Multicarrier Pulse Design and Iterative Equalization for Doubly-Dispersive Channels

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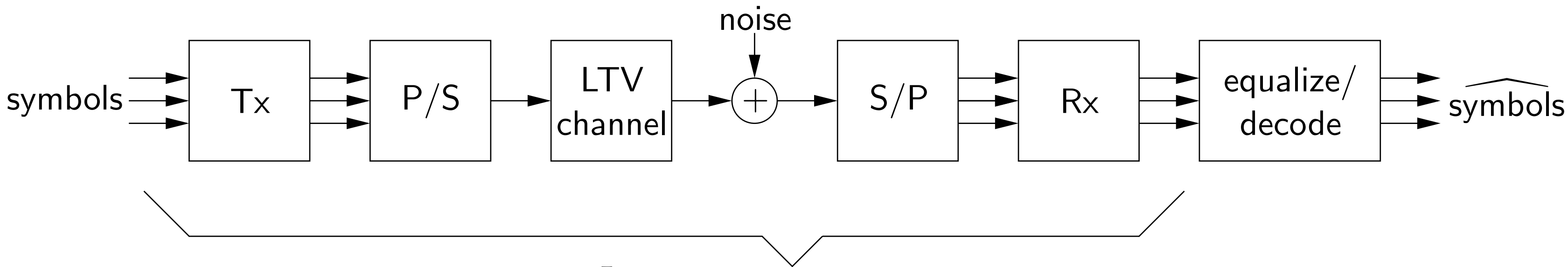
Multicarrier Modulation:



$$\boldsymbol{x}(i) = \sum_{j=-L_{\text{pre}}}^{L_{\text{pst}}} \boldsymbol{\mathcal{H}}(i,j)\boldsymbol{s}(i-j) + \boldsymbol{w}(i)$$

"LTV MIMO channel"

MIMO channel properties depend on Tx/Rx design. Consider:

1. CP-OFDM
2. PS-FDM
3. OQAM-OFDM

CP-OFDM:

- Tx/Rx are extended DFT matrices $\rightsquigarrow$ efficient implementation.
- Adequate guard $\Rightarrow$ no ISI (i.e., $\boldsymbol{\mathcal{H}}(i,j)\big|_{j\neq 0} = \mathbf{0}$).
- Zero Doppler $\Rightarrow$ no ICI (i.e., $\boldsymbol{\mathcal{H}}(i,0)$ diagonal).
- Doubly-dispersive challenges:

long channel $\rightarrow$ long prefix to reduce ISI $\rightarrow$
- short symbol $\rightarrow$ low Eff_{BW}
- long symbol $\rightarrow$ lots of ICI

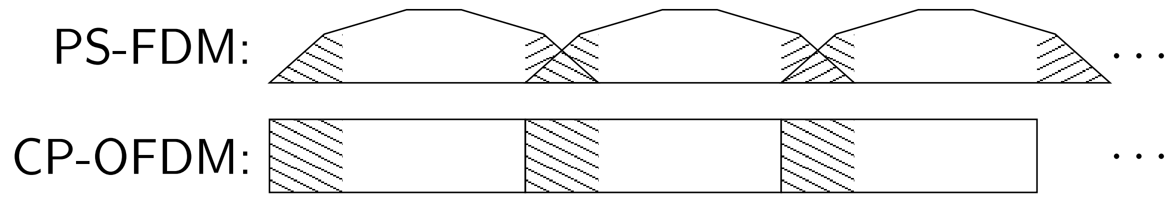
high Doppler $\rightarrow$ short symbol to reduce ICI $\rightarrow$
- short guard $\rightarrow$ lots of ISI
- long guard $\rightarrow$ low Eff_{BW}

- Conclusion:

Inherent tradeoff between { ISI, ICI, Eff_{BW} }!

PS-FDM:

- Essentially CP-OFDM with smooth overlapping windows at Tx/Rx.



Only $\mathcal{O}(N)$ complexity beyond CP-OFDM.

- Non-trivial pulses. . .
  - induce ISI/ICI for trivial chans.
  - reduce ISI/ICI for non-trivial chans (relative to CP-OFDM).

OQAM-OFDM:

- Offset-QAM used in conjunction with Tx/Rx filterbanks.
- Orthogonal/bi-orthogonal filterbanks...
  - maintain zero ISI/ICI for trivial channels
  - reduce ISI/ICI for non-trivial chans (relative to CP-OFDM).
- ISI/ICI suppression proportional to filterbank complexity.

  Much more complex than PS-FDM!

Why worry about ISI/ICI?

