# Chip-Rate Adaptive Two-Stage Receiver for Scrambled Multirate CDMA Downlink

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Fig. 1. Scrambled-CDMA transmitter model.

Abstract—In this paper we propose an adaptive two-stage receiver for a scrambled multirate DS-CDMA downlink transmitted via synchronous orthogonal short codes and subjected to time- and frequency-selective multipath fading. The first stage of our two-stage receiver (described in a previous publication [1]) consists of adaptive FIR equalization generating tentative hard decisions. The decisions are fed-forward to the second stage for further processing via adaptive decision-feedforward equalization (DFFE) or adaptive inter-chip interference cancellation (ICIC). (The ICIC receiver was introduced in [2].) Here we detail the adaptive DFFE and ICIC structures, which are based on low-complexity decision-directed LMS. For further complexity reduction, the ICIC performs maximal ratio combining of only the ICIC branches corresponding to the largest channel taps. Since ICIC removes both pre- and post-cursor interference, it outperforms the DFFE.

#### I. INTRODUCTION

In third generation mobile DS-CDMA systems, downlink multirate symbol streams are multiplexed using orthogonal short codes and then scrambled by a cell-specific long code prior to synchronous transmission, as shown in Fig.1. The multipath propagation channel creates inter-chip interference (ICI) in the received signal, which destroys the orthogonality among user codes, which in turn introduces multi-access interference (MAI) in the symbol estimates of matched-filter (MF) based detectors. Since the mobile terminals in these systems are costand power-limited, we desire a low-complexity solution.

The usual methods of multipath mitigation in CDMA (e.g., "blind minimum output energy" techniques [3]) rely on received signal cyclostationarity. In our application, however, the scrambling code destroys the cyclostationarity and so an alternative means of multipath mitigation is necessary. We focus on a two-stage receiver as shown in Fig. 2.







Fig. 3. Adaptive FIR equalizer first stage: (a) pilot trained AEAR-LMS, and (b) DD-LMS equalization.

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## A. First Stage

Frank and Visotsky [4] first proposed the use of a codemultiplexed pilot for equalizer adaptation in the scrambled CDMA downlink. Though Petre et al. [5] later extended this pilot-aided scheme to incorporate chip-fractional sampling, both [4], [5] update the equalizer at the symbol rate. Motivated by the potential for improved tracking performance, we considered a pilot-aided equalization algorithm, referred to as averaged-error average-regressor LMS (AEAR-LMS), that updates at the *chip* rate using an "error filtering" mechanism [1]. The mean transient response of the resulting algorithm corresponds to that of a third-order dynamical system (in contrast to standard LMS, which behaves as a first-order dynamical system [6]). Such higher-order LMS algorithms have demonstrated tracking performance superior to standard LMS [7].

The aforementioned pilot-aided scheme is intended for cold-start or loss-of-lock situations. When adequately reliable symbol estimates are obtained, the first stage switches to the decision-directed (DD) equalizer update algorithm, for which we assume all active user codes are known and employed in the equalizer update [1]. The chip-rate DD algorithm alleviates the MAI problem faced by the pilot-aided algorithm and consequently yields better performance. The receiver monitors pilot decision quality as a means of switching between AEAR-LMS and DD to update the FIR equalizer [1].

Whether in AEAR-LMS mode (see Fig.3(a)) or in DD mode (see Fig. 3(b)), the multiuser chip-rate sequence is detected and fed-forward to the second stage. In the block diagrams,  $\nu$ denotes the system delay and  $N_{\text{max}}$  the spreading factor of the lowest rate user. Note that that any linear chip-rate equalization structure could serve as the first stage in our two-stage receiver.

#### B. Second Stage

In this paper, we present the details of two adaptive secondstage algorithms: decision feed-forward equalization (DFFE) and inter-chip interference cancellation (ICIC), which was introduced in [2]. The second stage of our receiver uses tentative decisions—produced by the first stage—for interference cancellation.

linear equalizers reduce MAI by re-First-stage orthogonalizing the chip-rate signal prior to the de-spreading operation. It has been noticed in [8], [9], however, that nonlinear processing offers potential performance improvement. In [8], chips from a tentative decision of the current symbol are used as feedback in a decision feedback equalizer (DFE). However, ICI from only a single user is suppressed, and the algorithm is not amenable to adaptation. Our proposed algorithms are more reminiscent of [9], where the output of an FIR chip-level equalization stage is de-spread and soft-decoded to obtain estimates of all active users' symbol streams, which are then fed-forward to be re-spread and used as feedback information for re-processing the received signal via a chip-level DFFE stage. To reduce complexity and delay, our receiver foregoes the decoding step and instead feeds forward hard decisions.



