Iterative Equalization for Single Carrier Cyclic Prefix in Doubly-Dispersive Channels

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Background:

- First, consider communication over **time-dispersive** channels.

- Options:
  1. Single-Carrier Modulation with Time-Domain Equalization
     - $\mathcal{O}(N_h)$ operations/symbol for chan length $N_h$.
     - low peak-to-average-power ratio (PAPR).
  2. Multi-Carrier Modulation with Freq-Domain Equalization
     - $\mathcal{O}(\log N)$ operations/symbol for block length $N$.
     - high PAPR.
     - “OFDM.”
  3. Single-Carrier Modulation with Freq-Domain Equalization
     - $\mathcal{O}(\log N)$ operations/symbol for block length $N$.
     - low PAPR.
     - “single carrier cyclic prefix (**SCCP**).”
Single-Carrier Cyclic Prefix (SCCP):

- SCCP is like OFDM with both FFTs at the receiver.
- Freq-domain equalization requires only one mult-per-symbol if:
  1. cyclic prefix length > channel delay spread,
  2. channel time-invariant over the FFT-block interval.
- Our final goal, however, is communication over *time-dispersive and frequency-dispersive* channels.

*How can we handle SCCP with significant channel variation over the block interval?*
System Model:

\[ r = H_{tl} s + \nu \]

\[ x = \underbrace{F H_{tl} F^H}_{H_{df}} \underbrace{F s + F \nu}_{w} \]

where

\[ H_{tl} \] = circular-convolution matrix,

\[ H_{df} \] = “virtual-subcarrier” coupling matrix.

\[ \sim H_{df} \] diagonal iff channel is LTI and prefix-length is adequate.
Virtual-Subcarrier Coupling Matrix $\mathcal{H}_{df}$:

$$
\mathcal{H}_{df} = 
\begin{pmatrix}
    h_{df}(0,0) & h_{df}(-1,1) & \ldots & h_{df}(1-N,N-1) \\
    h_{df}(1,0) & h_{df}(0,1) & \ldots & h_{df}(2-N,N-1) \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{df}(N-1,0) & h_{df}(N-2,1) & \ldots & h_{df}(0,N-1)
\end{pmatrix}
$$

$$
h_{df}(\nu,k) := \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} h_{tl}(n,l)e^{-j\frac{2\pi}{N}n\nu}e^{-j\frac{2\pi}{N}lk}
$$

= response at carrier $k+\nu$ to an impulse applied at carrier $k$

$$
h_{tl}(n,l) := \text{response at time } n \text{ to an impulse applied at time } n-l
$$
Inter-Carrier Interference Mechanism:

Doppler Spread meets Finite Block Length:

\[
E \{ |h_{df}(\nu, k)|^2 \} = \left( \frac{I_{[0, 2\pi f_d]}(|\phi|) \sum I}{\sqrt{\left(2\pi f_d\right)^2 - \phi^2}} \right) \ast \left( \frac{\sin(\phi N/2)}{N \sin(\phi/2)} \right)^2 _{\phi = \frac{2\pi}{N} \nu}
\]

\[
= \text{Samples of} \ * \text{Samples of} \ * \text{Samples of}
\]

Note: Zero Doppler spread \(\Rightarrow\) Sample at sinc nulls \(\Rightarrow\) Zero ICI
Rayleigh $\mathbb{E}\{|h_{df}(\nu, k)|^2\}$ for $N = 128$ and $f_d = 0.03$: 
Rayleigh $\mathbb{E}\{ |h_{df}(\nu, \cdot)|^2 \}$ for $N = 128$ and various $f_d$: 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{chart.png}
\end{figure}
SCCP Equalization/Detection:

Objective: Recover finite-alphabet vector $s$ from $x = \mathcal{H}_{df} F s + w$.

Classical Strategies:

- ZF, LS: $\hat{s}_{zf} = \text{slice}\left[ F^H \mathcal{H}_{df}^{-1} x \right]$
- MMSE: $\hat{s}_{mmse} = \text{slice}\left[ F^H \mathcal{H}_{df}^H (\mathcal{H}_{df} \mathcal{H}_{df}^H + \sigma_w^2 I)^{-1} x \right]$
- MLSD: $\hat{s}_{mlsd} = \arg \max_s ||x - \mathcal{H}_{df} F s||^2$

With LTV channel: $\leadsto$ Equalization requires $\geq O(N^3)$ operations
$\leadsto$ Low-complexity advantage of SCCP is lost!
Linear Pre-Processing to Simplify Detection:

- Use linear pre-processing to simplify detection.
  - Want to make $H_{df}$ sparse
  - ICI-response “shortening”
  - Reminiscent of ISI-shortening for single-carrier MLSD
- Time-domain windowing = Doppler-domain convolution!

![Diagram showing before and after processing](image-url)
Max-SINR Window Coefficients:

- Say we allow $2D$ diagonals of controlled ICI.

