AMP-MMV Demo

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This demo file will provide several examples of how to use the various tools that make up the AMP-MMV software package. We realize that there is a lot to read in this demo, but taking the time to carefully follow the examples we've provided should provide the reader with a fairly thorough understanding of how to correctly operate AMP-MMV. The file tutorial.pdf in the top-level directory of this software package provides a more visually appealing version of this demo.

Note that in order for this demo script to work properly, all of the folders in the AMP-MMV software package must be added to MATLAB's path. This can be accomplished easily by running the script add2path.m, which can be found in the top-level directory of this software package.

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```
% Initialize the random number stream
RandStream.setDefaultStream(RandStream('mt19937ar', 'Seed', 0));
```

Example 1: A Simple Recovery

In this first example, we will demonstrate how to obtain a sample MMV recovery with just a few lines of code.

First, let's generate some synthetic data according to AMP-MMV's signal model. To do this, we will use the function `SIGNAL_GEN_FXN`, which requires as an input an object of the custom-defined class SigGenParams. (The class definition file can be found in the folder ClassDefs.) There are several ways to create objects of this class. The easiest is to use a constructor with no arguments, which will construct an object using all default values for the parameters of the signal model.

```
SigGenObj = SigGenParams();     % Call the default constructor
```

Now we can look at the parameter values associated with our object by using its print method, which will print the current configuration to the command window.

```
SigGenObj.print();      % Print the current signal model configuration
```

```
*****************************************
        Signal Generation Parameters
*****************************************
         Signal dimension (N): 1024
    Measurement dimension (M): 256
           # of timesteps (T): 4
```

```
      Measurement matrix type: IID Gaussian
Activity probability (lambda): 0.08
           Active mean (zeta): 0
     Active variance (sigma2): 1
      Innovation rate (alpha): 0.1
       Empirical SNR (SNRmdB): 25 dB
                  Data type: Complex-valued
```

By looking at the command window printout, we can see that the default signal model consists of a signal of length `N` = 1024, measurement vectors of length `M` = 256, and a total of `T` = 4 timesteps. The measurement matrix will be randomly constructed with IID Gaussian elements. The next several parameters specify the signal prior. We see that the apriori probability that a given coefficient `x_n(t)` is non-zero is given by `lambda` = 0.08. For those coefficients that are non-zero, they have a mean of `zeta` = 0, and a variance of `sigma2` = 1. The active coefficient amplitudes evolve over time with a correlation equal to 1 - `alpha` = 1 - 0.10 = 0.90. Finally, we see that the additive noise will have a variance chosen to yield an SNR of 25dB, and that the data generated will be complex-valued.

Having constructed the object that contains our model parameters, we can now pass it to `SIGNAL_GEN_FXN`, which will produce 6 outputs...

```
[x_true, y, A, support, K, sig2e] = signal_gen_fxn(SigGenObj);
```

...the first of which, `x_true`, is a 1-by-T cell array, with cell t containing the value of the true length-N signal at timestep t, i.e., `x_true{t}`. The second output, `y`, is a 1-by-T cell array of length-M measurements. The third output, `A`, is a 1-by-T cell array of M-by-N (identical) measurement matrices. (We defer for now the reason for giving duplicates of the measurement matrix.) The fourth output is a vector of indices of the non-zero coefficients of the true signal, and the fifth output, `K`, is the cardinality of the support (i.e., the number of true non-zeroes). The sixth and final output, `sig2e`, is the variance of the AWGN corrupting noise.

Now that we have a realization to work with, we can now use AMP-MMV to attempt to recover `x_true` from the noisy measurements, `y`. The easiest way to do this is to call `SP_MMV_WRAPPER_FXN`, which will try several different configurations of AMP-MMV (e.g., different message passing schedules, different numbers of smoothing iterations, etc.) until it finds a solution with a residual energy that is small enough.

In order to use `SP_MMV_WRAPPER_FXN`, we need at least `y`, `A`, and a third input, `Params`, which is an object of the class ModelParams, and contains parameters that specify the signal model. Since we have generated data according to our signal model, (and for now, we will assume perfect knowledge of the model parameters), we can construct a `Params` object easily by passing our SigGenParams object, `SigGenObj`, to the ModelParams constructor, along with `sig2e`, and it will copy over the required parameters:

```
Params = ModelParams(SigGenObj, sig2e);     % Construct ModelParams object
```

Now let's verify that our Params object was constructed correctly by printing its property values to the command window:

```
Params.print();     % Print the value of the Params object properties
```

```
**************************************
        Signal Model Parameters
**************************************
Activity probability (lambda): 0.08
           Active mean (zeta): 0
     Active variance (sigma2): 1
```

```
      Innovation rate (alpha): 0.1
        AWGN variance (sig2e): 0.00092579
```

Everything looks good here, so we are now ready to call SP_MMV_WRAPPER_FXN. Since we are not providing an optional fourth input of runtime configuration options, AMP-MMV will use a default runtime configuration.

```
% Obtain a recovery using AMP-MMV
[x_hat, v_hat, lambda_hat] = sp_mmv_wrapper_fxn(y, A, Params);
```

```
**********************************
      AMP-MMV Runtime Options
**********************************
Max. smoothing iterations: 5
Min. smoothing iterations: 5
   Max. AMP/BP iterations: 15
    Intra-frame algorithm: AMP
    EM parameter learning: Yes
             Update groups: [Default]
                 Verbosity: Verbose
               Warm-Start: No
                  epsilon: 1e-06
  f-to-theta approx (tau): Taylor series approx
**********************************************************************************
Running parallel schedule w/ Taylor approx. & smooth_iters = 5, min_iters = 5, inn
SP_MMV_FXN: Completed 1 iterations
Total elapsed time: 0.038629 s
Time-averaged residual energy: 0.166055
----------------------------------------------------------
SP_MMV_FXN: Completed 2 iterations
Total elapsed time: 0.052474 s
Time-averaged residual energy: 0.173272
----------------------------------------------------------
SP_MMV_FXN: Completed 3 iterations
Total elapsed time: 0.066453 s
Time-averaged residual energy: 0.173192
----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.080000
Updated value of 'alpha' for Group 1: 0.100000
Updated value of 'zeta' for Group 1: 0.0022397-0.0055706i
Updated value of 'sigma2' for Group 1: 0.975009
Updated value of 'sig2e': 0.001007
----------------------------------------------------------
SP_MMV_FXN: Completed 4 iterations
Total elapsed time: 0.086025 s
Time-averaged residual energy: 0.173271
----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.066407
Updated value of 'alpha' for Group 1: 0.099598
Updated value of 'zeta' for Group 1: 0.0022397-0.0055706i
Updated value of 'sigma2' for Group 1: 0.970842
Updated value of 'sig2e': 0.001034
----------------------------------------------------------
SP_MMV_FXN: Completed 5 iterations
Total elapsed time: 0.105013 s
Time-averaged residual energy: 0.173335
----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.066407
Updated value of 'alpha' for Group 1: 0.099598
Updated value of 'zeta' for Group 1: 0.0042776-0.010685i
Updated value of 'sigma2' for Group 1: 0.963186
Updated value of 'sig2e': 0.001045
```

