The Role of the Receive Filter in BSE/FSE Comparisons

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April 30, 1999

1 Introduction

White pepper ice cream.
In my mouth.
It stings my lips.
It's like an eclipse.
As if I'm the crossword puzzle but I can't fill in the blank. —Cibo Matto

In this pedagogical snack, we discuss the role of the receive filter (see Fig. 1) in comparisons between BSEs and FSEs. It is the authors' belief that frequently encountered questions regarding "proper" decimation of the FS channel response and "proper" specification of downsampled-noise power can be answered by consideration of the role played by the receive filter.

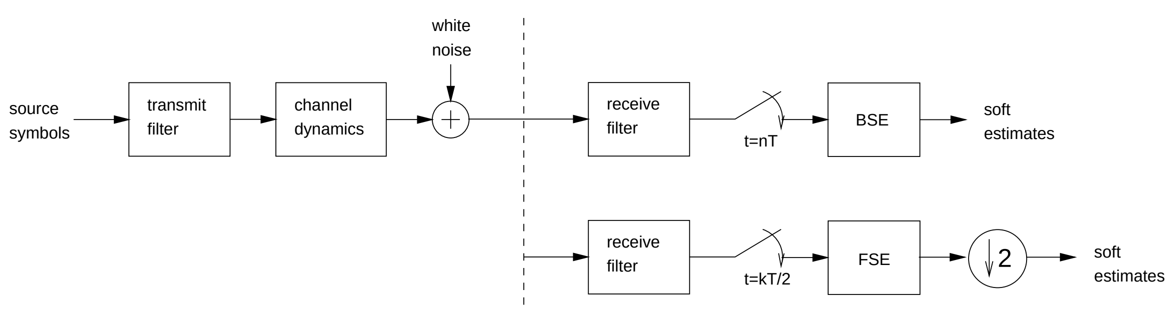

Figure 1: Typical communication system model.

2 System Models

Fig. 1 presents the typical single-user, linear, additive-noise communication channel with pulse-shaping at the transmitter and two receiver options: (i) a BSE, and (ii) a $T/2$-spaced FSE. Assuming that all the blocks in Fig. 1 are LTI, we can represent them in terms of their impulse responses, as shown in Fig. 2. Note that $h(t)$ represents the serial combination of the transmit filter and channel dynamics.

If we assume that the transmit and receive filters are bandlimited to $2\pi/T$ radians, as occurs with the typical choice of square-root raised cosine (SRRC) filtering, the linear system in Fig. 2 has the discrete-time multirate equivalent shown in Fig. 3. There we use the sequence $\{h_k\}$

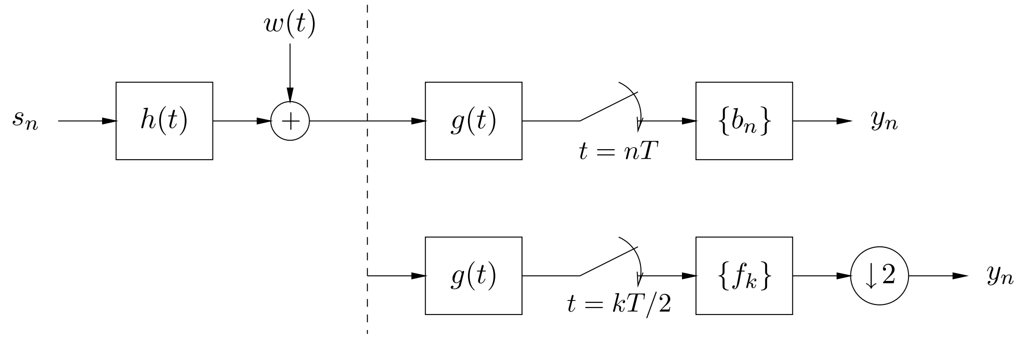

Figure 2: Continuous-time LTI system model.

to denote the fractionally-sampled sequence $\{h(kT/2 + \tau)\}$ (and likewise for other quantities). Throughout, subscripts "k" denote fractionally-spaced quantities, while subscripts "n" denote baud-spaced quantities.

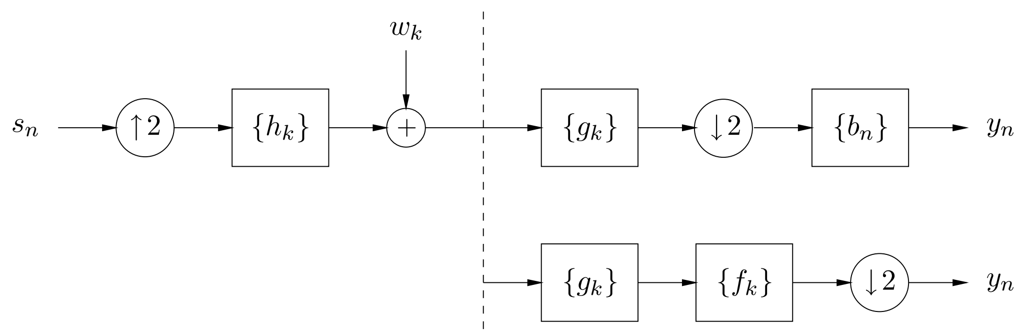

Figure 3: Discrete-time multirate LTI system model.

