Adaptive Chip-Rate Equalization of Downlink Multirate Wideband CDMA

Philip Schniter and Adam R. Margetts

Dept. of Electrical Engineering
The Ohio State University
Columbus, OH 43210
schniter.1@osu.edu, margetts@ieee.org

Abstract

We consider a downlink DS-CDMA system in which multirate user signals are transmitted via synchronous orthogonal short codes overlaid with a common scrambling sequence. The transmitted signal is subjected to significant time- and frequency-selective multipath fading.

In response to this scenario, a novel two-step receiver is proposed that combines chip-rate adaptive equalization with error filtering. In the first step, a code-multiplexed pilot is used to adapt the equalizer. The use of error filtering implies a third-order LMS algorithm which has significant advantages over standard LMS in tracking the time-varying channel. In the second step, decision-direction is used to improve the error signal used in adaptation, resulting in improved tracking performance. The performance of the adaptive receiver is studied through analysis and simulation.

1 Introduction

Data rates in the downlink of third generation mobile phone services are expected to be greater than uplink rates due to user-directed services such as internet browsing and video streaming. The mobile terminals in these systems must consume litter power. This motivates low-complexity mobile receivers offering enhanced downlink performance.

In third generation mobile DS-CDMA systems, the downlink multirate bit-streams are multiplexed using orthogonal short codes and then scrambled by a cell-specific long code prior to synchronous transmission, as shown in Fig. 1. The propagation channel is characterized by time- and frequency-selective multipath fading. This destroys the orthogonality among users which in turn substantially degrades the performance of the matched-filter based detector. The usual methods of multipath mitigation in CDMA (e.g., the "blind minimum output energy" techniques [1]) rely on received signal cyclostationarity. In our case, however, the scrambling code destroys the cyclostationarity and so an alternative means of multipath mitigation is required. We focus on adaptive chip-level linear equalization as a means of restoring orthogonality and hence reducing multi-access interference (MAI) in a time- and frequency-selective fading environment. Several linear and approximately minimum mean-squared error (MMSE) adaptive equalizers have been proposed (e.g., [2]–[8]), which update at the bit rate. In this paper we consider novel adaptive equalizer structures that *update at the chip rate* in hope of better tracking the true time-variant MMSE solution.

2 System model

Our received signal model is illustrated in Fig. 1 with the following definitions. K denotes the number of users, N_k the k^{th} user's spreading gain, $b_k(n)$ the k^{th} user's bit stream, $c_k(i)$ the k^{th} user's short code, and $s(i)$ the scrambling sequence. $\{h_i\}$ denotes the chip-spaced channel impulse response (assumed time-invariant for simplicity), M_h the channel length, and $w(i)$ the additive noise. Finally, $t(i)$ denotes the transmitted sequence and $r(i)$ the received sequence.

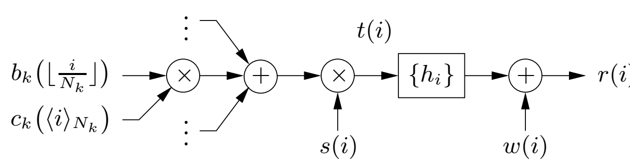

Figure 1: Synchronous Downlink Chip-Spaced Model

We make the following assumptions about the system:

A.1) *Circular, i.i.d., zero-mean, PSK scrambling*:

$$\forall i,\ |s(i)| = 1;\quad \mathrm{E}\{s(i)s^*(i+j)\} = \delta_j.$$

A.2) *Multi-rate orthonormal Walsh codes*:

$$\forall k,\ \ell \text{ s.t. } N_\ell \geq N_k,\ m \in \{0, \ldots, \tfrac{N_\ell}{N_k}-1\}, j:$$
$$\delta_{\ell-k} = \sum_{i=0}^{N_k-1} c_k^*(i) c_\ell(i+mN_k),\ \ |c_k(j)| = \frac{1}{\sqrt{N_k}}$$

A.3) *Constant pilot at "user" index $k=0$*:

$$\forall n,\ b_0(n) = b_0;\ \ c_0(i) = \begin{cases} \frac{1}{\sqrt{N_0}} & 0 \le i \le N_0 - 1 \\ 0 & \text{else} \end{cases}$$

A.4) *Circular, independent, zero-mean user bits ($k > 0$)*:

$$\forall n, m, k \neq 0,\ \ \mathrm{E}\{b_k(n) b_\ell^*(n+m)\} = P_k \delta_m \delta_{\ell-k}$$

where P_k is the symbol power of the k^{th} user.