```
------------------------------------------------------------
sp_mmv_fxn: Max # of smoothing iterations reached
Residual energy ratio (actual/expected): 0.731362
Returning recovery from parallel schedule w/ Taylor approx. & smooth_iters = 5, mi
*******************************************************************************
```

The first output, `x_hat`, is a 1-by-T cell array of MMSE signal estimates. The second output, `v_hat`, contains the posterior marginal variances of each MMSE estimate in a 1-by-T cell array, i.e., `v_hat{t}(n) = var{x_n(t) | y}`. The final output, `|lambda_hat`, is a length-N vector of posterior activity probabilities, i.e., `lambda_hat(n) = p(s_n = 1 | y)`.

In addition to returning these three outputs, `SP_MMV_WRAPPER_FXN` has printed some information to the command window. We see that the first things that it printed were the runtime options that it was using (the defaults) during execution. For example, we can see from the printout that the maximum number of smoothing iterations equaled the minimum number (5), and that during the AMP message passes that occurred within each frame, a maximum of 15 AMP iterations were allowed. The EM parameter learning procedure is enabled by default, and the algorithm is set to operate verbosely. We also see that it first attempted a parallel message passing schedule, and we can observe the EM parameter updates of each of the model parameters after each smoothing iteration. Since the residual energy obtained from using this initial schedule was below a pre-defined threshold, the algorithm terminates.

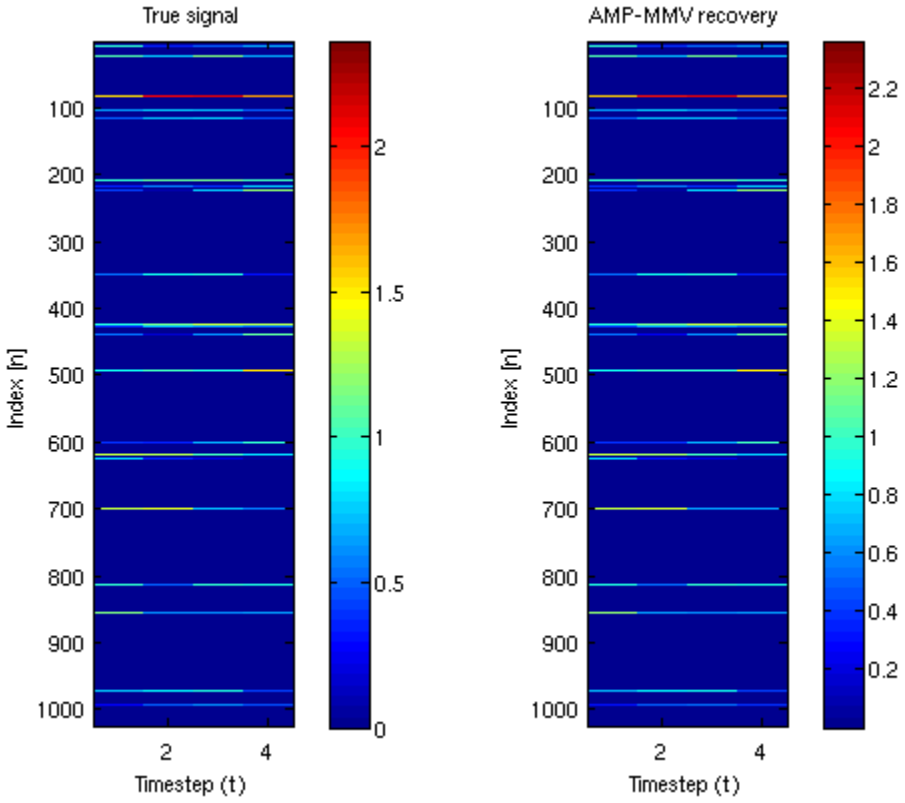
Now let's check the quality of the solution obtained using AMP-MMV. We'll use timestep-averaged normalized mean square error (TNMSE) and the normalized support error rate (NSER) as our performance metrics. As a support estimate, we will use a simple ML estimator that relies on our vector of posterior activity probabilities, `lambda_hat`.

```
s_hat = (lambda_hat > 1/2);     % ML support estimate

T = SigGenObj.T;        % Total # of timesteps

TNMSE = sum(sum(([x_true{:}]-[x_hat{:}]).^2, 1)./sum([x_true{:}].^2, 1))/T;
NSER = nser(support, find(s_hat == 1));

fprintf('TNMSE: %g dB\n', 10*log10(TNMSE));
fprintf('NSER: %g\n', NSER);
```

```
TNMSE: -25.6568 dB
NSER: 0
```

From the printouts to the command window, we see that AMP-MMV performed well on this problem in both TNMSE and NSER. To finish this example, let's visually observe the true signal and the recovered signal:

```
figure(1);
subplot(121); imagesc(abs([x_true{:}])); colorbar
xlabel('Timestep (t)'); ylabel('Index [n]'); title('True signal')
subplot(122); imagesc(abs([x_hat{:}])); colorbar
xlabel('Timestep (t)'); ylabel('Index [n]'); title('AMP-MMV recovery')
```



Example 2: Customizing AMP-MMV

In the previous example, we used default values for all of the functions that we called. In this example, we will illustrate how to choose non-default values for the parameters.

As before, we will generate synthetic data according to our signal model by calling `SIGNAL_GEN_FXN`. Once again, we start by constructing an object of the SigGenParams class. Previously, we called the SigGenParams constructor with no arguments, which created an object of all default values. Suppose now that we'd like to come up with a more challenging test case. Let's keep the signal dimension, `N`, the same, but reduce the number of measurements at each timestep, `M`, to 128, giving an undersampling ratio of `N/M = 8`. Let's likewise adjust the prior activity probability, `lambda`, so that `M/K = 2`, that is, `lambda = 0.0625`. We can use the following constructor call:

```
SigGenObj = SigGenParams('M', 128, 'lambda', 0.0625);
SigGenObj.print();
```

```
*****************************************
        Signal Generation Parameters
*****************************************
         Signal dimension (N): 1024
    Measurement dimension (M): 128
           # of timesteps (T): 4
      Measurement matrix type: IID Gaussian
Activity probability (lambda): 0.0625
           Active mean (zeta): 0
     Active variance (sigma2): 1
      Innovation rate (alpha): 0.1
       Empirical SNR (SNRmdB): 25 dB
```

```
Data type: Complex-valued
```

From the printout to the command window, we can see that `SigGenObj` has used our two custom settings, and left everything else at their default values. Suppose we realize now that we wanted to change the innovation rate, `alpha`, as well, to `alpha = 0.05`. We could just construct `SigGenObj` all over again, and include our custom choice of alpha in the constructor call. Alternatively, we can change the value of `alpha` for the object we just constructed:

```
SigGenObj.alpha = 0.05;
SigGenObj.print();
```

```
*****************************************
        Signal Generation Parameters
*****************************************
         Signal dimension (N): 1024
    Measurement dimension (M): 128
           # of timesteps (T): 4
      Measurement matrix type: IID Gaussian
Activity probability (lambda): 0.0625
           Active mean (zeta): 0
     Active variance (sigma2): 1
      Innovation rate (alpha): 0.05
       Empirical SNR (SNRmdB): 25 dB
                  Data type: Complex-valued
```

From the printout, we see that the value of `alpha` was successfully updated.

Now, let's generate our synthetic data by calling `SIGNAL_GEN_FXN`:

```
[x_true, y, A, support, K, sig2e] = signal_gen_fxn(SigGenObj);
```

As before, in order to obtain a recovery with AMP-MMV, we need to provide it with an initialization of the model parameters. Last time, we created a ModelParams object, `Params`, with the true parameter values as an initialization by using the constructor ModelParams(SigGenParams, sig2e). This time, let's start with an initialization that is a little bit off from the true values. To do this, we will use an alternative constructor, which has the syntax ModelParams(lambda, zeta, sigma2, alpha, sig2e). Here's how we'll initialize:

```
Params = ModelParams((1/4)*SigGenObj.lambda, 0, 2, 0.029, 1/2*sig2e);
Params.print();
```

```
*************************************
        Signal Model Parameters
*************************************
Activity probability (lambda): 0.015625
           Active mean (zeta): 0
     Active variance (sigma2): 2
      Innovation rate (alpha): 0.029
        AWGN variance (sig2e): 0.00068928
```

In the last example, we used default runtime options for AMP-MMV. This time, let's customize some of those options. To do this, we need to create an object of the class Options, which we will then pass to AMP-MMV. Since this is a more challenging example, let's increase the default maximum number of smoothing iterations to 10, and let's increase the maximum number of inner AMP iterations to 25. We can use the following constructor:

```
RunOpt = Options('smooth_iters', 10, 'inner_iters', 25);
RunOpt.print();
```

```
**********************************
      AMP-MMV Runtime Options
**********************************
Max. smoothing iterations: 10
Min. smoothing iterations: 5
   Max. AMP/BP iterations: 25
    Intra-frame algorithm: AMP
    EM parameter learning: Yes
             Update groups: [Default]
                 Verbosity: Silent
                Warm-Start: No
                   epsilon: 1e-06
  f-to-theta approx (tau): Taylor series approx
```

As we can see from the command window printout, our custom options appear within the `RunOpt` object we just created. Notice though that our constructed object has verbosity defaulting to silent. Let's change this so that we can see what AMP-MMV is doing while it runs.

```
RunOpt.verbose = true;
RunOpt.print();
```

```
**********************************
      AMP-MMV Runtime Options
**********************************
Max. smoothing iterations: 10
Min. smoothing iterations: 5
   Max. AMP/BP iterations: 25
    Intra-frame algorithm: AMP
    EM parameter learning: Yes
             Update groups: [Default]
                 Verbosity: Verbose
                Warm-Start: No
                   epsilon: 1e-06
  f-to-theta approx (tau): Taylor series approx
```

Now everything is ready for us to call AMP-MMV. As before, we'll use `SP_MMV_WRAPPER_FXN` to allow us to try multiple different schedules, only this time we will provide it with our custom runtime options.