Two schools of thought:

1. ISI/ICI-suppressing modulation $\longrightarrow \begin{cases} \text{simple est/detect} \\ \text{low Eff}_{\text{BW}} \\ \text{may lose diversity} \end{cases}$

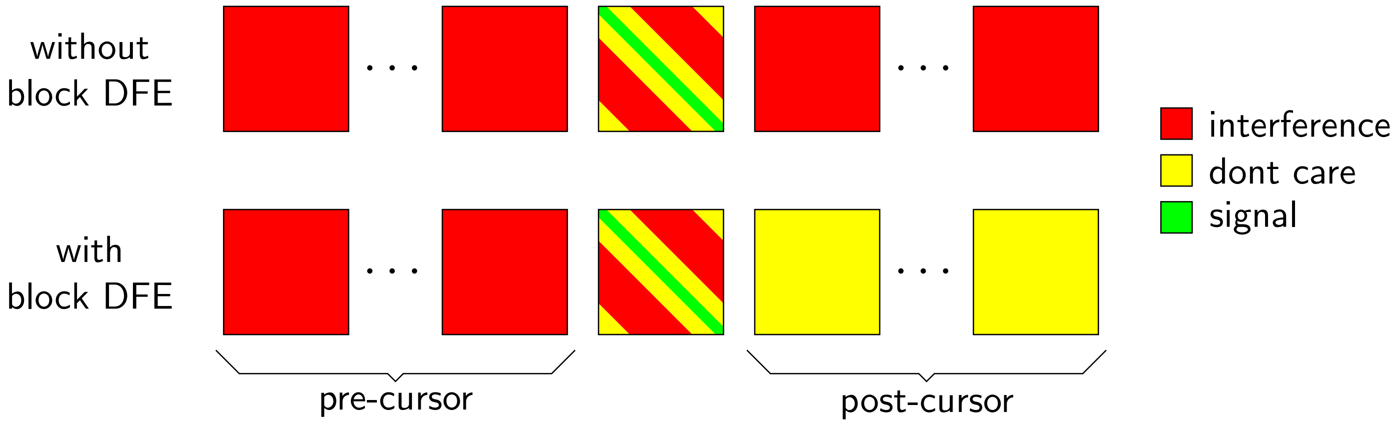
2. ISI/ICI-tolerating modulation $\longrightarrow \begin{cases} \text{high Eff}_{\text{BW}} \\ \text{complicated est/detect} \\ \text{can capture diversity} \end{cases}$.

Key Idea:

- $\rightsquigarrow$ *Design ICI/ISI pattern for simple estimation/detection.*
- $\rightsquigarrow$ *ICI/ISI shaping rather than ICI/ISI suppression.*
- $\rightsquigarrow$ *Leverage iterative equalization algs for banded systems.*

Pulse Design:

Target MIMO channel $\{\mathcal{H}(i,j)\}_{j=-L_{\text{pre}}}^{L_{\text{pst}}}$:



Maximizing SINR gives:

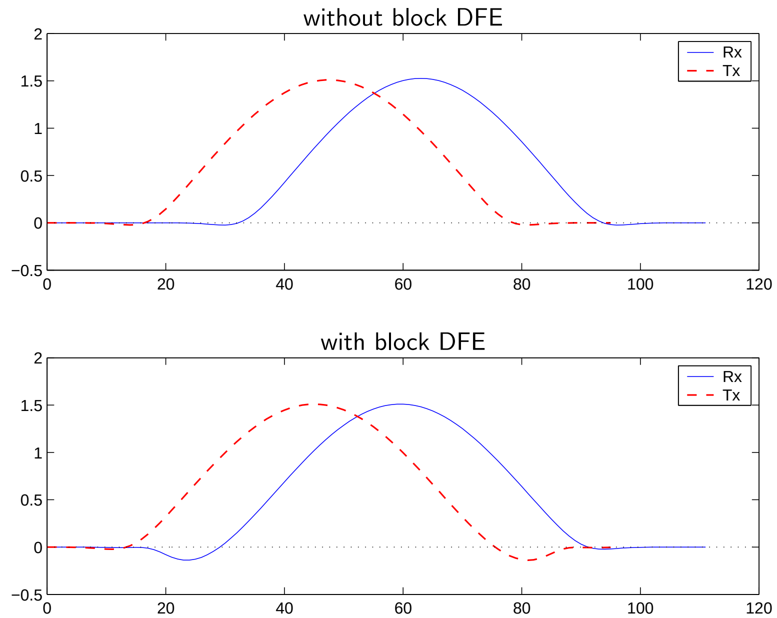
$$\boldsymbol{a}^{(i)} = \underset{\|\boldsymbol{a}\|^2=N_s}{\arg\max} \frac{\boldsymbol{a}^H \boldsymbol{P}_{\text{n}}(\boldsymbol{b}^{(i)})\boldsymbol{a}}{\boldsymbol{a}^H \boldsymbol{P}_{\text{d}}(\boldsymbol{b}^{(i)})\boldsymbol{a}} : \qquad \text{Tx pulse}$$