A.5) *Zero-mean, circular, white, Gaussian noise $w(i)$ with variance σ_w^2.*

The transmitted signal can be written as

$$t(i) = \left(\frac{b_0}{\sqrt{N_0}} + u(i) \right) s(i) \tag{1}$$

which from A.1)-A.4) is zero-mean uncorrelated with power

$$\sigma_t^2 \ = \ \mathrm{E} \left| \frac{b_0}{\sqrt{N_0}} + u(i) \right|^2 \mathrm{E}\, |s(i)|^2 \ = \ \frac{|b_0|^2}{N_0} + \sigma_u^2 \tag{2}$$

where

$$u(i) = \sum_{k=1}^{K} c_k\big(\langle i \rangle_{N_k}\big) b_k\big(\lfloor \tfrac{i}{N_k} \rfloor\big)$$

and

$$\sigma_u^2 \ = \ \sum_{k=1}^{K} \big| c_k\big(\langle j \rangle_{N_k}\big) \big|^2 \, \mathrm{E}\big[\big| b_k\big(\lfloor \tfrac{j}{N_k} \rfloor\big) \big|^2 \big] \ = \ \sum_{k=1}^{K} \frac{P_k}{N_k}$$

The chip-rate received signal is given by

$$r(i) = w(i) + \sum_{\ell=0}^{M_h-1} h_\ell \, t(i-\ell) \tag{3}$$

3 Equalization

In the first subsection, we state the optimal linear MMSE equalizer and SINR expression given the channel state information and additive noise power. In the second subsection, we derive a novel third-order LMS chip-rate adaptive equalizer, and in the third subsection, we discuss a decision directed equalization scheme.

3.1 Optimal MMSE solution

The MMSE chip equalizer that minimizes the cost

$$J_t^{(\nu)} = \mathrm{E} \left| \mathbf{f}^H \mathbf{r}(i+\nu) - t(i) \right|^2 \tag{4}$$

is given by [9]

$$\mathbf{f}_{t,*}^{(\nu)} = \sigma_t^2 \left(\sigma_t^2 \mathbf{H}\mathbf{H}^H + \sigma_w^2 \mathbf{I} \right)^{-1} \mathbf{H}\mathbf{e}_\nu \tag{5}$$

where $\mathbf{r}(i) = [r(i), r(i-1), \ldots, r(i-M_f+1)]^T$; $\mathbf{f} = [f_0, f_1, \ldots, f_{M_f-1}]^T$; $\mathbf{e}_\nu = \left[0 \ldots 0, 1, 0 \ldots 0\right]^T$, i.e., $\mathbf{e}_\nu$ is the unit vector with a one in the ν^{th} position, ($\nu \ge 0$);

$$\mathbf{H} = \begin{bmatrix} h_0 & & & \\ \vdots & h_0 & & \\ h_{M_h-1} & \vdots & \ddots & h_0 \\ & h_{M_h-1} & & \vdots \\ & & \ddots & h_{M_h-1} \end{bmatrix}^T$$

The signal to interference plus noise (SINR) expression for the bit estimate of the ℓ^{th} user can be shown to be [9]

$$\mathrm{SINR}_\ell \ = \ \frac{P_\ell |q_\nu|^2}{\sigma_w^2 \|\mathbf{f}\|^2 + \sigma_t^2 \sum_{m \neq \nu} |q_m|^2} \tag{6}$$

where $\{q_i\}$ is the channel/equalizer response defined by $q_i = \sum_j f_j h_{i-j}^*$. In (4) and (6), expectations are taken over the user bits, the scrambling code, and the additive noise. It is interesting to note that because of the random scrambling code, (6) does not depend on the ℓ^{th} user's spreading factor.