```
[x_hat, v_hat, lambda_hat] = sp_mmv_wrapper_fxn(y, A, Params, RunOpt);
```

```
*******************************************************************************
Running parallel schedule w/ Taylor approx. & smooth_iters = 10, min_iters = 5, in
SP_MMV_FXN: Completed 1 iterations
Total elapsed time: 0.056897 s
Time-averaged residual energy: 2.432096
-------------------------------------------------------
SP_MMV_FXN: Completed 2 iterations
Total elapsed time: 0.077258 s
Time-averaged residual energy: 0.100955
-------------------------------------------------------
SP_MMV_FXN: Completed 3 iterations
Total elapsed time: 0.091593 s
Time-averaged residual energy: 0.087364
-------------------------------------------------------
```

```
Updated value of 'lambda' for Group 1: 0.015625
Updated value of 'alpha' for Group 1: 0.029000
Updated value of 'zeta' for Group 1: 0.0014796-0.0084173i
Updated value of 'sigma2' for Group 1: 1.720171
Updated value of 'sig2e': 0.001307
-----------------------------------------------------------
SP_MMV_FXN: Completed 4 iterations
Total elapsed time: 0.109443 s
Time-averaged residual energy: 0.096756
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.053880
Updated value of 'alpha' for Group 1: 0.028812
Updated value of 'zeta' for Group 1: 0.0014796-0.0084173i
Updated value of 'sigma2' for Group 1: 1.708716
Updated value of 'sig2e': 0.001733
-----------------------------------------------------------
SP_MMV_FXN: Completed 5 iterations
Total elapsed time: 0.125749 s
Time-averaged residual energy: 0.105677
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.053880
Updated value of 'alpha' for Group 1: 0.028812
Updated value of 'zeta' for Group 1: 0.0025874-0.015758i
Updated value of 'sigma2' for Group 1: 1.675061
Updated value of 'sig2e': 0.002009
-----------------------------------------------------------
SP_MMV_FXN: Completed 6 iterations
Total elapsed time: 0.142151 s
Time-averaged residual energy: 0.106906
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.053733
Updated value of 'alpha' for Group 1: 0.028761
Updated value of 'zeta' for Group 1: 0.0025874-0.015758i
Updated value of 'sigma2' for Group 1: 1.672036
Updated value of 'sig2e': 0.002201
-----------------------------------------------------------
SP_MMV_FXN: Completed 7 iterations
Total elapsed time: 0.158620 s
Time-averaged residual energy: 0.108514
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.053733
Updated value of 'alpha' for Group 1: 0.028761
Updated value of 'zeta' for Group 1: 0.0038061-0.022855i
Updated value of 'sigma2' for Group 1: 1.662872
Updated value of 'sig2e': 0.002337
-----------------------------------------------------------
SP_MMV_FXN: Completed 8 iterations
Total elapsed time: 0.175435 s
Time-averaged residual energy: 0.109349
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.053716
Updated value of 'alpha' for Group 1: 0.028747
Updated value of 'zeta' for Group 1: 0.0038061-0.022855i
Updated value of 'sigma2' for Group 1: 1.662066
Updated value of 'sig2e': 0.002434
-----------------------------------------------------------
SP_MMV_FXN: Completed 9 iterations
Total elapsed time: 0.191859 s
Time-averaged residual energy: 0.110013
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.053716
Updated value of 'alpha' for Group 1: 0.028747
Updated value of 'zeta' for Group 1: 0.0049578-0.029565i
Updated value of 'sigma2' for Group 1: 1.655693
```

```
Updated value of 'sig2e': 0.002505
------------------------------------------------------------
SP_MMV_FXN: Completed 10 iterations
Total elapsed time: 0.208545 s
Time-averaged residual energy: 0.110454
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.053713
Updated value of 'alpha' for Group 1: 0.028743
Updated value of 'zeta' for Group 1: 0.0049578-0.029565i
Updated value of 'sigma2' for Group 1: 1.655443
Updated value of 'sig2e': 0.002555
------------------------------------------------------------
sp_mmv_fxn: Max # of smoothing iterations reached
Residual energy ratio (actual/expected): 1.25191
Running serial schedule w/ Taylor approx. & smooth_iters = 10, min_iters = 5, inne
SP_MMV_SERIAL_FXN: Completed 1 forward and 0 backward iterations
Total elapsed time: 0.011688 s
Time-averaged residual energy: 0.101638
------------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 1 forward and 1 backward iterations
Total elapsed time: 0.020478 s
Time-averaged residual energy: 0.095457
------------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 2 forward and 1 backward iterations
Total elapsed time: 0.032062 s
Time-averaged residual energy: 0.090395
------------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 2 forward and 2 backward iterations
Total elapsed time: 0.040997 s
Time-averaged residual energy: 0.084839
------------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 3 forward and 2 backward iterations
Total elapsed time: 0.052821 s
Time-averaged residual energy: 0.078083
------------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 3 forward and 3 backward iterations
Total elapsed time: 0.061631 s
Time-averaged residual energy: 0.079161
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.015625
Updated value of 'alpha' for Group 1: 0.029000
Updated value of 'zeta' for Group 1: 0.0058069-0.035203i
Updated steady-state variance for Group 1: 1.873277
Updated value of 'sig2e': 0.001286
------------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 4 forward and 3 backward iterations
Total elapsed time: 0.079272 s
Time-averaged residual energy: 0.096478
------------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 4 forward and 4 backward iterations
Total elapsed time: 0.087883 s
Time-averaged residual energy: 0.101428
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.054064
Updated value of 'alpha' for Group 1: 0.028511
Updated value of 'zeta' for Group 1: 0.0058069-0.035203i
Updated steady-state variance for Group 1: 1.840908
Updated value of 'sig2e': 0.001756
------------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 5 forward and 4 backward iterations
Total elapsed time: 0.103820 s
Time-averaged residual energy: 0.103547
------------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 5 forward and 5 backward iterations
```

