{N} \times \underset{\text{mtx inv}}{\mathcal{O}(D^2)} \rightsquigarrow \mathcal{O}(N)$.

Iterative ML Equalization:

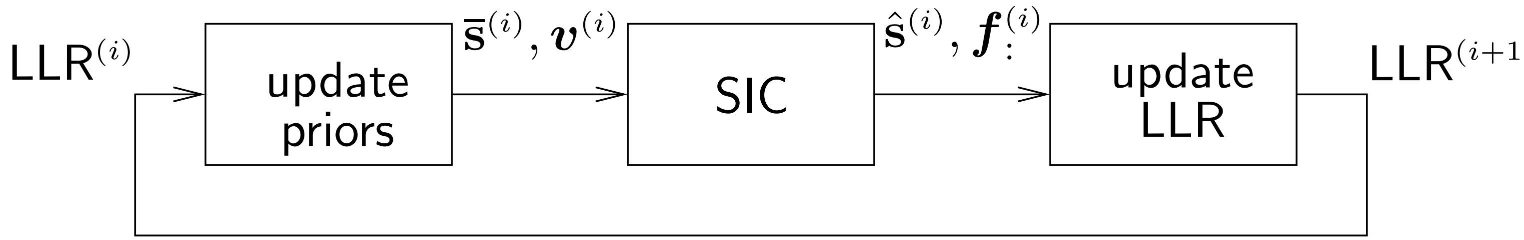


$$\boldsymbol{x}_k = \boldsymbol{\mathcal{H}}_k \boldsymbol{s}_k + \boldsymbol{w}_k$$



- Soft interference cancellation using mean of $\boldsymbol{s}_k$.
- Assuming Gaussian residual interference and using the covariance of $\boldsymbol{s}_k$, compute LLRs(s_k).
- Using LLRs(s_k), update mean/covariance of s_k.
- $k \longrightarrow \langle k+1 \rangle_N$.

Iterative ML Equalization (BPSK example):



$$\bar{s}_k^{(i)} := \widehat{\mathrm{E}\{s_k|\hat{s}_k\}} = \tanh(\mathsf{LLR}_k^{(i)}/2)$$