If the total transmitted signal $t(i)$ is available for training, we may use the standard LMS algorithm to adaptively minimize (4) and track (5) in time-varying channel conditions. In the CDMA systems under consideration, however, training comes in the form of a code-multiplexed pilot signal. In other words, the transmitted signal consists of a continuously-transmitted training signal superimposed with unknown user signals. Due to A.3), a chip-rate error signal is readily constructed as the difference between the descrambled equalizer output and a constant reference value, say, γ

$$J_p^{(\nu)} = \mathrm{E} \left| s^*(i) \mathbf{f}^H \mathbf{r}(i+\nu) - \gamma \right|^2 \tag{7}$$

which is minimized by [9]

$$\mathbf{f}_{p,*}^{(\nu)} = \gamma^* \frac{b_0}{\sqrt{N_0}} \left(\sigma_t^2 \mathbf{H}\mathbf{H}^H + \sigma_w^2 \mathbf{I} \right)^{-1} \mathbf{H}\mathbf{e}_\nu \tag{8}$$

Hence choosing

$$\gamma = \frac{\sigma_t^2}{\frac{b_0^*}{\sqrt{N_0}}} \tag{9}$$

sets $\mathbf{f}_{t,*}^{(\nu)} = \mathbf{f}_{p,*}^{(\nu)}$, i.e., proper choice of γ implies that a pilot-trained adaptive LMS equalizer will converge to the MMSE equalizer given by (5).

For clarity of presentation, we have assumed a single-channel system model. However, our analysis can be easily extended to a multichannel system, as would result from oversampling the received signal or adding additional receive antennas. In fact, the simulations in Section 4 correspond to 1/2-chip-spaced sampling.

The pilot signal could also be used to form channel estimates for use in rake combining. As we shall see, though, the performance of such adaptive rakes are inferior to our chip-level adaptive equalizer under low pilot-signal power scenarios.

3.2 Third order LMS

In typical bit-rate equalizer update schemes, the equalized signal is descrambled and then matched-filtered by the pilot code to generate soft pilot-bit estimates. Soft errors can then be calculated (once per bit) and used for equalizer adaptation. When perfectly equalized, the recovered user signals are orthogonal and hence the bit-rate equalizer updates are free of MAI. Before equalizer convergence, however, the recovered users signals are not orthogonal, hence the equalizer updates are corrupted by MAI.

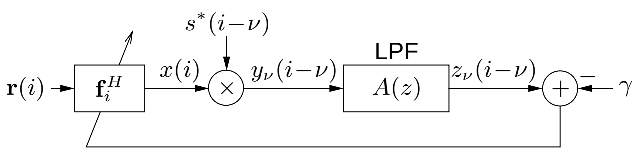

Figure 2: Chip-rate equalizer adaptation using output filtering.

Relative to bit-rate updating, chip-rate updating increases the update rate but employs an error signal corrupted by significantly higher levels of MAI. Nevertheless, the error-signal MAI is zero-mean and can be attenuated through lowpass filtering as shown in Fig. 2. Since lowering the cutoff frequency reduces MAI but slows the reaction to the error signal, the filter bandwidth should be optimized for a particular rate of channel variation and user load. As we shall see, this optimization can be performed on-the-fly. From Fig. 2, the instantaneous chip-rate error signal can be written

$$\hat{J}_{av}(i) \quad = \quad |z_\nu(i)-\gamma|^2 \tag{10}$$

Suppose $A(z)=\frac{\zeta}{1-G(z)}$ where ζ is a constant, where $A(z)$ and $G(z)$ have real valued coefficients, and where $G(z)$ is strictly causal. Define $z(i):=z_\nu(i)$ and $y(i):=y_\nu(i)$. Then $z(i)$ is obtained recursively such that

$$\begin{aligned} z(i) &= \zeta y(i)+\sum_{j=1}^{M_g} g_j z(i-j) \\ &= \zeta \sum_{m=0}^{M_f-1} f_m^*(i+\nu) r(i-m+\nu) s^*(i) \\ &\quad + \sum_{j=1}^{M_g} g_j z(i-j) \end{aligned}$$