$$\boldsymbol{b}^{(i+1)} = \underset{\|\boldsymbol{b}\|^2=N_s}{\arg\max} \frac{\boldsymbol{b}^H \boldsymbol{Q}_{\text{n}}(\boldsymbol{a}^{(i)})\boldsymbol{b}}{\boldsymbol{b}^H \boldsymbol{Q}_{\text{d}}(\boldsymbol{a}^{(i)})\boldsymbol{b}} : \qquad \text{Rx pulse}$$

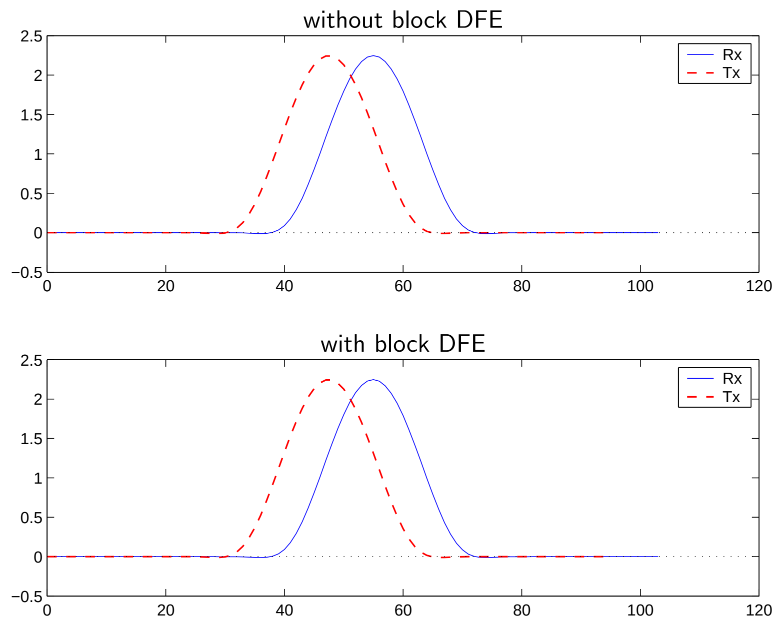
$\leadsto$ alternating between two generalized eigenvalue problems.

Example Max-SINR Pulses:

$N = 64,\ N_h = 32,\ \text{SNR} = 20\text{dB},\ f_\text{d} = 0.03$:



$N = 64,\ N_h = 16,\ \text{SNR} = 5\text{dB},\ f_\text{d} = 0.1$:



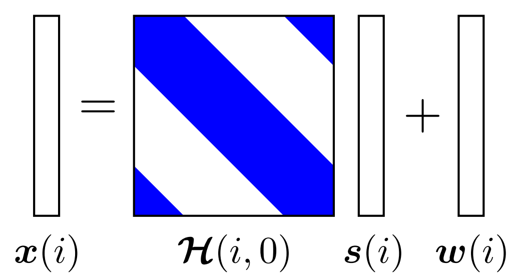
Iterative Equalization:

- We decouple equalization from decoding (for simplicity).
- System model:

$$\boldsymbol{x}(i) = \boldsymbol{\mathcal{H}}(i,0)\boldsymbol{s}(i) + \boldsymbol{w}(i) + \boldsymbol{\varepsilon}(i),$$