The next step in our development is the translation of the multirate BSE model to strictly baud-rate quantities. Using standard multirate theory[1] (see, e.g., [Johnson PROC 98]), it can be shown that the diagram in Fig. 4 is equivalent to the BSE portion of Fig. 3.

3 Role of BSE Receive Filter

The role of the BSE's receive filter is made explicit in Fig. 4. To gain intuition, we now analyze three specific choices of receive filter. The following assumptions will be made below: the source process $\{s_n\}$ is zero-mean and white with variance σ_s^2, and the noise process $\{w_k\}$ is zero-mean, white, and Gaussian with variance σ_w^2 and uncorrelated with the source.

Case 1: $g_k = \delta_k$. In this case, Fig. 4 reduces to Fig. 5. The appearance of w_n is justified by the fact that sub-sampling the $T/2$-spaced noise process generates a T-spaced noise process with identical statistics. Note that the magnitude response of this receiver is flat over $\omega \in (-2\pi/T, 2\pi/T]$, i.e., it has twice the bandwidth required for transmission of T-spaced data. Practically speaking, this choice of receive filter

[1] assuming "even" sub-sampling phase.

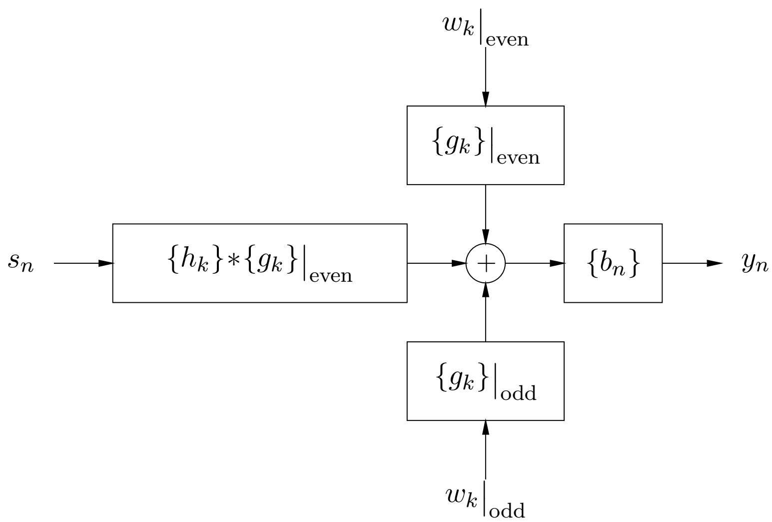

Figure 4: Discrete-time baud-rate BSE system model.

would normally lead to unnecessarily poor BSE performance since it unnecessarily admits out-of-band noise. Ironically, however, this system model is often chosen for BSE vs. FSE comparisons (e.g., [Johnson FOX-1 99]).

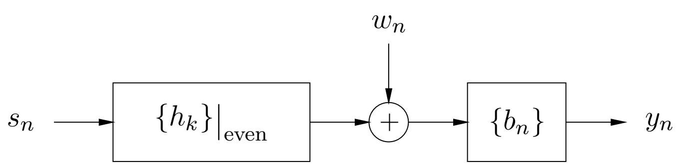

Figure 5: Baud-rate BSE system model for $g_k = \delta_k$.

Case 2: $g_k = \frac{1}{\sqrt{2}}(\delta_k + \delta_{k-1})$. In this case, Fig. 4 reduces to Fig. 6. The appearance of w_n is justified by the fact that the process $\{\frac{1}{\sqrt{2}}(w_k|_{\text{even}} + w_k|_{\text{odd}})\}$ is Gaussian and white with variance σ_w^2, thus identical to $\{w_n\}$. Note that the magnitude response of this receiver is lowpass with roughly the same bandwidth as the minimum required to transmit T-spaced data. Practically speaking, this choice of receive filter is expected to perform much better than $g_k = \delta_k$. In fact, a rough comparison of Fig.s 5 and 6 reveals that the latter has approximately twice the signal power of the former (for the same noise power). Though still not the (optimal) matched filter, this choice of received filter appears quite convenient from a simulation standpoint because it preserves the "additive white noise" feature and leads to a simple method of "converting" the FS channel response to a BS response.

