HOMEWORK ASSIGNMENT #4

Due Tues. Nov. 9, 1999 (in class)

1. *Polyphase/DFT Filterbank:*
In this problem, you will derive the equivalence between the uniformly modulated filterbank in Fig. 1 and its polyphase/DFT implementation in Fig. 2. Assume that the impulse response lengths of $H(z)$ and $K(z)$ both equal M, a multiple of N. The impulse responses of the polyphase filters $H^{(p)}(z)$ and $K^{(p)}(z)$ are related to those of $H(z)$ and $K(z)$ as follows.

$$\begin{aligned} h_\ell^{(p)} &= h_{\ell N+p} \\ k_\ell^{(p)} &= k_{\ell N+p} \end{aligned}$$

(a) Show the equivalence between the analysis banks in Fig. 1 and Fig. 2. (Hint: Using Fig. 1, derive an expression for $s_i(m)$ in terms of input $x(n)$ and filter coefficients $\{h_n\}$. Then convert to polyphase notation using $x^{(p)}(m)$ and $h_m^{(p)}$, and finally $w^{(p)}(m)$.)

(b) Show the equivalence between the synthesis banks in Fig. 1 and Fig. 2. (Hint: Using Fig. 1, derive an expression for $u(n)$ in terms of inputs $s_i(m)$ and filter coefficients $\{k_n\}$. Then convert to polyphase notation using $u^{(p)}(m)$ and $k_m^{(p)}$, and finally $v^{(p)}(m)$.)

(c) Implement the filterbank pairs of Fig. 1 and Fig. 2 in Matlab using $N = 8$, filters of length $M = 64$, and input data created via `x = randn(1,100)`. Using the following impulse response for both $H(z)$ and $K(z)$.

```
h = remez(M-1,[0,.8/N,1.2/N,1],[sqrt(N),sqrt(N),0,0]);
```

Plot the output from both filters as well as the M-delayed input as done in Fig. 3.

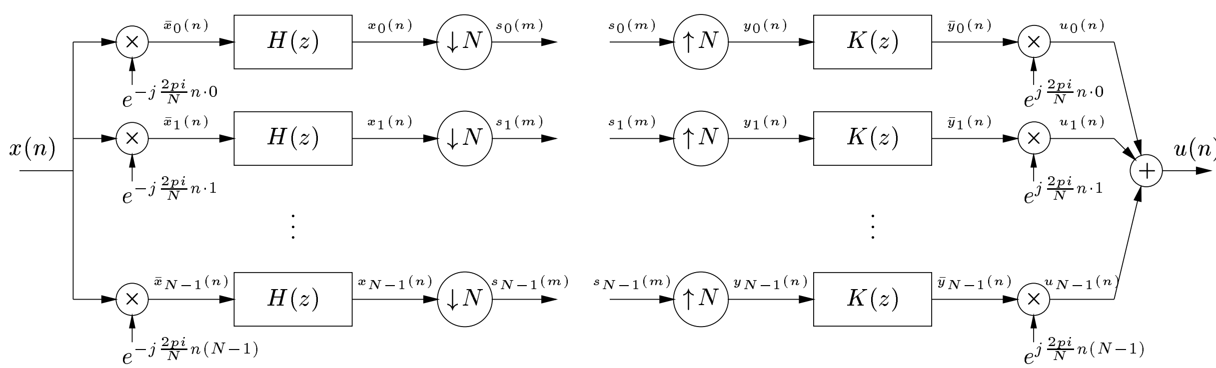

Figure 1: N-band uniformly-modulated analysis/synthesis filterbanks.