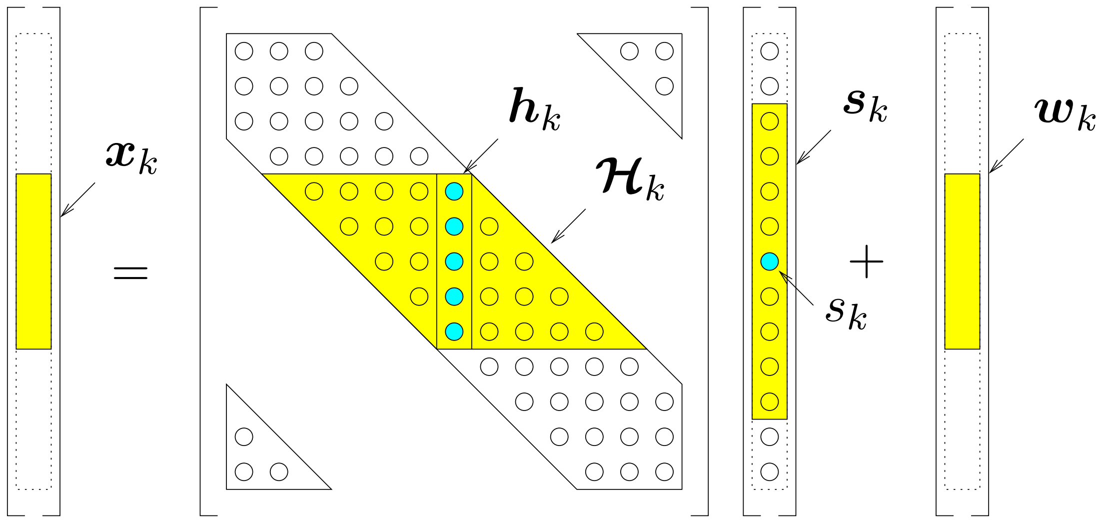
  where $\boldsymbol{\varepsilon}(i)$ represents ISI.

- With successful pulse designs...
  - ISI energy $\ll$ noise energy, so $\boldsymbol{\varepsilon}(i)$ can be ignored.
  - $\boldsymbol{\mathcal{H}}(i,0)$ has "circular-banded" structure.



- Employ iterative equalization tailored to circular-banded system.

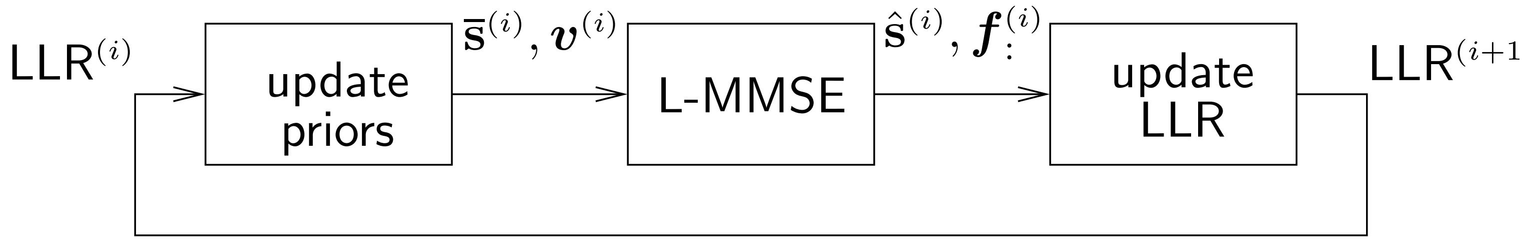
Iterative Equalization (cont.):



$$\boxed{\boldsymbol{x}_k \;=\; \boldsymbol{\mathcal{H}}_k \boldsymbol{s}_k + \boldsymbol{w}_k}$$

- L-MMSE estimate s_k from $\boldsymbol{x}_k$ using priors on interfering symbols in $\boldsymbol{s}_k$.
- generate posteriors $\mathrm{E}\{s_k|\hat{s}_k\}$ and $\mathrm{var}\{s_k|\hat{s}_k\}$.
- $k \leftarrow \langle k+1 \rangle_N$.

Iterative Equalization (cont.):



$$\begin{aligned}
\bar{s}_k^{(i)} &= \widehat{\mathrm{E}\{s_k|\hat{s}_k\}} = \tanh(\mathsf{LLR}_k^{(i)}/2) \\
v_k^{(i)} &= \widehat{\mathrm{var}(s_k|\hat{s}_k)} = 1 - (\bar{s}_k^{(i)})^2
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{f}_k^{(i)} &= \left(\boldsymbol{R}_w + \boldsymbol{\mathcal{H}}_k \,\mathcal{D}(\boldsymbol{v}_k^{(i)})\boldsymbol{\mathcal{H}}_k^H - \boldsymbol{h}_k v_k^{(i)} \boldsymbol{h}_k^H\right)^{-1} \boldsymbol{h}_k \\
\hat{s}_k^{(i)} &= \boldsymbol{f}_k^{(i)H}\left(\boldsymbol{x}_k - \boldsymbol{\mathcal{H}}_k \bar{\boldsymbol{s}}_k^{(i)} + \boldsymbol{h}_k \bar{s}_k^{(i)}\right)
\end{aligned}$$

$$\mathsf{LLR}_k^{(i+1)} = \mathsf{LLR}_k^{(i)} + 4\,\mathrm{Re}(\hat{s}_k^{(i)})/(1 - \boldsymbol{h}_k^H \boldsymbol{f}_k^{(i)})$$

