Pulse-Shaped FDM for Doubly-Dispersive Channels

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**Multicarrier Modulation:**

![Multicarrier Modulation Diagram]

\[
x(i) = \sum_{j=-L_{\text{pre}}}^{L_{\text{pst}}} \mathcal{H}(i, j) s(i - j) + w(i)
\]

“LTV MIMO channel”

- **Modulator:** multicarrier symbols \( \{s(i)\} \rightarrow \) waveforms,
- **Demodulator:** waveforms \( \rightarrow \) multicarrier observations \( \{x(i)\} \).

*How should we design modulator & demodulator?*

*How should we design equalizer/decoder?*
The Doubly-Dispersive Channel:

- We focus on time-frequency (i.e., doubly) dispersive channels.
- No fixed eigenbasis for these channels, so ISI/ICI is unavoidable in the absence of transmitter channel knowledge.
- Without dispersion, Nyquist theory specifies a maximum of 1 symbol/sec/Hz for interference-free modulation/demodulation.
- Roughly, as symbol/sub-carrier spacings are increased,
  - ISI/ICI decreases (good!), but
  - modulation efficiency decreases (bad!).

\[ \sim \text{Inherent tradeoff between modulation efficiency and ISI/ICI.} \]
Modulator Design Philosophy:

- Traditionally, (bi)orthogonal pulse families:
  - Zero ISI/ICI in non-dispersive (i.e., trivial) channels.
  - Low ISI/ICI in non-trivial channels ⇔ low modulation efficiency.
- Our approach: non-(bi)orthogonal pulse families:
  - We don’t expect trivial channels, so why design for them?
  - We do expect to have an equalizer, so why not leverage it?
- Main ideas:
  - Shape, rather than suppress, ISI/ICI.
  - Design waveforms to yield a target ISI/ICI response that
    - is attainable (i.e., suited to the typical channel),
    - allows low-complexity equalization/decoding.
  - An outage-capacity analysis suggests that shaping has advantages over suppression. (More later...)
Pulse-Shaped FDM:

- Like CP-OFDM but with smooth overlapping mod/demod pulses.

\[
\begin{align*}
\text{PS-FDM:} & \quad \ldots \\
\text{CP-OFDM:} & \quad \ldots
\end{align*}
\]

- Complexity on par with CP-OFDM.
- Since ISI/ICI always present, no explicit need for a guard interval.
  - Higher modulation efficiency than OFDM.
  - Possible to overload the signal space (i.e., >1 symbol/sec/Hz), though equalization/decoding becomes more challenging.
Pulse Design:

Target MIMO channel \( \{ \mathcal{H}(i, -L_{\text{pre}}), \ldots, \mathcal{H}(i, L_{\text{pst}}) \} \):

Without block DFE:

\[
\begin{array}{ccccccc}
\text{interference} & \ldots & \text{don’t care} & \text{signal} & \ldots & \text{interference}
\end{array}
\]

With block DFE:

\[
\begin{array}{ccccccc}
\text{interference} & \ldots & \text{don’t care} & \text{signal} & \ldots & \text{interference}
\end{array}
\]

Joint SINR-maximizing pulses \( \{a, b\} \):

\[
\begin{align*}
a^{(i)} &= \arg \max_{\|a\|^2 = N_s} \frac{a^H P_s(b^{(i)})a}{a^H P_{ni}(b^{(i)})a} & \text{Tx pulse} \\
b^{(i+1)} &= \arg \max_{\|b\|^2 = N_s} \frac{b^H Q_s(a^{(i)})b}{b^H Q_{ni}(a^{(i)})b} & \text{Rx pulse}
\end{align*}
\]

\( \sim \) alternate between two generalized eigenvalue problems.
Typical Max-SINR Pulse Shapes:

\[ N = 64 \text{ carriers, } \quad T_{\text{ISI}} = \frac{T_s}{2}, \quad \eta_o = 1 \text{ sym/sec/Hz.} \]
Outage Capacity:

- Definition of outage capacity $C_o$ via probability $P_o$:
  \[ P_o := \Pr\{\mathcal{I}^{(j)} < C_o\} \]

- Example setup with $M = 2$, $L_{\text{pre}} = 1$, $L_{\text{pst}} = 1$:

\[
\begin{bmatrix}
  x^{(1)} \\
  x^{(0)}
\end{bmatrix} =
\begin{bmatrix}
  H(1, -1) & H(1, 0) & H(1, 1) \\
  H(0, -1) & H(0, 0) & H(0, 1)
\end{bmatrix}
\begin{bmatrix}
  s(2) \\
  s(1) \\
  s(0) \\
  s(-1)
\end{bmatrix} +
\begin{bmatrix}
  w(1) \\
  w(0)
\end{bmatrix}
\]

\[
\begin{bmatrix}
  x^{(1)} \\
  x^{(0)}
\end{bmatrix} =
\begin{bmatrix}
  H(1, 0) & H(1, 1) \\
  H(0, -1) & H(0, 0)
\end{bmatrix}
\begin{bmatrix}
  s(1) \\
  s(0)
\end{bmatrix} +
\begin{bmatrix}
  H(1, -1) \\
  H(0, 1)
\end{bmatrix}
\begin{bmatrix}
  s(2) \\
  s(-1)
\end{bmatrix} +
\begin{bmatrix}
  w(1) \\
  w(0)
\end{bmatrix}
\]

- Mutual info (bits/sec/Hz) between Gaussian $s^{(j)}$ and $x^{(j)}$

\[
\mathcal{I}^{(j)} = \frac{1}{MN_s} \log_2 \det \left( I_{MN} + H^{(j)H} R_v^{-1} H^{(j)} \right)
\]

where $N_s = B T_s$ and $M$ is # of m.c. symbols in a code block.
Outage Capacity vs $f_d T_s$ for various $D$: 

$\Rightarrow$ Max-SINR pulse designs based on an ICI radius of $\approx f_d T_s$ have a capacity advantage at higher Dopplers!
Equalizer/Decoder Structure:

Serial Config:

Turbo Config:
Iterative Equalization:

• System model:

\[ x(i) = H(i, 0)s(i) + w(i) + \varepsilon(i), \]

where \( \varepsilon(i) \) represents ISI.