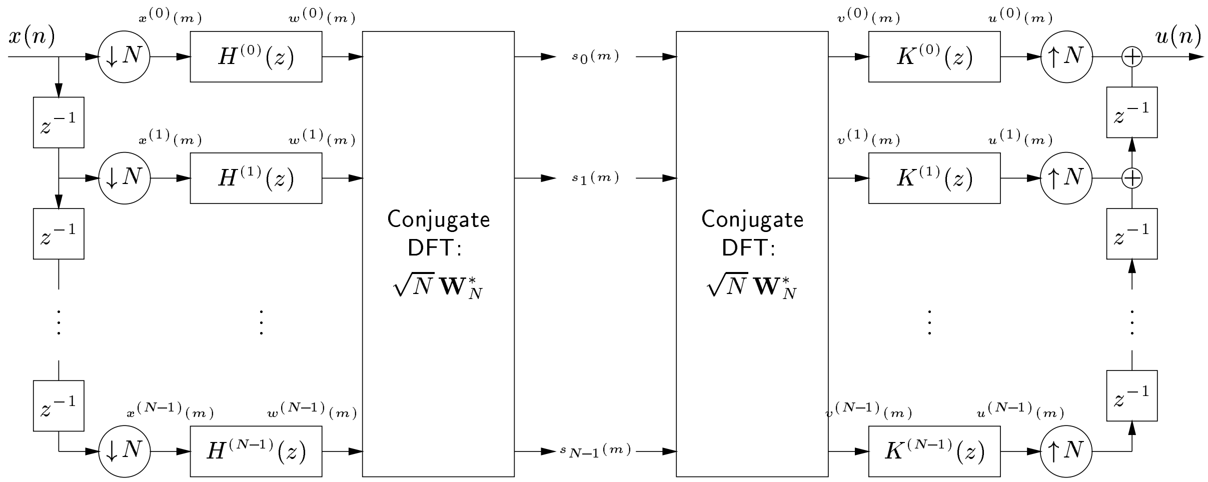

Figure 2: Polyphase/DFT implementation of N-band uniformly modulated analysis/synthesis filterbanks.

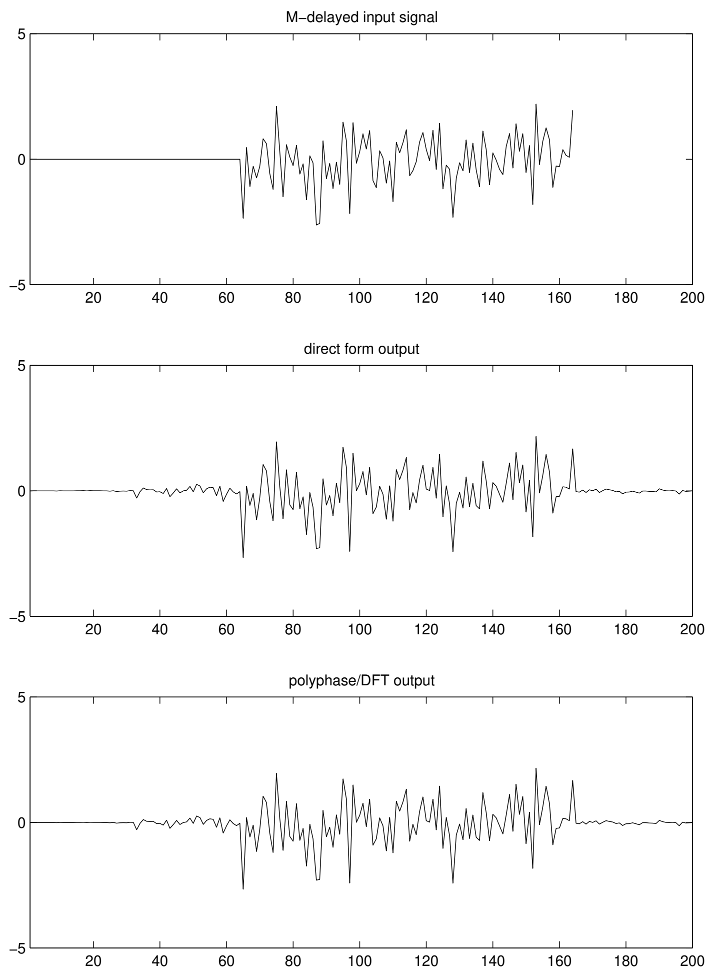

Figure 3: Matlab filterbanks simulation outputs.