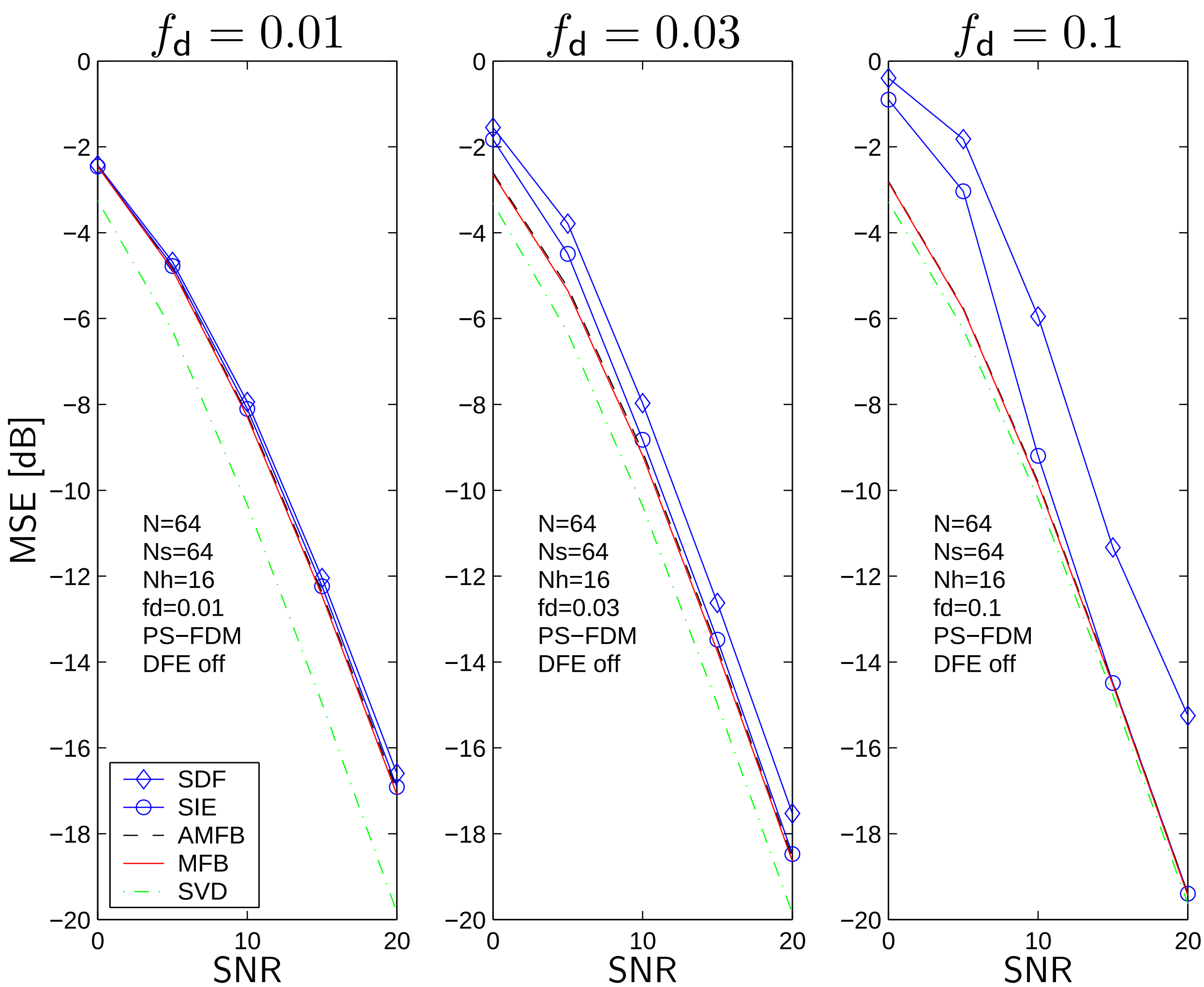
Complexity: $\underset{\text{iters}}{M} \times \underset{\text{syms}}{N} \times \underset{\text{mtx inv}}{\mathcal{O}(D^2)} \rightsquigarrow \mathcal{O}(N)$.