$$v_k^{(i)} := \widehat{\mathrm{var}(s_k|\hat{s}_k)} = 1 - (\bar{s}_k^{(i)})^2$$

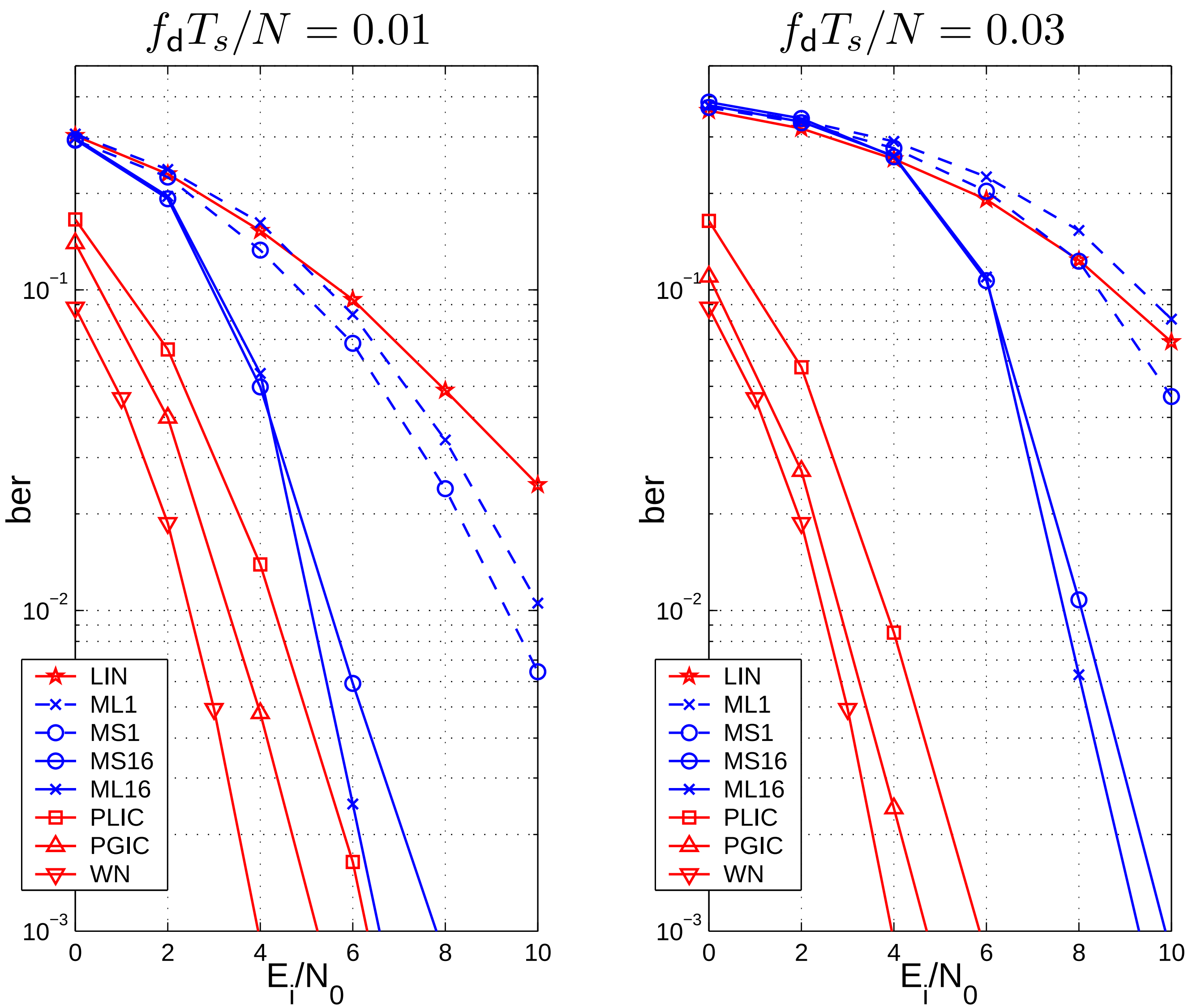
$$\boldsymbol{y}_k^{(i)} = \boldsymbol{x}_k - \boldsymbol{\mathcal{H}}_k \bar{\boldsymbol{s}}_k^{(i)}$$

$$g_k^{(i)} = \boldsymbol{y}_k^{(i)H} \big(\boldsymbol{R}_w + \boldsymbol{\mathcal{H}}_k \, \mathcal{D}(\boldsymbol{v}_k^{(i)}) \boldsymbol{\mathcal{H}}_k^H\big)^{-1} \boldsymbol{h}_k$$