2. *MPEG Prototype Filter Design:*
Here we focus on prototype filter design for the MPEG-style cosine-modulated filterbank. The notes derived the following expression for the transfer function of the composite system and derived $\{a_i\}$ and $\{c_i\}$ which result in real-valued filter coefficients and near-perfect reconstruction.

$$\begin{aligned} Q(z) &= U(z)/X(z) \\ &= \sum_{n=0}^{2M-2} \frac{2}{N} \left(\sum_{i=0}^{N-1} \mathrm{Re}(a_i c_i) \cos\left(\pi \frac{2i+1}{2N} n\right) - \mathrm{Im}(a_i c_i) \sin\left(\pi \frac{2i+1}{2N} n\right) \right) \left(\sum_{k=0}^{M-1} h_k h_{n-k} \right) z^{-n}, \end{aligned}$$

Above, N is the number of sub-bands, M is the prototype filter impulse response length, and $\{h_k\}$ is the prototype filter impulse response. Recall that in MPEG, $N = 32$ and $M = 513$.

(a) Assuming a unit-variance white input process $\{x(n)\}$, derive an expression for reconstruction error variance

$$\sigma_e^2 = \mathrm{E}\{|u(n) - x(n-M)|^2\}$$

in terms of the impulse response of the prototype filter $\{q_n\}$.

(b) Using the MPEG prototype filter coefficients in the file[1] `h_mpeg.mat`, plot in dB:

   i. the prototype filter DTFT magnitude $|H(\omega)|$ over $0 \leq \omega \leq \pi$,

   ii. the prototype filter DTFT magnitude $|H(\omega)|$ over $0 \leq \omega \leq \frac{2\pi}{N}$ (to better see the passband) superimposed on the ideal magnitude response, and

   iii. the composite system DTFT magnitude $|Q(\omega)|$ over $0 \leq \omega \leq \pi$,

   and calculate σ_e^2. An example appears in Fig. 4.

(c) Using the `remez` filter design command, attempt to design a length-513 FIR filter with similar passband response but better composite response than the MPEG filter. Can you? Plot the same graphs as in (b) for your best design, and compute σ_e^2. (Hint: make minor adjustments to the passband and stopband cutoff frequencies so that the composite's passband response alternates between $\pm\epsilon$ dB for some very small ϵ.)

(d) Attempt (c) using the `firls` filter design command. Does this seem to be a better design technique?

[1] See the course web page.