Simulation Setup:

channel	WSSUS Rayleigh
f_{d}	**chip**-normalized Doppler
$N_h = 16$	delay spread [chips]
$N = 64$	BPSK syms per FDM symbol
$N_s = N$	FDM symbol interval [chips]
$N_a = \frac{3}{2} N_s$	Tx pulse duration [chips]
$N_b = N_a + \frac{N_h}{2}$	Rx pulse duration [chips]
$D = \lceil f_{\mathsf{d}} N \rceil + 1$	radius of neighboring-ICI
$5000 \cdot N$	BPSK syms per data point

SIE	soft cancellation of neighboring ICI
SDF	hard cancellation of neighboring ICI
AMFB	perfect cancellation of neighboring ICI
MFB	perfect cancellation of all ISI/ICI
SVD	pulses = singular vecs of channel conv mtx

Performance vs. Doppler:



AMFB = MFB : undesired ISI/ICI is negligible.

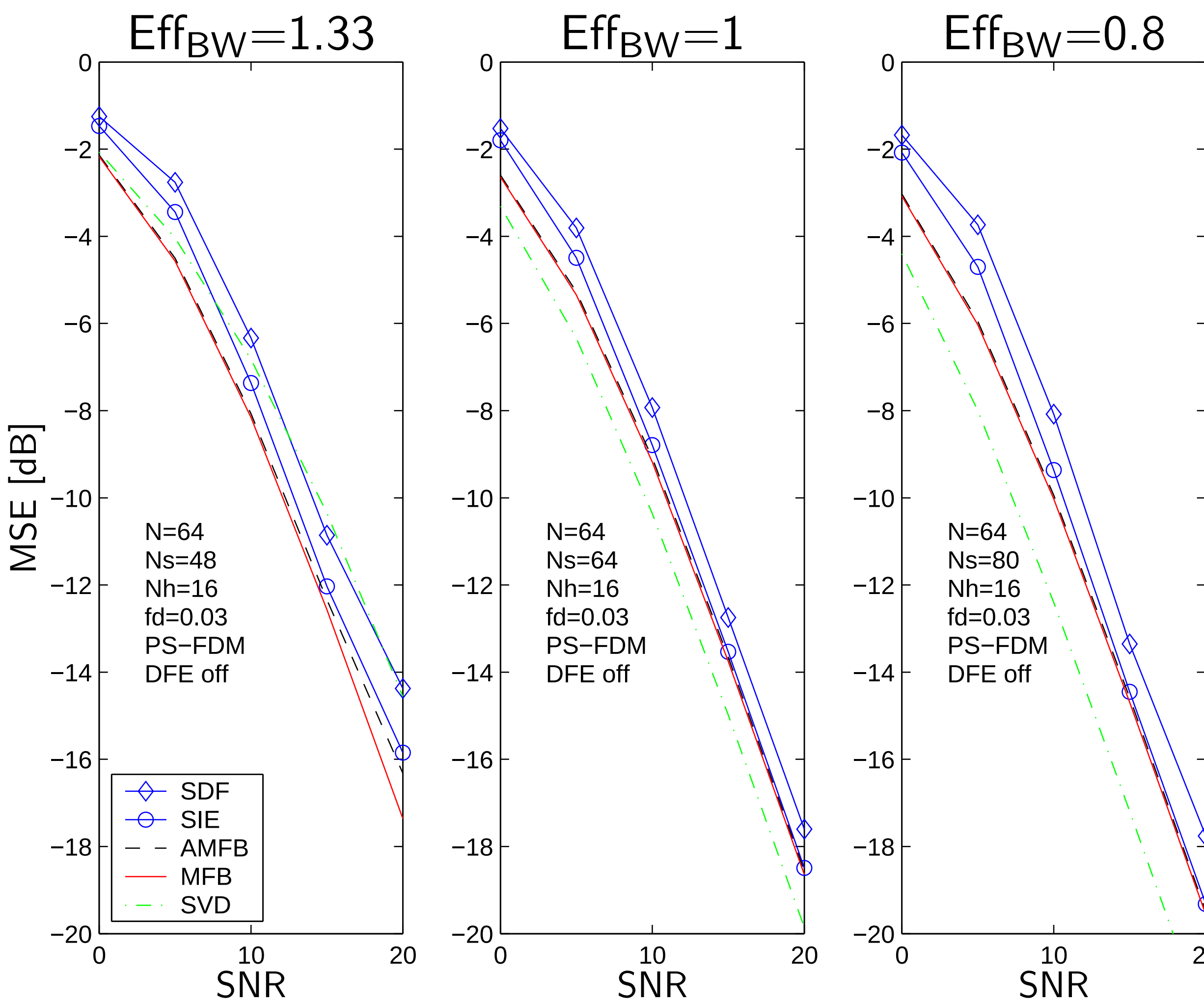
SIE ≈ AMFB : lower bound nearly achieved!

SDF > AMFB : error propagation.

PS-MFB > SVD-MFB : SVD scheme needs CSI at Tx and lots of computation.