```
Total elapsed time: 0.112598 s
Time-averaged residual energy: 0.105613
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.054064
Updated value of 'alpha' for Group 1: 0.028511
Updated value of 'zeta' for Group 1: 0.0068213-0.041078i
Updated steady-state variance for Group 1: 1.783917
Updated value of 'sig2e': 0.002046
-----------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 6 forward and 5 backward iterations
Total elapsed time: 0.128337 s
Time-averaged residual energy: 0.106937
-----------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 6 forward and 6 backward iterations
Total elapsed time: 0.136907 s
Time-averaged residual energy: 0.107282
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.053720
Updated value of 'alpha' for Group 1: 0.028349
Updated value of 'zeta' for Group 1: 0.0068213-0.041078i
Updated steady-state variance for Group 1: 1.773553
Updated value of 'sig2e': 0.002245
-----------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 7 forward and 6 backward iterations
Total elapsed time: 0.153022 s
Time-averaged residual energy: 0.108250
-----------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 7 forward and 7 backward iterations
Total elapsed time: 0.161612 s
Time-averaged residual energy: 0.108402
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.053720
Updated value of 'alpha' for Group 1: 0.028349
Updated value of 'zeta' for Group 1: 0.0078092-0.046801i
Updated steady-state variance for Group 1: 1.761148
Updated value of 'sig2e': 0.002382
-----------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 8 forward and 7 backward iterations
Total elapsed time: 0.177543 s
Time-averaged residual energy: 0.109112
-----------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 8 forward and 8 backward iterations
Total elapsed time: 0.185972 s
Time-averaged residual energy: 0.109211
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.053713
Updated value of 'alpha' for Group 1: 0.028287
Updated value of 'zeta' for Group 1: 0.0078092-0.046801i
Updated steady-state variance for Group 1: 1.757162
Updated value of 'sig2e': 0.002478
-----------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 9 forward and 8 backward iterations
Total elapsed time: 0.201879 s
Time-averaged residual energy: 0.109722
-----------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 9 forward and 9 backward iterations
Total elapsed time: 0.210374 s
Time-averaged residual energy: 0.109788
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.053713
Updated value of 'alpha' for Group 1: 0.028287
Updated value of 'zeta' for Group 1: 0.0087583-0.052284i
Updated steady-state variance for Group 1: 1.747922
Updated value of 'sig2e': 0.002543
```

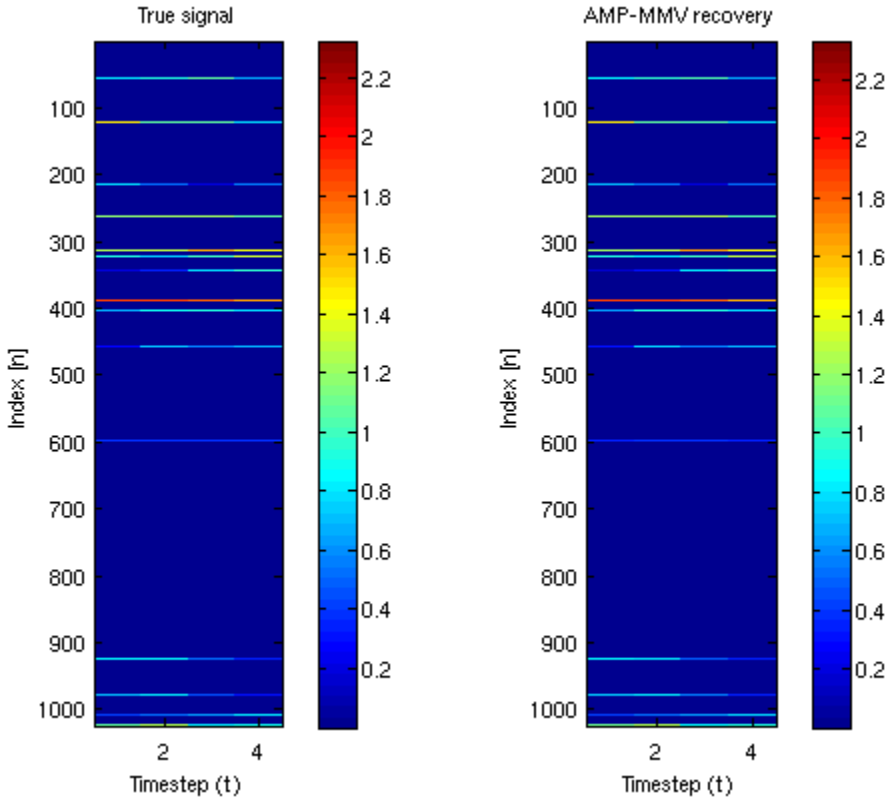
```
------------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 10 forward and 9 backward iterations
Total elapsed time: 0.226213 s
Time-averaged residual energy: 0.110164
------------------------------------------------------------
SP_MMV_SERIAL_FXN: Completed 10 forward and 10 backward iterations
Total elapsed time: 0.234753 s
Time-averaged residual energy: 0.110226
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.053711
Updated value of 'alpha' for Group 1: 0.028258
Updated value of 'zeta' for Group 1: 0.0087583-0.052284i
Updated steady-state variance for Group 1: 1.746129
Updated value of 'sig2e': 0.002588
------------------------------------------------------------
sp_mmv_serial_fxn: Max # of smoothing iterations reached
Residual energy ratio (actual/expected): 1.24933
Returning recovery from serial schedule w/ Taylor approx. & smooth_iters = 10, min
********************************************************************************
```

Once again, we see that SP_MMV_WRAPPER_FXN first tried a parallel message passing schedule, however this time, the resultant residual energy was not low enough, given our initialization of sig2e, to allow AMP-MMV to terminate. Therefore, it next attempted a serial message passing schedule, also with 10 smoothing iterations. This time, the residual energy is low enough, and so SP_MMV_WRAPPER_FXN reports that it is returning the recovery from the serial message passing schedule.

Once more, let's compute our performance metrics, and look at a plot of our recovery:

```
s_hat = (lambda_hat > 1/2);
TNMSE = sum(sum(([x_true{:}]-[x_hat{:}]).^2, 1)./sum([x_true{:}].^2, 1))/T;
NSER = nser(support, find(s_hat == 1));
fprintf('TNMSE: %g dB\n', 10*log10(TNMSE));
fprintf('NSER: %g\n', NSER);
figure(1);
subplot(121); imagesc(abs([x_true{:}])); colorbar
xlabel('Timestep (t)'); ylabel('Index [n]'); title('True signal')
subplot(122); imagesc(abs([x_hat{:}])); colorbar
xlabel('Timestep (t)'); ylabel('Index [n]'); title('AMP-MMV recovery')
```

```
TNMSE: -35.1386 dB
NSER: 0
```



With a TNMSE of -35 dB, and an NSER of 0, we can see that AMP-MMV performed quite well, despite the fact that we initialized it with inaccurate model parameters, which it refined during execution.

Example 3: Using a particular schedule, and running the SKS

In previous experiments, we used `SP_MMV_WRAPPER_FXN` to allow us to cycle through multiple different configurations of AMP-MMV if needed, changing the schedule between serial and parallel, and adjusting the number of smoothing iterations. In this example, we will illustrate how to obtain a recovery using AMP-MMV only once, by calling the function that implements the specific message passing schedule we wish to use. We will also demonstrate how to obtain a lower bound on the achievable TNMSE performance by running the support-aware Kalman smoother.

If we are only interested in the parallel message passing schedule, then we can call the function `SP_MMV_FXN`, which accepts the same arguments as `SP_MMV_WRAPPER_FXN`. Alternatively, if we are interested in the serial message passing schedule, which likewise uses the same arguments as `SP_MMV_WRAPPER_FXN`, then the appropriate function call is `SP_MMV_SERIAL_FXN`.

For this example, let's use the parallel message passing schedule. First let's generate a signal according to our signal model, with some customizations of the default signal model. Let's also try generating real-valued, rather than complex-valued data

```
SigGenObj = SigGenParams('M', 160, 'lambda', 0.0625, 'zeta', 1, ...
    'version', 'R');
SigGenObj.print();
[x_true, y, A, support, K, sig2e] = signal_gen_fxn(SigGenObj);
```

```
*****************************************
        Signal Generation Parameters
*****************************************
         Signal dimension (N): 1024
    Measurement dimension (M): 160
           # of timesteps (T): 4
      Measurement matrix type: IID Gaussian
Activity probability (lambda): 0.0625
           Active mean (zeta): 1
     Active variance (sigma2): 1
      Innovation rate (alpha): 0.1
       Empirical SNR (SNRmdB): 25 dB
                  Data type: Real-valued
```

Now we need to create our ModelParams object. Let's provide AMP-MMV with an inaccurate initialization once again. Let's also create a custom Options object with an increased number of smoothing iterations. We'll bump this maximum number of smoothing iterations way up so that we can observe the parameter estimates move towards their true values.

```
Params = ModelParams(1/5*SigGenObj.lambda, 0, 1, 0.50, 1e-3);
Params.print();

RunOpt = Options('smooth_iters', 70, 'verbose', true);
RunOpt.print();
```

```
*************************************
        Signal Model Parameters
*************************************
Activity probability (lambda): 0.0125
           Active mean (zeta): 0
     Active variance (sigma2): 1
      Innovation rate (alpha): 0.5
        AWGN variance (sig2e): 0.001
*********************************
      AMP-MMV Runtime Options
*********************************
Max. smoothing iterations: 70
Min. smoothing iterations: 5
   Max. AMP/BP iterations: 15
    Intra-frame algorithm: AMP
    EM parameter learning: Yes
            Update groups: [Default]
                Verbosity: Verbose
               Warm-Start: No
                  epsilon: 1e-06
  f-to-theta approx (tau): Taylor series approx
```

Now we are ready to directly call the parallel message passing schedule implementation of AMP-MMV:

```
[x_hat, v_hat, lambda_hat] = sp_mmv_fxn(y, A, Params, RunOpt);
```

```
SP_MMV_FXN: Completed 1 iterations
Total elapsed time: 0.025256 s
Time-averaged residual energy: 37.571589
------------------------------------------------------------
SP_MMV_FXN: Completed 2 iterations
Total elapsed time: 0.050387 s
Time-averaged residual energy: 3.251477
------------------------------------------------------------
```