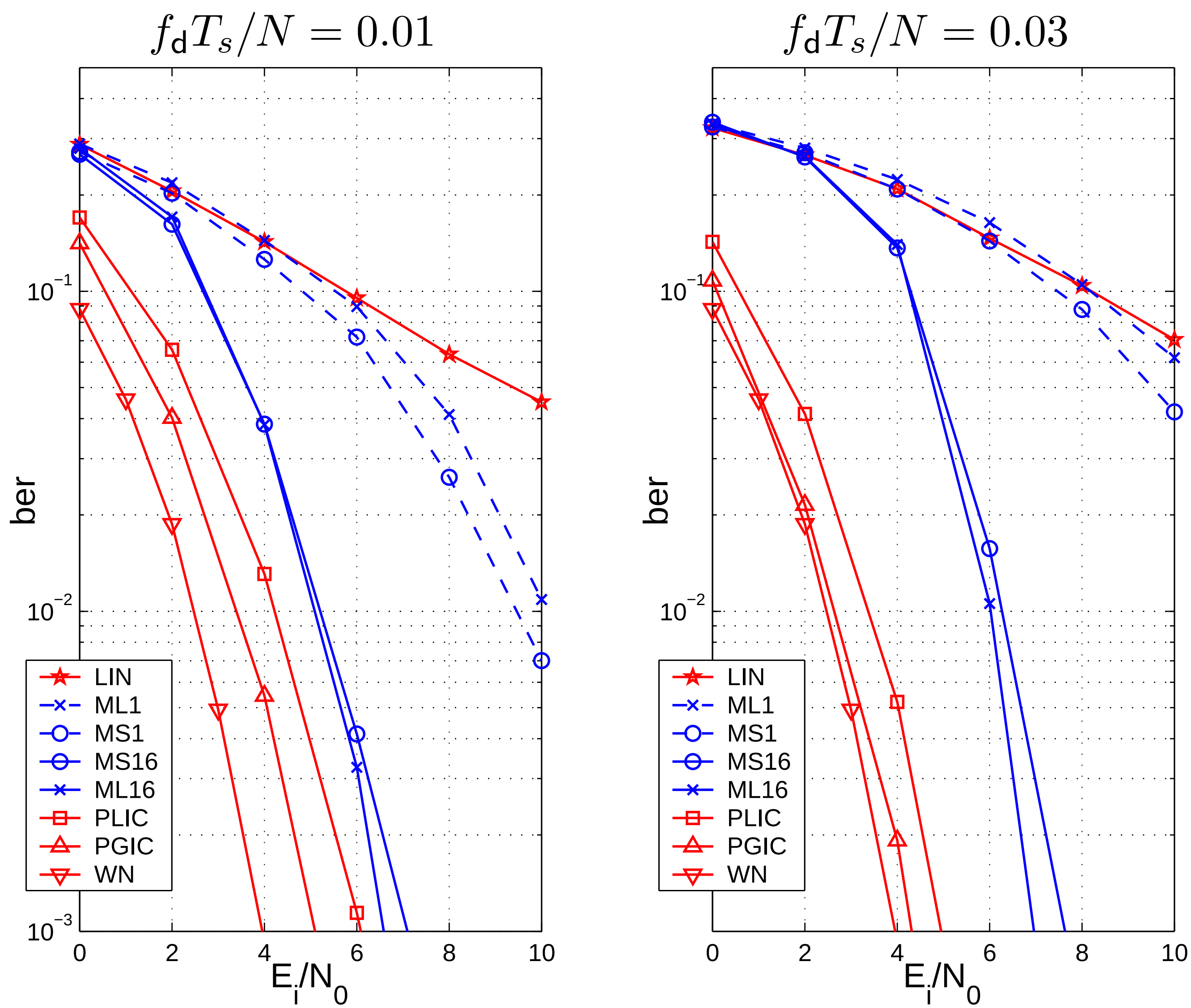
$$\mathsf{LLR}_k^{(i+1)} = \mathsf{LLR}_k^{(i)} + 2\,\mathrm{Re}(g_k)$$

Complexity: $\underset{\text{iters}}{M} \times \underset{\text{syms}}{N} \times \underset{\text{mtx inv}}{\mathcal{O}(D^2)} \rightsquigarrow \mathcal{O}(N)$.

BER for Transmitter/Receiver Pulses:



BER for Transmitter-Only Pulses:



Simulation Details:

Channel/Modulator:

- WSSUS Rayleigh fading, uniform delay profile, length $T_s/2$.
- $N = 32$ carriers, no guard interval, QPSK.
- $(7, 5)$ rate-$\frac{1}{2}$ (non-systematic) convolutional code.
  Coded/interleaved over blocks of 40 multicarrier symbols.

Equalizer/Decoder:

- ICI radius $D = \lceil f_\text{d} T_s \rceil + 1$.
- 16 turbo iterations, BCJR SISO decoder.

Bounds:

- LIN: joint-linear-MMSE estimation followed by decoding.
- PGIC: *all* ISI/ICI known, PLIC: *neighboring* ICI known.
- WN: no ISI/ICI and no fading

Conclusions:

- Considered interference *shaping* (not suppression) to design FFT-based PS-FDM scheme for doubly dispersive channels.
- Neighboring-ICI can be mitigated using *low-complexity* iterative equalization/decoding.
- Postcursor-ISI can be mitigated using block decision feedback, though this is not necessary if delay spread $\leq T_s/2$.
- Equalization/decoding can be done single-shot or turbo.
- BER performance close to perfect-interference cancellation bounds, and far beyond one-shot linear equalization plus decoding.
- Outage-capacity analysis suggests performance advantages over interference-suppressing designs in coded systems.