(Note: same performance with/without block DFE.)

Performance vs. Eff_{BW}:

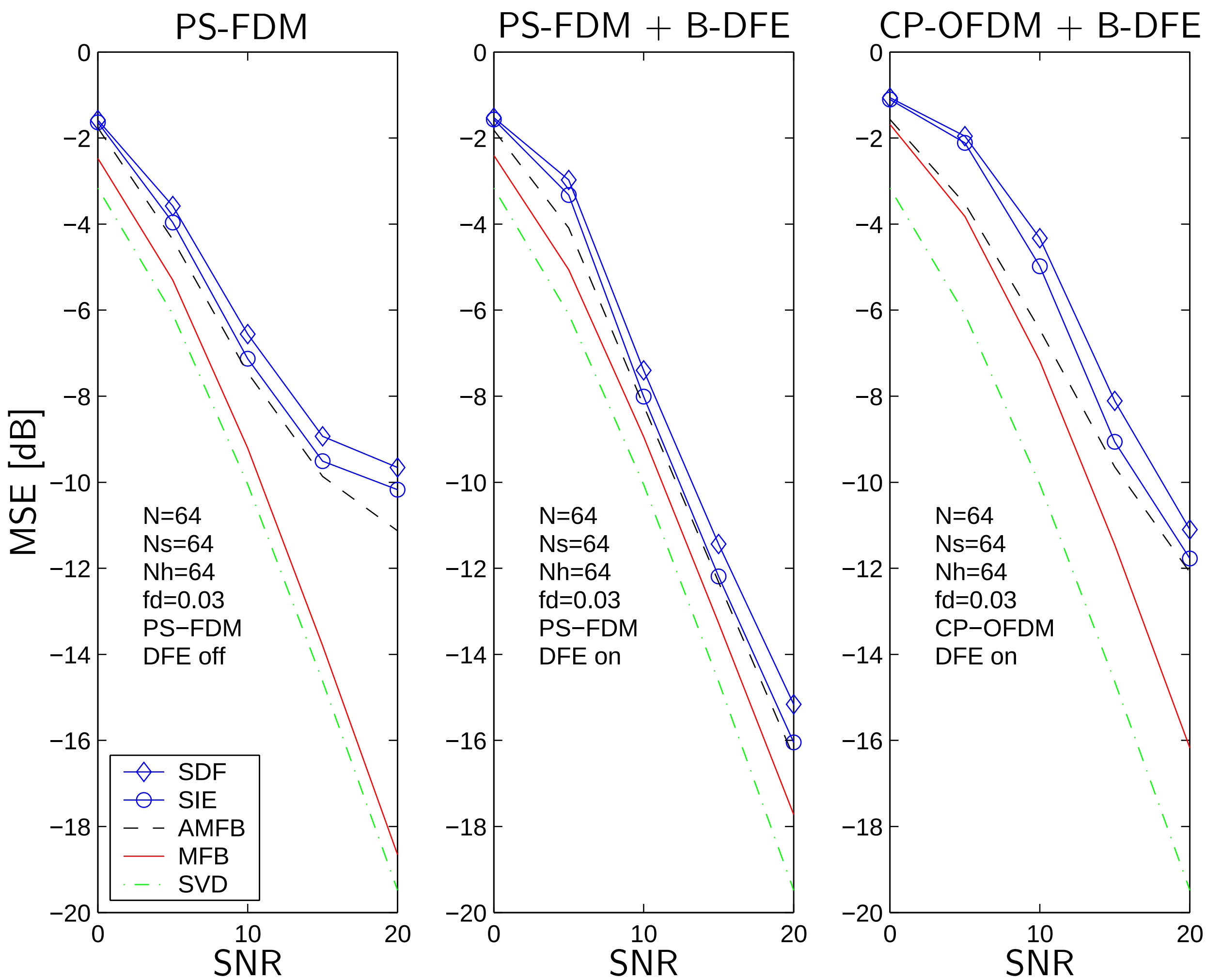


Relative to $\text{Eff}_{\text{BW}} = 1\ldots$

- 1 dB benefit when $\text{Eff}_{\text{BW}} = 0.8$.
- 2-3 dB loss when $\text{Eff}_{\text{BW}} = 1.3$.

(Note: same performance with/without block DFE.)