```
SP_MMV_FXN: Completed 3 iterations
Total elapsed time: 0.075848 s
Time-averaged residual energy: 25.175767
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.012500
Updated value of 'alpha' for Group 1: 0.500000
Updated value of 'zeta' for Group 1: 0.035917
Updated value of 'sigma2' for Group 1: 0.871366
Updated value of 'sig2e': 0.211323
------------------------------------------------------------
SP_MMV_FXN: Completed 4 iterations
Total elapsed time: 0.098856 s
Time-averaged residual energy: 3.457915
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.024905
Updated value of 'alpha' for Group 1: 0.482918
Updated value of 'zeta' for Group 1: 0.035917
Updated value of 'sigma2' for Group 1: 0.831727
Updated value of 'sig2e': 0.255929
------------------------------------------------------------
SP_MMV_FXN: Completed 5 iterations
Total elapsed time: 0.115387 s
Time-averaged residual energy: 20.038591
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.024905
Updated value of 'alpha' for Group 1: 0.482918
Updated value of 'zeta' for Group 1: 0.071914
Updated value of 'sigma2' for Group 1: 0.843570
Updated value of 'sig2e': 0.192565
------------------------------------------------------------
SP_MMV_FXN: Completed 6 iterations
Total elapsed time: 0.132557 s
Time-averaged residual energy: 8.822881
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.034923
Updated value of 'alpha' for Group 1: 0.474286
Updated value of 'zeta' for Group 1: 0.071914
Updated value of 'sigma2' for Group 1: 0.823606
Updated value of 'sig2e': 0.134117
------------------------------------------------------------
SP_MMV_FXN: Completed 7 iterations
Total elapsed time: 0.150713 s
Time-averaged residual energy: 9.096523
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.034923
Updated value of 'alpha' for Group 1: 0.474286
Updated value of 'zeta' for Group 1: 0.10362
Updated value of 'sigma2' for Group 1: 0.814196
Updated value of 'sig2e': 0.120013
------------------------------------------------------------
SP_MMV_FXN: Completed 8 iterations
Total elapsed time: 0.168696 s
Time-averaged residual energy: 5.775555
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.041775
Updated value of 'alpha' for Group 1: 0.462948
Updated value of 'zeta' for Group 1: 0.10362
Updated value of 'sigma2' for Group 1: 0.788617
Updated value of 'sig2e': 0.099559
------------------------------------------------------------
SP_MMV_FXN: Completed 9 iterations
Total elapsed time: 0.186767 s
Time-averaged residual energy: 6.404340
------------------------------------------------------------
```

```
Updated value of 'lambda' for Group 1: 0.041775
Updated value of 'alpha' for Group 1: 0.462948
Updated value of 'zeta' for Group 1: 0.12925
Updated value of 'sigma2' for Group 1: 0.769712
Updated value of 'sig2e': 0.095520
------------------------------------------------------------
SP_MMV_FXN: Completed 10 iterations
Total elapsed time: 0.205871 s
Time-averaged residual energy: 4.631725
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.046216
Updated value of 'alpha' for Group 1: 0.450849
Updated value of 'zeta' for Group 1: 0.12925
Updated value of 'sigma2' for Group 1: 0.743481
Updated value of 'sig2e': 0.086856
------------------------------------------------------------
SP_MMV_FXN: Completed 11 iterations
Total elapsed time: 0.223284 s
Time-averaged residual energy: 5.246462
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.046216
Updated value of 'alpha' for Group 1: 0.450849
Updated value of 'zeta' for Group 1: 0.1519
Updated value of 'sigma2' for Group 1: 0.721696
Updated value of 'sig2e': 0.083801
------------------------------------------------------------
SP_MMV_FXN: Completed 12 iterations
Total elapsed time: 0.240678 s
Time-averaged residual energy: 4.065438
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.051105
Updated value of 'alpha' for Group 1: 0.438995
Updated value of 'zeta' for Group 1: 0.1519
Updated value of 'sigma2' for Group 1: 0.697140
Updated value of 'sig2e': 0.076595
------------------------------------------------------------
SP_MMV_FXN: Completed 13 iterations
Total elapsed time: 0.257308 s
Time-averaged residual energy: 4.289547
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.051105
Updated value of 'alpha' for Group 1: 0.438995
Updated value of 'zeta' for Group 1: 0.17221
Updated value of 'sigma2' for Group 1: 0.675453
Updated value of 'sig2e': 0.071838
------------------------------------------------------------
SP_MMV_FXN: Completed 14 iterations
Total elapsed time: 0.273600 s
Time-averaged residual energy: 3.497187
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.055785
Updated value of 'alpha' for Group 1: 0.427594
Updated value of 'zeta' for Group 1: 0.17221
Updated value of 'sigma2' for Group 1: 0.652917
Updated value of 'sig2e': 0.066238
------------------------------------------------------------
SP_MMV_FXN: Completed 15 iterations
Total elapsed time: 0.288989 s
Time-averaged residual energy: 3.438987
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.055785
Updated value of 'alpha' for Group 1: 0.427594
Updated value of 'zeta' for Group 1: 0.19051
Updated value of 'sigma2' for Group 1: 0.630706
```

```
Updated value of 'sig2e': 0.061069
-----------------------------------------------------------
SP_MMV_FXN: Completed 16 iterations
Total elapsed time: 0.303918 s
Time-averaged residual energy: 3.037180
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.059301
Updated value of 'alpha' for Group 1: 0.416331
Updated value of 'zeta' for Group 1: 0.19051
Updated value of 'sigma2' for Group 1: 0.609516
Updated value of 'sig2e': 0.055596
-----------------------------------------------------------
SP_MMV_FXN: Completed 17 iterations
Total elapsed time: 0.318426 s
Time-averaged residual energy: 2.919550
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.059301
Updated value of 'alpha' for Group 1: 0.416331
Updated value of 'zeta' for Group 1: 0.2054
Updated value of 'sigma2' for Group 1: 0.586927
Updated value of 'sig2e': 0.051224
-----------------------------------------------------------
SP_MMV_FXN: Completed 18 iterations
Total elapsed time: 0.331713 s
Time-averaged residual energy: 2.727802
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.061478
Updated value of 'alpha' for Group 1: 0.405275
Updated value of 'zeta' for Group 1: 0.2054
Updated value of 'sigma2' for Group 1: 0.567183
Updated value of 'sig2e': 0.048383
-----------------------------------------------------------
SP_MMV_FXN: Completed 19 iterations
Total elapsed time: 0.344105 s
Time-averaged residual energy: 2.603785
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.061478
Updated value of 'alpha' for Group 1: 0.405275
Updated value of 'zeta' for Group 1: 0.21619
Updated value of 'sigma2' for Group 1: 0.545471
Updated value of 'sig2e': 0.046867
-----------------------------------------------------------
SP_MMV_FXN: Completed 20 iterations
Total elapsed time: 0.356690 s
Time-averaged residual energy: 2.454229
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.063323
Updated value of 'alpha' for Group 1: 0.394644
Updated value of 'zeta' for Group 1: 0.21619
Updated value of 'sigma2' for Group 1: 0.527466
Updated value of 'sig2e': 0.046019
-----------------------------------------------------------
SP_MMV_FXN: Completed 21 iterations
Total elapsed time: 0.369378 s
Time-averaged residual energy: 2.352970
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.063323
Updated value of 'alpha' for Group 1: 0.394644
Updated value of 'zeta' for Group 1: 0.22454
Updated value of 'sigma2' for Group 1: 0.508693
Updated value of 'sig2e': 0.045310
-----------------------------------------------------------
SP_MMV_FXN: Completed 22 iterations
Total elapsed time: 0.382518 s
```

```
Time-averaged residual energy: 2.244928
-------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.064830
Updated value of 'alpha' for Group 1: 0.384863
Updated value of 'zeta' for Group 1: 0.22454
Updated value of 'sigma2' for Group 1: 0.492925
Updated value of 'sig2e': 0.044556
-------------------------------------------------------
SP_MMV_FXN: Completed 23 iterations
Total elapsed time: 0.395591 s
Time-averaged residual energy: 2.169290
-------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.064830
Updated value of 'alpha' for Group 1: 0.384863
Updated value of 'zeta' for Group 1: 0.23186
Updated value of 'sigma2' for Group 1: 0.477453
Updated value of 'sig2e': 0.043615
-------------------------------------------------------
SP_MMV_FXN: Completed 24 iterations
Total elapsed time: 0.408707 s
Time-averaged residual energy: 2.085301
-------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.065993
Updated value of 'alpha' for Group 1: 0.375939
Updated value of 'zeta' for Group 1: 0.23186
Updated value of 'sigma2' for Group 1: 0.463683
Updated value of 'sig2e': 0.042722
-------------------------------------------------------
SP_MMV_FXN: Completed 25 iterations
Total elapsed time: 0.422181 s
Time-averaged residual energy: 2.008562
-------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.065993
Updated value of 'alpha' for Group 1: 0.375939
Updated value of 'zeta' for Group 1: 0.23847
Updated value of 'sigma2' for Group 1: 0.450916
Updated value of 'sig2e': 0.041726
-------------------------------------------------------
SP_MMV_FXN: Completed 26 iterations
Total elapsed time: 0.436171 s
Time-averaged residual energy: 1.912886
-------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.066838
Updated value of 'alpha' for Group 1: 0.367724
Updated value of 'zeta' for Group 1: 0.23847
Updated value of 'sigma2' for Group 1: 0.438721
Updated value of 'sig2e': 0.040134
-------------------------------------------------------
SP_MMV_FXN: Completed 27 iterations
Total elapsed time: 0.450345 s
Time-averaged residual energy: 1.866082
-------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.066838
Updated value of 'alpha' for Group 1: 0.367724
Updated value of 'zeta' for Group 1: 0.24502
Updated value of 'sigma2' for Group 1: 0.427930
Updated value of 'sig2e': 0.038075
-------------------------------------------------------
SP_MMV_FXN: Completed 28 iterations
Total elapsed time: 0.464592 s
Time-averaged residual energy: 1.822188
-------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.066683
Updated value of 'alpha' for Group 1: 0.360134
```