To derive the gradient, we realize that

$$\frac{\partial \hat{J}_{av}(i)}{\partial f_\ell^*(i+\nu)} \quad = \quad \big(z(i)-\gamma\big)^* \frac{\partial z(i)}{\partial f_\ell^*(i+\nu)}$$

For convenience we define

$$\alpha_\ell(i) \quad := \quad \frac{\partial z(i)}{\partial f_\ell^*(i+\nu)}, \qquad 0 \le \ell \le M_f-1$$

If we assume, due to small μ, that $f_\ell^*(i+\nu) \approx f_\ell^*(i+\nu-j)$, for $j \in \{1,\ldots,M_g\}$ then

$$\frac{\partial z(i-j)}{\partial f_\ell^*(i+\nu)} \quad \approx \quad \frac{\partial z(i-j)}{\partial f_\ell^*(i-j+\nu)} \;=\; \alpha_\ell(i-j)$$

and we obtain the recursion

$$\alpha_\ell(i) \quad = \quad \zeta r(i-\ell+\nu) s^*(i) + \sum_{j=1}^{M_g} g_j \alpha_\ell(i-j) \tag{11}$$

Note that $\alpha_\ell(i)$ is obtained by delaying the received signal by ℓ, then de-scrambling and filtering.

Defining $\boldsymbol{\alpha}(i)=[\alpha_0(i),\ldots,\alpha_{M_f-1}(i)]^t$, the equalizer update is

$$\mathbf{f}(i+1) \;=\; \mathbf{f}(i)-\mu\cdot\boldsymbol{\alpha}(i-\nu)\big(z(i-\nu)-\gamma\big)^* \tag{12}$$

where $\boldsymbol{\alpha}(i)$ is computed from (11). By using a single-pole lowpass filter, i.e., $A(z)=\frac{1-\rho}{1-\rho z^{-1}}$, the filter bandwidth can be made readily adjustable, and the resulting algorithm takes the form

$$\begin{aligned} \boldsymbol{\alpha}(i) &= (1-\rho)\mathbf{r}(i)s^*(i-\nu)+\rho\boldsymbol{\alpha}(i-1) && (13) \\ e(i) &= (1-\rho)\Big(\mathbf{f}^H(i)\mathbf{r}(i)s^*(i-\nu)-\gamma\Big) \\ &\quad +\rho e(i-1) && (14) \\ \mathbf{f}(i+1) &= \mathbf{f}(i)-\mu\boldsymbol{\alpha}(i)e^*(i) && (15) \end{aligned}$$

As is evident from (13)-(15), the incorporation of single-pole "matched filtering" is a form of filtered-error/filtered-regressor LMS [10]. This particular algorithm can be described as a third-order dynamical system, which has known advantages over standard (first-order) LMS in regards to tracking a Rayleigh-fading channel [11]. The tracking behavior of this algorithm is a function of two adjustable parameters, μ and ρ. Simulation studies under various operating conditions suggest that fixing ρ (within a suitable range) and adjusting μ yields performance very close to that obtained through joint optimization of both parameters. (See Fig. 3). Automatic adjustment of μ can be accomplished using an adaptive step-size procedure (e.g., [12]), implying that this scheme should work well under a wide range of mobility conditions.

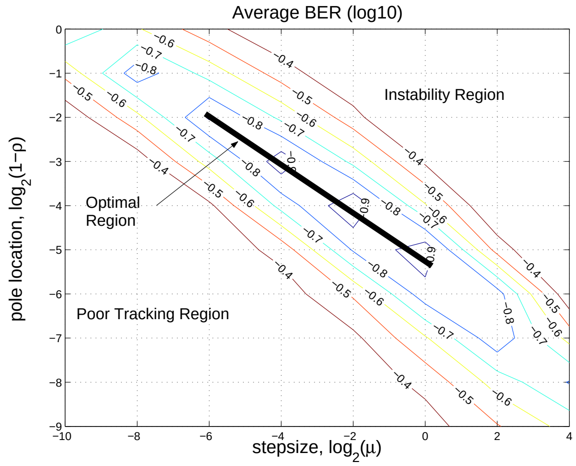

Figure 3: BER versus equalizer pole location and step-size.