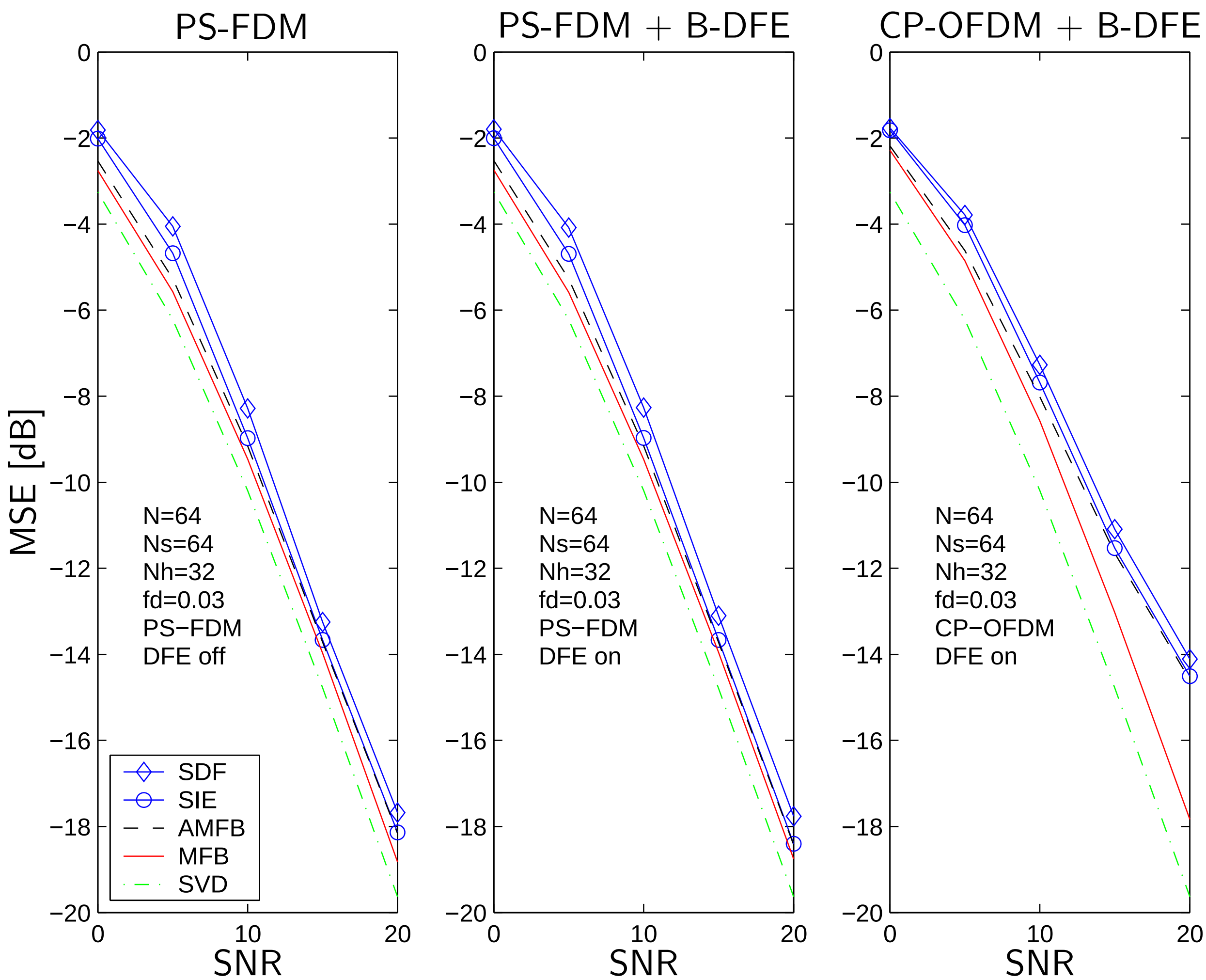
Very long delay spread ($N_h = 64 = N$):



Relative to PS-FDM *with* block-DFE. . .

- PS-FDM *without* block-DFE breaks down.
- CP-OFDM *with* block-DFE has 5 dB loss.

Long delay spread $(N_h = 32 = N/2)$:



Relative to PS-FDM *with* block-DFE. . .

- PS-FDM *without* block-DFE has 0.5 dB loss.
- CP-OFDM *with* block-DFE has 4 dB loss.

Summary:

- Pulse-shaped FDM:
  - ICI/ISI-shaping for efficient equalization/detection.
  - max-SINR pulse design based on fading statistics & SNR.
  - Complexity on par with CP-OFDM.
- Iterative equalization algorithm for "circular-banded" system:
  - Soft cancellation of neighboring ICI.
  - MSE performance near MFB.
  - $\mathcal{O}(N)$ complexity.
- Together...
  - MSE performance near SVD-MFB.
  - BW efficient; capable of over-loaded operation.
  - Can add block-DFE for very long delay spreads (i.e., $N_h \geq N$).