Case 3: $g_k = \text{SRRC}$. For this receive filter, the effective noise will not be white, and there is no simplification of Fig. 4. When the transmit filter is known but the channel dynamics are completely unknown, this is perhaps the best fixed approximation to the (optimal) matched filter ($g_k = h^*_{-k}$). As with the lowpass filter in Case 2, we

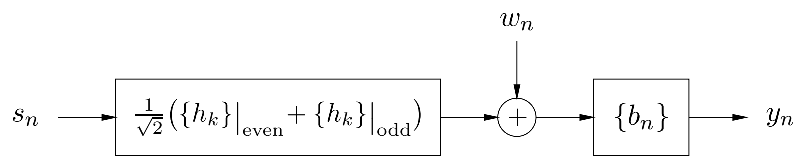

Figure 6: Baud-rate BSE system model for $g_k = \frac{1}{\sqrt{2}}(\delta_k + \delta_{k-1})$.

expect a signal-power to noise-power ratio gain of roughly two relative to the allpass filter in Case 1.

To conclude, we expect BSE performance to strongly depend on the choice of receiver filtering; appropriate lowpass filtering boosts the effective SNR by about 3dB. Though Case 1 depicts a setup frequently used in BSE vs. FSE comparisons, we expect it to generate unfair results and contribute to the myth that "FSEs are always better". Case 3, which seems a good choice from the perspective of BSE performance, should also be applied with caution: it would be unfair to use a receive filter with greater computational complexity that the intended application is likely to support.

4 Role of FSE Receive Filter

Though it is possible for a $T/2$-spaced FSE to perform the combined roles of approximate matched filter and equalizer, we should ask ourselves whether this configuration would be fair for BSE vs./ FSE comparison and/or good design practice.

Fig. 7 presents simulation results demonstrating that FSE performance benefits from even simple (e.g., Case 2) receive filtering, especially for short equalizer lengths. Thus, we conclude that "fair" comparisons should allow the FSE access to the same[2] receive filter as used for the BSE. Though it is a bit harder to ascertain what is "good design practice," we should keep the following in mind: implementation cost of fixed coefficient FIR filters is much less than that of full-precision adaptive FIR filters, and implementation cost can be further reduced by taking advantage of coefficient symmetries and/or numerical precision requirements (e.g., filters with power-of-two coefficients).

5 Conclusion

Based on performance and implementation considerations, it seems appropriate that BSE vs. FSE comparisons are conducted using the diagram in Fig. 3, where $\{g_k\}$ is chosen as an approximation to the optimal matched filter with appropriate computational complexity. Other considerations in BSE vs. FSE comparisons include proper selection of channel model and timing recovery (if any).

[2]Note, however, that the BSE may suffer more from poor choices of receive filter than the FSE.

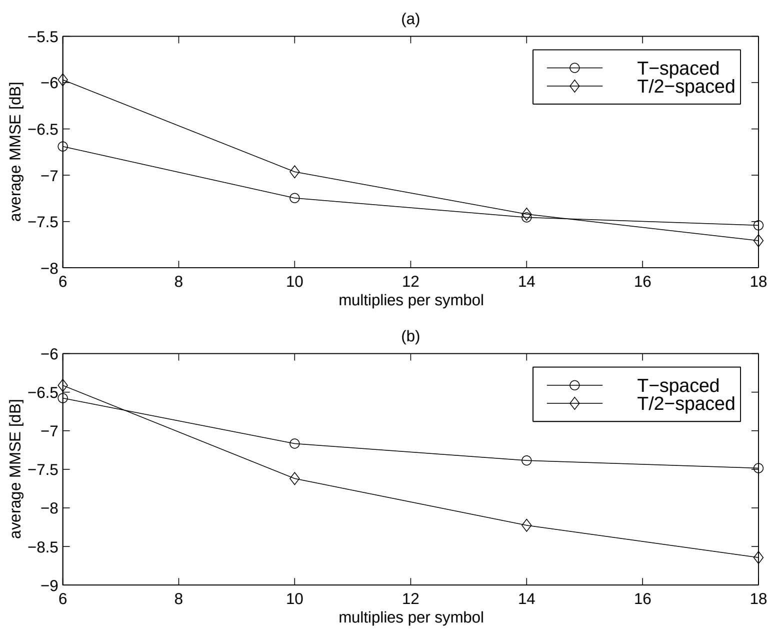

Figure 7: Comparison of BSE and FSE using Case-2 BSE receive filter and (a) no FSE receive filtering, (b) Case-2 FSE receive filtering. Details: $\sigma_s^2/\sigma_w^2 = 10$dB, $\alpha = 11.5\%$, channel duration $4T$, average of 500 simulations.