- Max-SINR window coefficients $b_\star$ are

$$b_\star = \text{gen-evec}_{\text{max}}\left( A \odot R^*, \ diag(R + \sigma^2 I) - A \odot R^* \right)$$

where, for WSSUS Rayleigh fading,

$$[A]_{m,n} = \frac{\sin\left(\frac{\pi}{N}(2D + 1)(n - m)\right)}{N \sin\left(\frac{\pi}{N}(n - m)\right)}$$

$$[R]_{n,m} = J_0\left(2\pi f_d(n - m)\right) \sum_{l=0}^{N_h-1} \sigma_l^2$$

- Note that $b_\star$ is a function of $\left\{D, N, f_d, \frac{\sum \sigma_l^2}{\sigma^2}\right\}$
Windowed-System Model:

- Apply windowing before first receiver DFT:
  \[\tilde{x} = F \mathcal{D}(b)r\]
  \[= F \mathcal{D}(b)F^H F r = C(\beta) x\text{ for } \beta = \frac{Fb}{\sqrt{N}}\]
  \[= C(\beta) \mathcal{H}_{df} F s + C(\beta) w\]
  nearly banded

- **Goal:**
  Estimate finite-alphabet \(\{s_0, \ldots, s_{N-1}\}\) given \(\mathcal{H}_{df}, \beta\), and \(\tilde{x}\).

- **Approach:**
  Leverage sparse \(\mathcal{H}_{df}\) to estimate \(t\), then relate \(t \rightarrow s\).
Iterative MMSE Estimation:

Block Iteration:

$L$-MMSE step for each $k$:

\[
\tilde{x}_k = \tilde{h}_k \tilde{H}_k + t_k C_k + w
\]
Algorithm requiring $O(D^2 \log N)$ operations/symbol:

\[ L^{(0)}(s_k) = 0 \quad \forall k \]

\[ \text{for } i = 0 \ldots, \]
\[ \text{for } k = 0 \ldots N - 1, \]
\[ \tilde{s}_k^{(i+1)} = \tanh(L^{(i+1)}(s_k)/2) \]
\[ v_k^{(i+1)} = 1 - (\tilde{s}_k^{(i+1)})^2 \]
\[ \text{end} \]
\[ \bar{t}^{(i)} = F\tilde{s}^{(i)} \]
\[ \text{for } k = 0 \ldots N - 1, \]
\[ g_k^{(i)} = -\hat{H}_k F \mathcal{D}(\mathbf{v}^{(i)}) F^H \hat{H}_k^H + \sigma^2 C_k C_k^H \]^{-1} \hat{H}_k F \mathcal{D}(\mathbf{v}^{(i)}) F^H i_k \]
\[ \hat{t}_k^{(i)} = \bar{t}_k^{(i)} + g_k^{(i)H}(x_k - \hat{H}_k \bar{t}^{(i)}) \]
\[ \text{end} \]
\[ Q^{(i)} = F^H \left( \sum_{k=0}^{N-1} \hat{H}_k g_k^{(i)H} i_k^H \right) F \]
\[ P^{(i)} = F^H \left( \sum_{k=0}^{N-1} C_k C_k^H g_k^{(i)H} i_k^H \right) F \]
\[ \hat{s}^{(i)} = F^H \hat{t}^{(i)} \]
\[ \text{for } k = 0 \ldots N - 1, \]
\[ L^{(i+1)}(s_k) = L^{(i)}(s_k) + 4 \frac{\text{Re}\{Q^{(i)}_{k,k}(\hat{s}_k^{(i)} - \tilde{s}_k^{(i)})\} + |Q^{(i)}_{k,k}|^2 \tilde{s}_k^{(i)}}{q_k^{(i)H} \mathcal{D}(\mathbf{v}^{(i)}) q_k^{(i)} - |Q^{(i)}_{k,k}|^2 v_k^{(i)} + \sigma^2 \|P_k^{(i)}\|^2} \]
\[ \text{end} \]
end
Uncoded-SER versus SNR:

- Left: $fd=0.001$, $D=1$, $N=128$
- Middle: $fd=0.01$, $D=3$, $N=128$
- Right: $fd=0.03$, $D=5$, $N=128$

Legend:
- LIN
- ITER
- AMFB
- MFB
Observations:

• Classical (Joint Linear) MMSE:
  – $O(N^2)$ operations/symbol.
  – Worst performance.

• Iterative MMSE:
  – $O(\log N)$ operations/symbol.
  – $\sim 2$dB from MFB
  – Easily combined with decoding algorithm (i.e., turbo eq).

• Approximate MFB:
  – Uses sparse $\tilde{H}_{df}$ with perfect interference cancellation.

• MFB:
  – Uses true $\tilde{H}_{df}$ with perfect interference cancellation.
Summary:

- SCCP reception complicated by time-selectivity.
- Proposed a two-stage SCCP receiver for doubly-selective channels:
  1. SINR-optimal windowing,
  2. Iterative MMSE estimation.
- Like classical SCCP receivers, requires $O(\log N)$ operations/symbol.
- Uncoded error rate is $\sim 2$dB from MFB.
- Soft decoding can be easily incorporated.