3.3 Decision-directed adaptation

Assuming reasonable SNR levels, the pilot-based adaptation scheme tracks the channel reasonably well and provides an output signal from which reliable bit decisions can be obtained. Equalizer tracking could be significantly improved, however, if we could somehow reduce the high level of MAI in the output signal.

With this in mind, we propose a two-stage adaptation scheme. The first stage uses the pilot-trained algorithm from Section 3.2 and is intended for "cold startup" conditions, i.e., when the channel is completely unknown. The second stage uses tentative bit decisions (in addition to the pilot) to adapt a delayed version of the equalizer, as shown in Fig. 4. The tentative decisions are obtained by despreading and detecting the output of the "current" equalizer $\hat{\mathbf{f}}_i$, whose values can be predicted from the delayed equalizer $\mathbf{f}_{i-N_0}$. Joint detection requires, in the worst case, a delay of N_0 chips, where N_0 is the spreading gain of the lowest-rate user. Arguing that, for typical mobile velocities, the equalizer taps experience relatively little change over a span of N_0 chips, the prediction can be accomplished by simply copying $\mathbf{f}_{i-N_0}$ to $\hat{\mathbf{f}}_i$. For best performance, final bit decisions should be made from the delayed output $x(i-N_0)$.

It should be emphasized that our decision-directed (DD) scheme is quite robust to tentative decision errors. In the worst case—a tentative bit error rate of 50%—the MAI power in the DD training signal $\hat{t}(i-N_0-\nu)$ will be no more than twice that in the pilot-only training signal (assuming BPSK and equal user powers for simplicity): in the DD case, decision errors of magnitude 2 are made half the time, while in the pilot case, errors of magnitude 1 are present all the time (since we ignore the user bits altogether). Using the same reasoning, the DD algorithm will have less MAI than the pilot-only algorithm when the tentative BER is below 25%. This implies that BER=0.25 is an appropriate threshold for switching from pilot to DD.

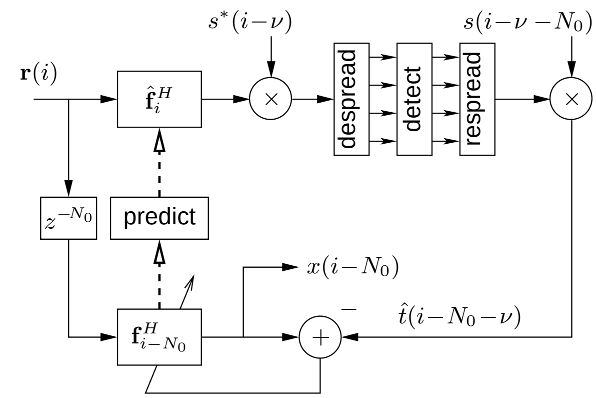

Figure 4: Decision-directed adaptive equalization.

4 Simulations

In all simulations we assume a 1/2-chip-spaced, 1/2-loaded, synchronous DS-CDMA downlink consisting of one user at each of the following spreading factors: $\{4, 8, 16, 32, 64, 128, 256\}$. Users transmit unit power BPSK, and pilot power is one percent of total transmitted power σ_t^2. A Rayleigh-fading channel is used where the chip-spaced rays have power profile $\{0, -3, -6, -9\}$ dB and total power equal to one. Velocity is 60 km/hr, chipping rate is 3.84 Mcps, carrier frequency is 2 GHz, and square-root raised-cosine chip pulsing shaping has excess bandwidth 0.22. The performances in Figs. 5–6 are averaged across users.