```
Updated value of 'zeta' for Group 1: 0.24502
Updated value of 'sigma2' for Group 1: 0.417050
Updated value of 'sig2e': 0.036023
------------------------------------------------------------
SP_MMV_FXN: Completed 29 iterations
Total elapsed time: 0.478330 s
Time-averaged residual energy: 1.733974
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.066683
Updated value of 'alpha' for Group 1: 0.360134
Updated value of 'zeta' for Group 1: 0.25102
Updated value of 'sigma2' for Group 1: 0.407966
Updated value of 'sig2e': 0.034221
------------------------------------------------------------
SP_MMV_FXN: Completed 30 iterations
Total elapsed time: 0.492000 s
Time-averaged residual energy: 1.590650
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.068066
Updated value of 'alpha' for Group 1: 0.353297
Updated value of 'zeta' for Group 1: 0.25102
Updated value of 'sigma2' for Group 1: 0.398465
Updated value of 'sig2e': 0.032787
------------------------------------------------------------
SP_MMV_FXN: Completed 31 iterations
Total elapsed time: 0.505564 s
Time-averaged residual energy: 1.487443
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.068066
Updated value of 'alpha' for Group 1: 0.353297
Updated value of 'zeta' for Group 1: 0.2579
Updated value of 'sigma2' for Group 1: 0.392297
Updated value of 'sig2e': 0.031253
------------------------------------------------------------
SP_MMV_FXN: Completed 32 iterations
Total elapsed time: 0.518096 s
Time-averaged residual energy: 1.428560
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.068558
Updated value of 'alpha' for Group 1: 0.347332
Updated value of 'zeta' for Group 1: 0.2579
Updated value of 'sigma2' for Group 1: 0.384200
Updated value of 'sig2e': 0.030036
------------------------------------------------------------
SP_MMV_FXN: Completed 33 iterations
Total elapsed time: 0.530682 s
Time-averaged residual energy: 1.370005
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.068558
Updated value of 'alpha' for Group 1: 0.347332
Updated value of 'zeta' for Group 1: 0.26483
Updated value of 'sigma2' for Group 1: 0.379887
Updated value of 'sig2e': 0.029278
------------------------------------------------------------
SP_MMV_FXN: Completed 34 iterations
Total elapsed time: 0.543226 s
Time-averaged residual energy: 1.322940
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.069036
Updated value of 'alpha' for Group 1: 0.341981
Updated value of 'zeta' for Group 1: 0.26483
Updated value of 'sigma2' for Group 1: 0.372757
Updated value of 'sig2e': 0.028695
------------------------------------------------------------
```

```
SP_MMV_FXN: Completed 35 iterations
Total elapsed time: 0.556051 s
Time-averaged residual energy: 1.282074
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.069036
Updated value of 'alpha' for Group 1: 0.341981
Updated value of 'zeta' for Group 1: 0.27159
Updated value of 'sigma2' for Group 1: 0.370081
Updated value of 'sig2e': 0.028143
-----------------------------------------------------------
SP_MMV_FXN: Completed 36 iterations
Total elapsed time: 0.569401 s
Time-averaged residual energy: 1.257515
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.068963
Updated value of 'alpha' for Group 1: 0.337243
Updated value of 'zeta' for Group 1: 0.27159
Updated value of 'sigma2' for Group 1: 0.363852
Updated value of 'sig2e': 0.027365
-----------------------------------------------------------
SP_MMV_FXN: Completed 37 iterations
Total elapsed time: 0.581911 s
Time-averaged residual energy: 1.223247
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.068963
Updated value of 'alpha' for Group 1: 0.337243
Updated value of 'zeta' for Group 1: 0.27787
Updated value of 'sigma2' for Group 1: 0.362583
Updated value of 'sig2e': 0.026613
-----------------------------------------------------------
SP_MMV_FXN: Completed 38 iterations
Total elapsed time: 0.594422 s
Time-averaged residual energy: 1.194271
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.069079
Updated value of 'alpha' for Group 1: 0.333010
Updated value of 'zeta' for Group 1: 0.27787
Updated value of 'sigma2' for Group 1: 0.357067
Updated value of 'sig2e': 0.025851
-----------------------------------------------------------
SP_MMV_FXN: Completed 39 iterations
Total elapsed time: 0.606915 s
Time-averaged residual energy: 1.165853
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.069079
Updated value of 'alpha' for Group 1: 0.333010
Updated value of 'zeta' for Group 1: 0.28388
Updated value of 'sigma2' for Group 1: 0.356760
Updated value of 'sig2e': 0.025219
-----------------------------------------------------------
SP_MMV_FXN: Completed 40 iterations
Total elapsed time: 0.619576 s
Time-averaged residual energy: 1.147213
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.069520
Updated value of 'alpha' for Group 1: 0.329081
Updated value of 'zeta' for Group 1: 0.28388
Updated value of 'sigma2' for Group 1: 0.351672
Updated value of 'sig2e': 0.024554
-----------------------------------------------------------
SP_MMV_FXN: Completed 41 iterations
Total elapsed time: 0.632052 s
Time-averaged residual energy: 1.124605
-----------------------------------------------------------
```

```
Updated value of 'lambda' for Group 1: 0.069520
Updated value of 'alpha' for Group 1: 0.329081
Updated value of 'zeta' for Group 1: 0.28935
Updated value of 'sigma2' for Group 1: 0.351771
Updated value of 'sig2e': 0.023957
-----------------------------------------------------------
SP_MMV_FXN: Completed 42 iterations
Total elapsed time: 0.644533 s
Time-averaged residual energy: 1.106094
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.070019
Updated value of 'alpha' for Group 1: 0.325342
Updated value of 'zeta' for Group 1: 0.28935
Updated value of 'sigma2' for Group 1: 0.346950
Updated value of 'sig2e': 0.023366
-----------------------------------------------------------
SP_MMV_FXN: Completed 43 iterations
Total elapsed time: 0.656937 s
Time-averaged residual energy: 1.081729
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.070019
Updated value of 'alpha' for Group 1: 0.325342
Updated value of 'zeta' for Group 1: 0.29416
Updated value of 'sigma2' for Group 1: 0.347349
Updated value of 'sig2e': 0.022900
-----------------------------------------------------------
SP_MMV_FXN: Completed 44 iterations
Total elapsed time: 0.669641 s
Time-averaged residual energy: 1.063222
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.070472
Updated value of 'alpha' for Group 1: 0.321801
Updated value of 'zeta' for Group 1: 0.29416
Updated value of 'sigma2' for Group 1: 0.342798
Updated value of 'sig2e': 0.022485
-----------------------------------------------------------
SP_MMV_FXN: Completed 45 iterations
Total elapsed time: 0.682093 s
Time-averaged residual energy: 1.041249
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.070472
Updated value of 'alpha' for Group 1: 0.321801
Updated value of 'zeta' for Group 1: 0.29843
Updated value of 'sigma2' for Group 1: 0.343561
Updated value of 'sig2e': 0.022186
-----------------------------------------------------------
SP_MMV_FXN: Completed 46 iterations
Total elapsed time: 0.694626 s
Time-averaged residual energy: 1.026957
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.070844
Updated value of 'alpha' for Group 1: 0.318471
Updated value of 'zeta' for Group 1: 0.29843
Updated value of 'sigma2' for Group 1: 0.339290
Updated value of 'sig2e': 0.021913
-----------------------------------------------------------
SP_MMV_FXN: Completed 47 iterations
Total elapsed time: 0.707042 s
Time-averaged residual energy: 1.009478
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.070844
Updated value of 'alpha' for Group 1: 0.318471
Updated value of 'zeta' for Group 1: 0.30228
Updated value of 'sigma2' for Group 1: 0.340435
```