• With successful pulse designs...

  1. ISI energy \( \ll \) noise energy, so \( \varepsilon(i) \) can be ignored.
  2. \( H(i, 0) \) has a banded structure.

• Exploit banded channel and finite alphabet symbols.
  - Iterative MMSE Equalization (IMSE)
  - Iterative ML Equalization (IMLE)
Iterative MMSE Equalization (IMSE):

\[ x_k = H_k s_k + w_k \]

- L-MMSE estimate \( s_k \) from \( x_k \) using mean & covariance of \( s_k \).
- Assuming Gaussian error, compute LLRs\( (s_k) \).
- Using LLRs\( (s_k) \), update mean/covariance of \( s_k \).
- \( k \rightarrow (k + 1)_N \).
Iterative MMSE Equalization (BPSK example):

\[ \bar{s}_k^{(i)} := \text{E}\{s_k|\hat{s}_k\} = \tanh(\text{LLR}_k^{(i)}/2) \]
\[ v_k^{(i)} := \text{var}(s_k|\hat{s}_k) = 1 - (\bar{s}_k^{(i)})^2 \]
\[ f_k^{(i)} = \left( R_w + \mathcal{H}_k \mathcal{D}(v_k^{(i)})\mathcal{H}_k^H + h_k h_k^H \right)^{-1} h_k \]
\[ \hat{s}_k^{(i)} = f_k^{(i) H} (x_k - \mathcal{H}_k \bar{s}_k^{(i)}) \]
\[ \text{LLR}_k^{(i+1)} = \text{LLR}_k^{(i)} + 4 \text{Re}(\hat{s}_k^{(i)})/(1 - h_k^H f_k^{(i)}) \]

**Complexity:** \( M \times N \times O(D^2) \sim O(N) \).
Iterative ML Equalization:

\[ x_k = \mathcal{H}_k s_k + w_k \]

- Soft interference cancellation using mean of \( s_k \).
- Assuming Gaussian residual interference and using the covariance of \( s_k \), compute LLRs(\( s_k \)).
- Using LLRs(\( s_k \)), update mean/covariance of \( s_k \).
- \( k \rightarrow \langle k + 1 \rangle_N \).
Iterative ML Equalization (BPSK example):

\[
\begin{align*}
\tilde{s}^{(i)}_k &:= \mathbb{E}\{s_k|\hat{s}_k\} = \tanh(\text{LLR}^{(i)}_k/2) \\
v^{(i)}_k &:= \text{var}(s_k|\hat{s}_k) = 1 - (\tilde{s}^{(i)}_k)^2 \\
y^{(i)}_k &= x_k - H_k \tilde{s}^{(i)}_k \\
g^{(i)}_k &= y^{(i)}_k H \left( R_w + H_k D(v^{(i)}_k) H_k^H \right)^{-1} h_k \\
\text{LLR}^{(i+1)}_k &= \text{LLR}^{(i)}_k + 2 \text{Re}(g_k)
\end{align*}
\]

Complexity: $M \times \text{N} \times \mathcal{O}(D^2) \sim \mathcal{O}(N)$. 

\[\text{M} \times \text{N} \times \mathcal{O}(D^2) \sim \mathcal{O}(N).\]
BER for Transmitter/Receiver Pulses:

\[ f_d T_s / N = 0.01 \]

\[ f_d T_s / N = 0.03 \]
BER for Transmitter-Only Pulses:

\[ f_d T_s / N = 0.01 \]

\[ f_d T_s / N = 0.03 \]
Simulation Details:

Channel/Modulator:

• WSSUS Rayleigh fading, uniform delay profile, length $T_s/2$.
• $N = 32$ carriers, no guard interval, QPSK.
• $(7, 5)$ rate-$\frac{1}{2}$ (non-systematic) convolutional code. Coded/interleaved over blocks of 40 multicarrier symbols.

Equalizer/Decoder:

• ICI radius $D = \lceil f_d T_s \rceil + 1$.
• 16 turbo iterations, BCJR SISO decoder.

Bounds:

• LIN: joint-linear-MMSE estimation followed by decoding.
• PGIC: all ISI/ICI known, PLIC: neighboring ICI known.
• WN: no ISI/ICI and no fading
Conclusions:

- Considered interference *shaping* (not suppression) to design FFT-based PS-FDM scheme for doubly dispersive channels.

- Neighboring-ICI can be mitigated using low-complexity iterative equalization/decoding.

- Postcursor-ISI can be mitigated using block decision feedback, though this is not necessary if delay spread $\leq T_s/2$.

- Equalization/decoding can be done single-shot or turbo.

- BER performance close to perfect-interference cancellation bounds, and far beyond one-shot linear equalization plus decoding.

- Outage-capacity analysis suggests performance advantages over interference-suppressing designs in coded systems.