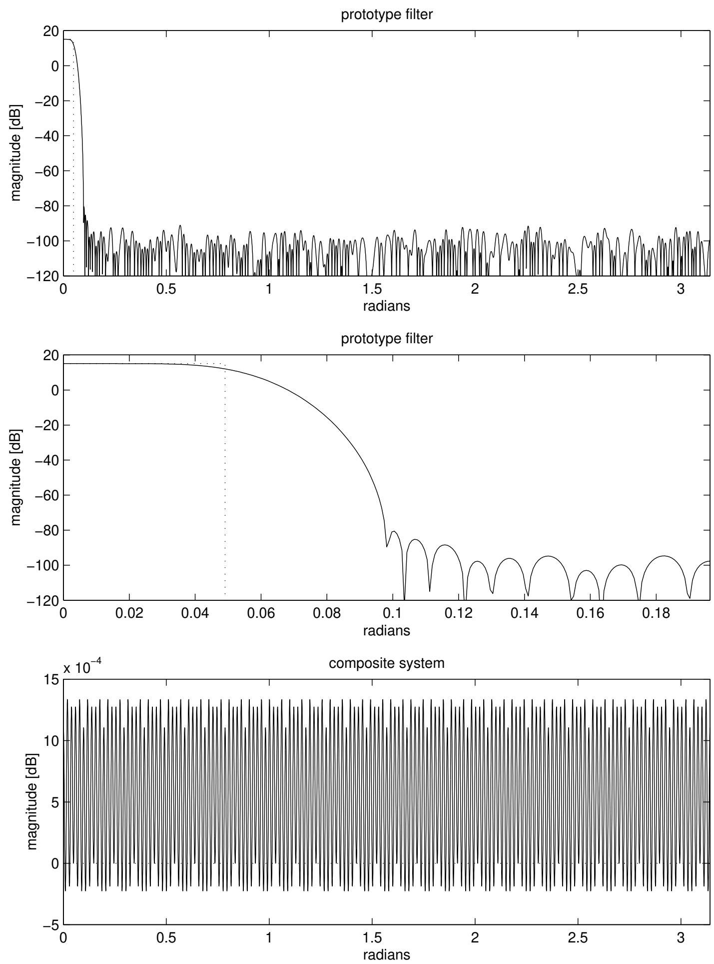

Figure 4: Prototype-filter and composite-system DTFT magnitude responses.

3. *MPEG Filterbank Implementation:*
Here we will implement the polyphase/DCT version of the MPEG filterbank illustrated in Fig.s 25-26 of the subband coding notes.

(a) Implement the filterbank using the prototype filter in `h_mpeg.mat` and an input generated by `x = randn(1,10000)`. (Hints: To generate the full output sequence, zero-pad the beginning of the input record and zero-pad the end by M plus some so that the length of the padded record is a multiple of N. Don't implement the DCT until everything else works.)

Plot the output $u(n)$ and the reconstruction error $e(n) = u(n) - x(n-M)$ for comparison with an M-delayed version of the input $x(n)$. See Fig. 5 for an example.

(b) Calculate the mean-squared reconstruction error (MSRE):

$$\text{MSRE} = \frac{1}{L} \sum_{n=M}^{M+L-1} |u(n) - x(n-M)|^2$$

where L is the length of the input record. How does it compare to σ_e^2?

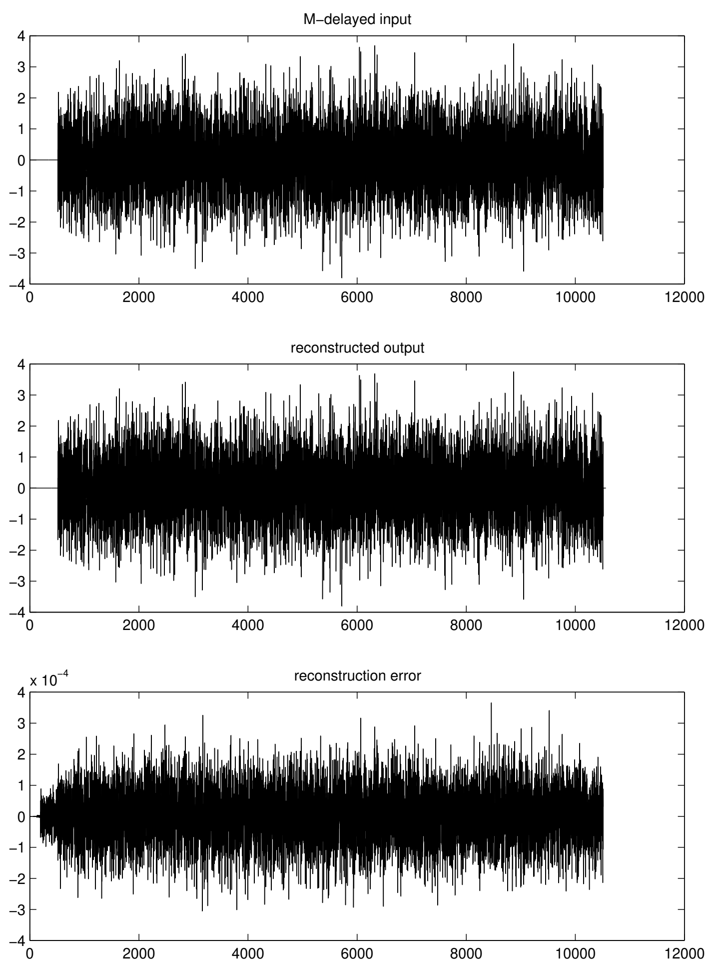

Figure 5: Input, output, and reconstruction error for MPEG filterbank.