```
Updated value of 'sig2e': 0.021719
-----------------------------------------------------------
SP_MMV_FXN: Completed 48 iterations
Total elapsed time: 0.719712 s
Time-averaged residual energy: 0.999727
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.071131
Updated value of 'alpha' for Group 1: 0.315351
Updated value of 'zeta' for Group 1: 0.30228
Updated value of 'sigma2' for Group 1: 0.336436
Updated value of 'sig2e': 0.021525
-----------------------------------------------------------
SP_MMV_FXN: Completed 49 iterations
Total elapsed time: 0.732153 s
Time-averaged residual energy: 0.985724
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.071131
Updated value of 'alpha' for Group 1: 0.315351
Updated value of 'zeta' for Group 1: 0.30579
Updated value of 'sigma2' for Group 1: 0.337936
Updated value of 'sig2e': 0.021388
-----------------------------------------------------------
SP_MMV_FXN: Completed 50 iterations
Total elapsed time: 0.744673 s
Time-averaged residual energy: 0.978986
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.071353
Updated value of 'alpha' for Group 1: 0.312428
Updated value of 'zeta' for Group 1: 0.30579
Updated value of 'sigma2' for Group 1: 0.334185
Updated value of 'sig2e': 0.021239
-----------------------------------------------------------
SP_MMV_FXN: Completed 51 iterations
Total elapsed time: 0.757142 s
Time-averaged residual energy: 0.966861
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.071353
Updated value of 'alpha' for Group 1: 0.312428
Updated value of 'zeta' for Group 1: 0.30898
Updated value of 'sigma2' for Group 1: 0.335997
Updated value of 'sig2e': 0.021135
-----------------------------------------------------------
SP_MMV_FXN: Completed 52 iterations
Total elapsed time: 0.769800 s
Time-averaged residual energy: 0.961591
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.071536
Updated value of 'alpha' for Group 1: 0.309684
Updated value of 'zeta' for Group 1: 0.30898
Updated value of 'sigma2' for Group 1: 0.332471
Updated value of 'sig2e': 0.021016
-----------------------------------------------------------
SP_MMV_FXN: Completed 53 iterations
Total elapsed time: 0.782258 s
Time-averaged residual energy: 0.950075
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.071536
Updated value of 'alpha' for Group 1: 0.309684
Updated value of 'zeta' for Group 1: 0.31192
Updated value of 'sigma2' for Group 1: 0.334563
Updated value of 'sig2e': 0.020937
-----------------------------------------------------------
SP_MMV_FXN: Completed 54 iterations
Total elapsed time: 0.794781 s
```

```
Time-averaged residual energy: 0.945038
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.071697
Updated value of 'alpha' for Group 1: 0.307108
Updated value of 'zeta' for Group 1: 0.31192
Updated value of 'sigma2' for Group 1: 0.331242
Updated value of 'sig2e': 0.020841
-----------------------------------------------------------
SP_MMV_FXN: Completed 55 iterations
Total elapsed time: 0.807184 s
Time-averaged residual energy: 0.933030
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.071697
Updated value of 'alpha' for Group 1: 0.307108
Updated value of 'zeta' for Group 1: 0.3147
Updated value of 'sigma2' for Group 1: 0.333603
Updated value of 'sig2e': 0.020781
-----------------------------------------------------------
SP_MMV_FXN: Completed 56 iterations
Total elapsed time: 0.819892 s
Time-averaged residual energy: 0.926962
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.071853
Updated value of 'alpha' for Group 1: 0.304696
Updated value of 'zeta' for Group 1: 0.3147
Updated value of 'sigma2' for Group 1: 0.330482
Updated value of 'sig2e': 0.020705
-----------------------------------------------------------
SP_MMV_FXN: Completed 57 iterations
Total elapsed time: 0.832314 s
Time-averaged residual energy: 0.913060
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.071853
Updated value of 'alpha' for Group 1: 0.304696
Updated value of 'zeta' for Group 1: 0.31742
Updated value of 'sigma2' for Group 1: 0.333129
Updated value of 'sig2e': 0.020658
-----------------------------------------------------------
SP_MMV_FXN: Completed 58 iterations
Total elapsed time: 0.844843 s
Time-averaged residual energy: 0.904160
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.072012
Updated value of 'alpha' for Group 1: 0.302458
Updated value of 'zeta' for Group 1: 0.31742
Updated value of 'sigma2' for Group 1: 0.330217
Updated value of 'sig2e': 0.020590
-----------------------------------------------------------
SP_MMV_FXN: Completed 59 iterations
Total elapsed time: 0.857258 s
Time-averaged residual energy: 0.886368
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.072012
Updated value of 'alpha' for Group 1: 0.302458
Updated value of 'zeta' for Group 1: 0.32029
Updated value of 'sigma2' for Group 1: 0.333211
Updated value of 'sig2e': 0.020534
-----------------------------------------------------------
SP_MMV_FXN: Completed 60 iterations
Total elapsed time: 0.870000 s
Time-averaged residual energy: 0.873161
-----------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.072087
Updated value of 'alpha' for Group 1: 0.300418
```

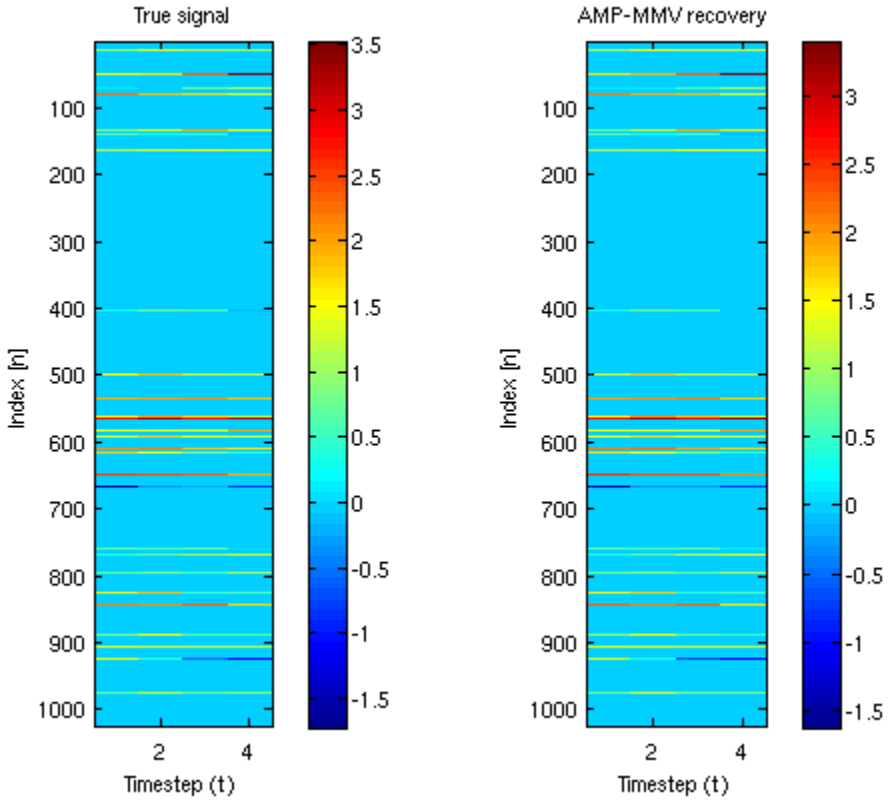
```
Updated value of 'zeta' for Group 1: 0.32029
Updated value of 'sigma2' for Group 1: 0.330540
Updated value of 'sig2e': 0.020422
------------------------------------------------------------
SP_MMV_FXN: Completed 61 iterations
Total elapsed time: 0.882450 s
Time-averaged residual energy: 0.854081
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.072087
Updated value of 'alpha' for Group 1: 0.300418
Updated value of 'zeta' for Group 1: 0.32351
Updated value of 'sigma2' for Group 1: 0.333979
Updated value of 'sig2e': 0.020224
------------------------------------------------------------
SP_MMV_FXN: Completed 62 iterations
Total elapsed time: 0.895507 s
Time-averaged residual energy: 0.838597
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.071465
Updated value of 'alpha' for Group 1: 0.298595
Updated value of 'zeta' for Group 1: 0.32351
Updated value of 'sigma2' for Group 1: 0.331574
Updated value of 'sig2e': 0.019783
------------------------------------------------------------
SP_MMV_FXN: Completed 63 iterations
Total elapsed time: 0.908817 s
Time-averaged residual energy: 0.794747
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.071465
Updated value of 'alpha' for Group 1: 0.298595
Updated value of 'zeta' for Group 1: 0.32773
Updated value of 'sigma2' for Group 1: 0.335717
Updated value of 'sig2e': 0.019162
------------------------------------------------------------
SP_MMV_FXN: Completed 64 iterations
Total elapsed time: 0.923262 s
Time-averaged residual energy: 0.729865
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.069404
Updated value of 'alpha' for Group 1: 0.297168
Updated value of 'zeta' for Group 1: 0.32773
Updated value of 'sigma2' for Group 1: 0.333813
Updated value of 'sig2e': 0.018059
------------------------------------------------------------
SP_MMV_FXN: Completed 65 iterations
Total elapsed time: 0.938013 s
Time-averaged residual energy: 0.645099
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.069404
Updated value of 'alpha' for Group 1: 0.297168
Updated value of 'zeta' for Group 1: 0.33478
Updated value of 'sigma2' for Group 1: 0.340463
Updated value of 'sig2e': 0.016139
------------------------------------------------------------
SP_MMV_FXN: Completed 66 iterations
Total elapsed time: 0.952775 s
Time-averaged residual energy: 0.515320
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.067385
Updated value of 'alpha' for Group 1: 0.296802
Updated value of 'zeta' for Group 1: 0.33478
Updated value of 'sigma2' for Group 1: 0.339964
Updated value of 'sig2e': 0.013314
------------------------------------------------------------
```