Fig. 4. Channel model at (a) chip-fractional-rate and (b) chip-rate.

We extend the DFFE concept from [9] to an *adaptive DFFE* and compare its performance to the *adaptive ICIC* structure. While DFFE eliminates post-cursor ICI, ICIC eliminates both post- and *pre*-cursor ICI in the received signal so that subsequent de-spreading removes all MAI in final symbol estimates. Adaptive ICIC employs a low-complexity LMS channel identification algorithm trained with chip-decisions that are fed-forward from the first stage.

## II. SYSTEM MODEL

Our transmitted signal model is illustrated in Fig. 1 with the following definitions. K denotes the number of users,  $N_k$  the  $k^{th}$  user's spreading gain,  $\{b_k(n), n \in \mathbb{Z}\}$  the  $k^{th}$ user's symbol stream,  $\{c_k(i), i = 0 \dots N_k - 1\}$  the  $k^{th}$  user's short code (where i is the chip index),  $\{u(i)\}$  the multiuser sequence,  $\{v(i)\}$  the multiuser-plus-pilot sequence,  $\{s(i)\}$  the scrambling sequence, and  $\{t_i\}$  the transmitted sequence. In the sequel, we will use  $\{\delta_i\}$  to denote the Kronecker delta sequence,  $(\cdot)^*$  to denote the complex conjugate,  $(\cdot)^T$  to denote the transpose, and  $(\cdot)^H$  the Hermitian transpose.

Figure 4(a) describes the discrete-time chip-fractionallyspaced channel model using P samples per chip, where  $\{h_m\}$  denotes the chip-fractional impulse response of the channel and pulse-shaping filters and where  $\{w_m\}$  denotes additive circular-Gaussian channel noise. The chip-fractional received signal can be written as (see, e.g., [10] for a detailed development of this multirate fractionally-sampled channel representation):

$$r_m = \sum_{\ell} t_{\ell} h_{m-\ell P} + w_m. \tag{1}$$

A chip-spaced model follows from definition of the vectors  $\boldsymbol{h}_i := [h_{iP+P-1}, \dots, h_{iP}]^T$ ,  $\boldsymbol{w}_i := [w_{iP+P-1}, \dots, w_{iP}]^T$ , and  $\boldsymbol{r}_i := [r_{iP+P-1}, \dots, r_{iP}]^T$ , as in Fig. 4(b). Then

$$\boldsymbol{r}_i = \sum_{\ell=0}^{L_h} \boldsymbol{h}_\ell t_{i-\ell} + \boldsymbol{w}_i, \qquad (2)$$

Since the DFE feed-forward filter employs a window of  $L_f+1$  chip-spaced samples, we define  $\boldsymbol{r}(i) := [\boldsymbol{r}_i^T, \dots, \boldsymbol{r}_{i-L_f}^T]^T$ ,  $\boldsymbol{t}(i) := [t_i, \dots, t_{i-L_f}]^T$ ,  $\boldsymbol{w}(i) := [\boldsymbol{w}_i^T, \dots, \boldsymbol{w}_{i-L_f}^T]^T$ , and the block-Toeplitz matrix

$$oldsymbol{H} \hspace{.1in}:=\hspace{.1in} \left[ egin{array}{cccc} oldsymbol{h}_0 \cdots oldsymbol{h}_{L_h} \ dots \ oldsymbol{h}_0 \cdots oldsymbol{h}_{L_h} \end{array} 
ight]$$



Fig. 5. Adaptive chip-rate DFFE.

so that

$$\boldsymbol{r}(i) = \boldsymbol{H}\boldsymbol{t}(i) + \boldsymbol{w}(i). \tag{3}$$

For simplicity of presentation, the system model (2) assumes that the channel is fixed, which is approximately true over short time periods. The simulations, however, are conducted using time- and frequency-selective channels.

We make the following assumptions about our system.