Figures 5 and 6 show that DD adaptation significantly increases SINR and BER performance relative to pilot-only adaptation and approaches the performance of MMSE-optimal (non-adaptive) equalization. From Fig. 6 we note that the DD algorithm fails when SNR$<$0 dB. This is consistent with the reasoning in Section 3.3 since, for the first stage pilot-based algorithm, SNR$<$0 dB corresponds to BER$>$0.25. Also shown in Figs. 5 & 6 are the performances of the optimal MMSE equalizer and optimal rake receiver. Unlike the adaptive algorithms we have derived, these optimal receivers assume perfect knowledge of the time-variant channel.

Figures 5 and 6 demonstrate that the pilot-based adaptive equalization scheme (13)-(15) outperforms the classical adaptive rake receiver in time- and frequency-selective multipath fading. The adaptive rake receiver used a pilot-based estimation of channel taps in which descrambled outputs were filtered using single-pole filters whose pole locations were BER-optimized through simulation.

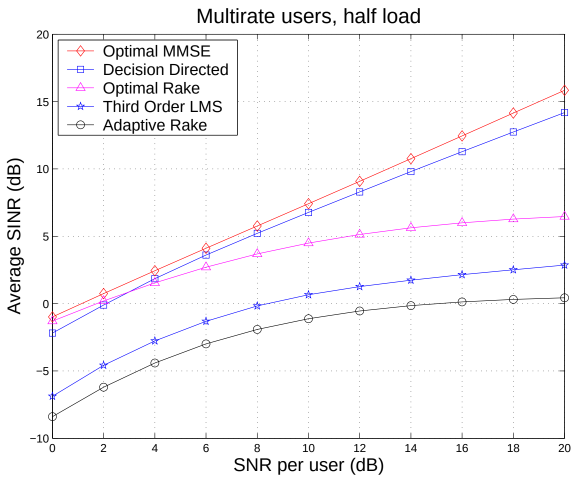

Figure 5: Average SINR versus SNR.

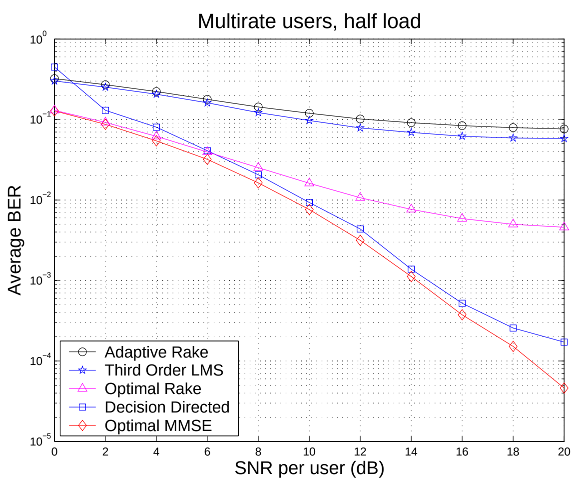

Figure 6: Average uncoded BER versus SNR.

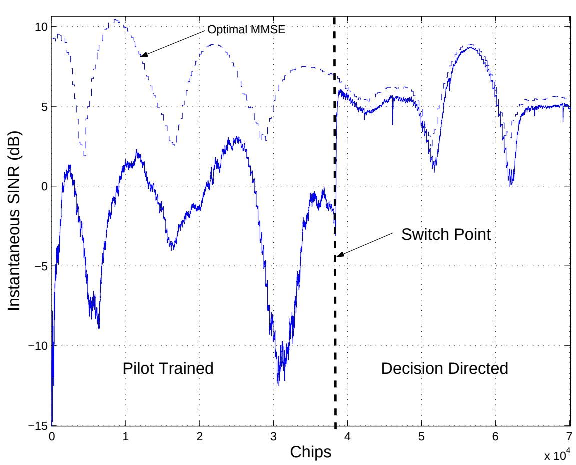

Figure 7: SINR trajectory from cold start to DD-tracking.

Figure 7 shows a prototypical SINR trajectory. From cold start, the pilot-based algorithm first converges then tracks the time-varying channel. After DD is incorporated, the equalizer converges closer to the optimal solution and then continues to track it.

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