```
SP_MMV_FXN: Completed 67 iterations
Total elapsed time: 0.965315 s
Time-averaged residual energy: 0.442852
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.067385
Updated value of 'alpha' for Group 1: 0.296802
Updated value of 'zeta' for Group 1: 0.34855
Updated value of 'sigma2' for Group 1: 0.350472
Updated value of 'sig2e': 0.011519
------------------------------------------------------------
SP_MMV_FXN: Completed 68 iterations
Total elapsed time: 0.977814 s
Time-averaged residual energy: 0.396004
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.067883
Updated value of 'alpha' for Group 1: 0.297143
Updated value of 'zeta' for Group 1: 0.34855
Updated value of 'sigma2' for Group 1: 0.350949
Updated value of 'sig2e': 0.010267
------------------------------------------------------------
SP_MMV_FXN: Completed 69 iterations
Total elapsed time: 0.990193 s
Time-averaged residual energy: 0.357496
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.067883
Updated value of 'alpha' for Group 1: 0.297143
Updated value of 'zeta' for Group 1: 0.36459
Updated value of 'sigma2' for Group 1: 0.362317
Updated value of 'sig2e': 0.009395
------------------------------------------------------------
SP_MMV_FXN: Completed 70 iterations
Total elapsed time: 1.002706 s
Time-averaged residual energy: 0.331105
------------------------------------------------------------
Updated value of 'lambda' for Group 1: 0.068498
Updated value of 'alpha' for Group 1: 0.297507
Updated value of 'zeta' for Group 1: 0.36459
Updated value of 'sigma2' for Group 1: 0.362844
Updated value of 'sig2e': 0.008762
------------------------------------------------------------
sp_mmv_fxn: Max # of smoothing iterations reached
```

And let's compute our performance metrics on the recovery we obtained using this schedule, and plot the recovery:

```
s_hat = (lambda_hat > 1/2);
TNMSE = sum(sum(([x_true{:}]-[x_hat{:}]).^2, 1)./sum([x_true{:}].^2, 1))/T;
NSER = nser(support, find(s_hat == 1));
fprintf('TNMSE: %g dB\n', 10*log10(TNMSE));
fprintf('NSER: %g\n', NSER);
figure(1);
subplot(121); imagesc([x_true{:}]); colorbar
xlabel('Timestep (t)'); ylabel('Index [n]'); title('True signal')
subplot(122); imagesc([x_hat{:}]); colorbar
xlabel('Timestep (t)'); ylabel('Index [n]'); title('AMP-MMV recovery')
```

```
TNMSE: -24.5377 dB
NSER: 0.0140845
```



Now let's run the support-aware Kalman smoother so that we can see how well AMP-MMV performed compared to this genie-aided bound. The function that we will call is `GENIE_MULTI_FRAME_FXN`. In order to obtain a meaningful bound, we will provide the function with the true underlying model parameters. Let's construct this ModelParams object by using the constructor that simply copies over the information from our SigGenParams object, `SigGenObj`:

```
SKSParams = ModelParams(SigGenObj, sig2e);
SKSParams.print();
```

```
****************************************
        Signal Model Parameters
****************************************
Activity probability (lambda): 0.0625
           Active mean (zeta): 1
     Active variance (sigma2): 1
      Innovation rate (alpha): 0.1
        AWGN variance (sig2e): 0.0026064
```

We will also create a custom Options object that tells `GENIE_MULTI_FRAME_FXN` not to use the AMP algorithm to perform message passing within each frame, but rather use the slower (but theoretically grounded) exact belief propagation algorithm.

```
SKSRunOpt = Options('smooth_iters', 200, 'alg', 'BP');
SKSRunOpt.print();
```

```
**********************************
      AMP-MMV Runtime Options
**********************************
```

```
Max. smoothing iterations: 200
Min. smoothing iterations: 5
   Max. AMP/BP iterations: 15
    Intra-frame algorithm: Gaussian BP
    EM parameter learning: Yes
            Update groups: [Default]
                Verbosity: Silent
               Warm-Start: No
                  epsilon: 1e-06
  f-to-theta approx (tau): Taylor series approx
```

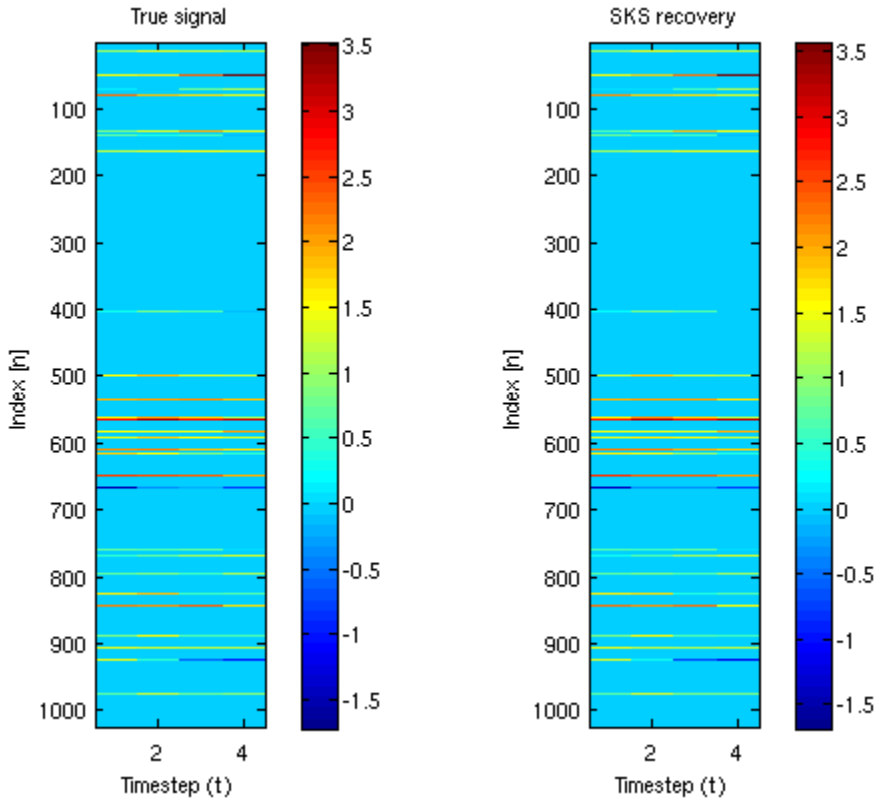
Now, recalling that `support` is the variable that contains the true signal support, let's run the SKS:

```
x_sks = genie_multi_frame_fxn(y, A, support, SKSParams, SKSRunOpt);
```

Now let's see how AMP-MMV stacked up against the SKS by calculating the TNMSE of the SKS recovery (note that the NSER for the SKS will always be zero)

```
TNMSE = sum(sum(([x_true{:}]-[x_sks{:}]).^2, 1)./sum([x_true{:}].^2, 1))/T;
fprintf('TNMSE: %g dB\n', 10*log10(TNMSE));
figure(1);
subplot(121); imagesc([x_true{:}]); colorbar
xlabel('Timestep (t)'); ylabel('Index [n]'); title('True signal')
subplot(122); imagesc([x_sks{:}]); colorbar
xlabel('Timestep (t)'); ylabel('Index [n]'); title('SKS recovery')
```

```
TNMSE: -25.1342 dB
```



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