- (A1) Circular, i.i.d., zero-mean, PSK scrambling  $\{s(i)\}$ .
- (A2) Multirate orthonormal Walsh codes.
- (A3) Constant pilot at user index k=0.
- (A4) Circular, independent, zero-mean user symbols.
- (A5) Zero-mean, circular, white, Gaussian noise  $\{w_m\}$  with variance  $\sigma_w^2$ , independent of  $\{b_k(n)\}\$  and  $\{s(i)\}$ .

# **III. DECISION FEEDFORWARD EQUALIZATION**

In this section we propose an adaptive chip-rate DFFE structure for the second stage of our receiver. The DFFE is illustrated in Fig. 5, where  $f_{\rm ff}$  denotes the feed-forward filter (FFF),  $\boldsymbol{f}_{\mathsf{fb}}$  denotes the feed-back filter (FBF), and  $N_{\max}$ denotes the lowest-rate user's spreading gain. We use the term DFFE rather than DFE because decisions are *fed forward* from, rather than fed back to, the first-stage. In the following discussion, we use *cursor* to denote the overall delay of the channel/feed-forward-filter and denote it by  $\nu$ . Due to the relative delay between  $\{r_{i-N_{\text{max}}}\}$  and  $\{\hat{t}_{i-N_{\text{max}}-\nu}\}$ , the overall delay of the first-stage channel/equalizer is also  $\nu$ .

In conventional DFE, post-cursor interference cancellation is accomplished by subtracting FBF-filtered past-decisions at the decision-device input. In a multirate CDMA system, however, the decision-making process incurs a delay of  $N_{\text{max}}$ chips, which can be much greater than the delay spread of the channel. In this case, the feedback signal would not cancel interference present at the decision-device input and the benefit of DFE would vanish. As an alternative, tentative decisions could be fed-forward from a previous stage to replace this "DFE feedback" signal, as shown in Fig. 5. Final bit decisions are made by de-scrambling and de-spreading  $x_2(i)$ , the output of the DFFE. The effectiveness of DFFE post-cursor interference cancellation, however, is not related to DFFE output signal quality but rather the quality of the tentative decisions produced by the previous-stage; this is an important difference between DFFE and DFE.

LMS adaptation is readily applied to update the DFFE feed-forward and feed-back filter weights. [Note from (A1)



Fig. 6. Adaptive ICIC branch corresponding to cursor *j*.

and (A4) that  $\{t_i\}$  is stationary.] The overall system delay is  $N_{\max} + \nu$  and, defining  $\tilde{\boldsymbol{f}}(i) := [\boldsymbol{f}_{\text{ff}}^T(i), \boldsymbol{f}_{\text{fb}}^T(i)]^T$ ,  $\hat{\boldsymbol{t}}(i) := [\hat{\boldsymbol{t}}_i, \dots, \hat{\boldsymbol{t}}_{i-L_b+1}]^T$ , and  $\tilde{\boldsymbol{r}}(i) := [\boldsymbol{r}^T(i), \hat{\boldsymbol{t}}(i-\nu-1)^T]^T$ , where  $L_b$  is the feedback-filter length, the adaptive step-size LMS [11] update equations are:

$$e(i) = \tilde{\boldsymbol{f}}^{H}(i)\tilde{\boldsymbol{r}}(i-N_{\max}) - \hat{t}(i-N_{\max}-\nu), (4)$$
  

$$\psi(i+1) = \left[\boldsymbol{I} - \mu_{i}\tilde{\boldsymbol{r}}(i-N_{\max})\tilde{\boldsymbol{r}}^{H}(i-N_{\max})\right]\psi(i)$$
  

$$-\tilde{\boldsymbol{r}}(i-N_{\max})e^{*}(i), \qquad (5)$$

$$\mu_{i+1} = \mu_{i+1} - \zeta \Re \left\{ \psi^{-}(i) r(i - N_{\max}) e^{*}(i) \right\}, (6)$$

$$\tilde{f}(i+1) = \tilde{f}(i) - \mu_{i} \tilde{r}(i - N_{\max}) e^{*}(i)$$
(7)

$$f(i+1) = f(i) - \mu_i r(i - N_{max}) e(i)$$
. (7)  
For completeness, we state the well-known MMSE-DFF

equalizer solution given perfect channel knowledge (denoted in the simulations by Max-SINR+DFE) [12]:

$$f_{\rm ff}^{(\rm MMSE)} = \left(H(I - M^T M)H^H + \frac{\sigma_w^2}{\sigma_t^2}I\right)^{-1}He_{\nu},(8)$$

$$f_{\rm fb}^{(\rm MMSE)} = -MH^H f_{\rm ff}^{(\rm MMSE)}, \qquad (9)$$

where  $M = \begin{bmatrix} \mathbf{0}_{L_b \times \nu+1} & \mathbf{I}_{L_b \times L_b} & \mathbf{0}_{L_b \times L_h + L_f - \nu - L_b} \end{bmatrix}$ , and  $e_{\nu}$  is a unit vector of zeros with a one in the  $\nu^{th}$  position  $0 \leq 1$  $\nu \le L_h + L_f + 1.$ 

# **IV. INTER-CHIP INTERFERENCE CANCELLATION**

The ICIC uses chip-decisions fed forward from the first stage to cancel both pre- and post-cursor interference in the received signal. While DFFE employed a single cursor " $\nu$ ," ICIC diversity-combines statistics from multiple cursors "j" where  $j \in \{0, \ldots, L_h\}$ . In ICIC,  $\nu$  will still refer to the overall delay of the first-stage channel/equalizer. To recover the energy of the desired chip  $t_{i-D}$  using cursor choice j, where D := $N_{\max} + \nu + L_h$ , the tentative sequence  $\{\hat{t}_{i-N_{\max}} - \nu\}$  is filtered using the impulse response  $\{\hat{h}_0, \dots, \hat{h}_{j-1}, 0, \hat{h}_{j+1}, \dots, \hat{h}_{L_h}\}$ and subtracted from the received signal  $\{r_{i-N_{\text{max}}}\}$ . Equivalently, we could filter using the complete impulse response  $\{\hat{h}_{\ell}\}_{\ell=0}^{\hat{L}_h}$  and subtract out the effect of the unwanted tap  $\hat{h}_j$ , as in Fig. 6. The output of the  $j^{th}$  ICI-cancellation branch is

$$\boldsymbol{y}_{j}(i) = \boldsymbol{r}_{i-D+j} - \sum_{\ell \neq j} \hat{\boldsymbol{h}}_{\ell} \hat{t}_{i-D+j-\ell}, \qquad (10)$$
$$= \boldsymbol{h}_{j} t_{i-D} + \sum_{\ell \neq j} \left( \boldsymbol{h}_{\ell} t_{i-D+j-\ell} - \hat{\boldsymbol{h}}_{\ell} \hat{t}_{i-D+j-\ell} \right)$$
$$+ \boldsymbol{w}_{i-D+j}. \qquad (11)$$

$$-\boldsymbol{w}_{i-D+j}.$$



Fig. 7. Adaptive ICIC with maximum-ratio combining.

If perfect channel-tap estimates and correct bit-decisions are available; i.e.,  $\hat{h}_{\ell} = h_{\ell} \forall \ell$  and  $\hat{t}_k = t_k \forall k \neq i - D$ , then (11) implies  $\boldsymbol{y}_j(i) = \boldsymbol{h}_j t_{i-D} + \boldsymbol{w}_{i-D+j}$ , in which case we have an interference-free estimate of the transmitted chip sequence. Since it is desirable to take advantage of the diversity provided by the channel, the ICIC branches corresponding to different cursors are maximum-ratio combined to form the ICIC output  $x_2(i)$  (see Fig. 7). Assuming perfect ICI cancellation and perfect channel estimates,

$$x_2(i) = \|\boldsymbol{h}\|^2 t(i-D) + \sum_{j=0}^{L_h} h_j^* w(i-D+j), \quad (12)$$

where  $\boldsymbol{h} = [\boldsymbol{h}_0^T, \dots, \boldsymbol{h}_{L_h}^T]^T$ . The quantity  $x_2(i)$  in (12) can be recognized as the (ICI-free) matched-filter output in AWGN. Final bit decisions can then be obtained by de-spreading  $x_2(i)$ , where MAI would be perfectly removed due to the code-orthogonality assumption (A2). While, in practice, channel estimation errors and tentative decision errors will degrade the quality of  $x_2(i)$ , our simulation results suggest that second-stage ICIC processing results in a net benefit at all SNRs.

The channel estimate of the ICIC can be made adaptive via LMS update using the tentative decisions as training:

$$\boldsymbol{e}(i) = \hat{\boldsymbol{H}}^{T}(i)\,\hat{\boldsymbol{t}}(i-N_{\max}-\nu) - \boldsymbol{r}_{i-N_{\max}-\nu} \quad (13)$$
$$\hat{\boldsymbol{H}}(i+1) = \hat{\boldsymbol{H}}(i) - \mu \boldsymbol{e}^{T}(i) \otimes \hat{\boldsymbol{t}}^{*}(i-N_{\max}-\nu).(14)$$

where  $\otimes$  denotes the Kronecker product and  $\hat{H}(i) = [\hat{h}_0(i), \dots, \hat{h}_{L_b}(i)]^T$ .

We can take advantage of sparse channels, i.e., those with only a few significant multipaths, by making the ICIC combining operation itself sparse; only the branches corresponding to the largest channel coefficients need to be combined to form  $x_2(i)$ .

# V. SIMULATIONS

In the simulations we assume a 1/2-chip spaced, 1/2loaded, synchronous DS-CDMA downlink consisting of one user at each of the following spreading factors:  $\{4, 8, 16, 32, 64, 128, 256\}$ . Users transmit BPSK symbols with equal power  $\{P_k\}_{k=1}^{K} = 1$  and the pilot has spreading factor of 256 with power  $P_0 = 4$ . SNR-per-user is defined as  $P_k/\sigma_w^2$ . A "typical-urban" Rayleigh-fading channel [13] is used where channel rays have the power-delay profile that spans approximately 2  $\mu$ s. The chipping rate is 3.84 Mcps, the carrier frequency is 2 GHz, and the square-root raised-cosine chip waveform has excess bandwidth 0.22. Except where noted, the plots show uncoded BER performance averaged across all users. As a lower limit to uncoded BER, the plots show the matched filter (single-user) bound (MFB) for spreading factors 4 (dashed) and 256 (solid)—see [14] for the details of computing the MFB for scrambled CDMA.

Fig.8 shows the performance of the first stage. The adaptive rake employs pilot-based channel estimation in which descrambled pilot-matched-filter outputs were averaged using single-pole filters whose pole locations were BER-optimized through simulation. The equalizers span 25 chips (50 taps) with system delay  $\nu = 21$ , and the adaptive rake spans the entire 28-chip channel delay spread (i.e., 56 taps). The AEAR-LMS (AEAR) algorithm [1] outperforms the adaptive rake at moderate to high SNR, and with switching to DD mode (DD) enabled, BER is significantly reduced at all SNR levels. The max-SINR (max-SINR) receiver maximizes the signal to interference plus noise ratio (SINR) in the symbol estimates. Unlike the adaptive algorithms, the max-SINR receiver assumes perfect knowledge of the time-variant channel.

Fig. 9 and Fig. 10 show the performances of two-stage receivers with DFFE and ICIC as the second stage, respectively. The ICIC receiver offers superior performance due to its ability to cancel both pre- and post-cursor interference. At 16 dB SNR, the ICIC receiver reduces BER by more than an order of magnitude over first-stage processing. The Max-SINR+DFFE and Max-SINR+ICIC curves show performance with perfect channel knowledge for both stages. Sparse ICIC performance is shown in Fig. 11, where ICIC combines only the L largest channel taps. Performance with L = 15 is nearly the same as with  $L = L_h$  (i.e., a non-sparse implementation).

## VI. CONCLUSIONS

We proposed a two-stage adaptive receiver for the scrambled multirate CDMA downlink with a FIR equalizer first stage and an ICI-canceling second stage. We found, through simulation, that the two-stage adaptive ICIC receiver outperforms singlestage and two-stage DFFE receivers. The superior performance is attributed to ICIC's ability to attenuate both pre- and postcursor ICI in the received chip-rate signal.



Fig. 8. Average uncoded BER vs SNR for one-stage receiver.



Fig. 9. Average uncoded BER vs SNR for two-stage receiver with DFFE second stage.



Fig. 10. Average uncoded BER vs SNR for two-stage receiver with ICIC second stage.



Fig. 11. Average uncoded BER vs SNR for sparse ICIC.

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