

# Analysis of Modified SMI Method for Adaptive Array Weight Control

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**Abstract**—This paper characterizes the performance of the diagonally loaded sample matrix inverse (SMI) algorithm versus the number  $K$  of snapshots used in the covariance matrix estimate by providing  $O(1/K)$  statistics (bias and variance) of the array weights, output powers, and output power ratios such as SINR and INR. The approach accommodates wide-band signals. Monte Carlo simulations verify the theoretical analysis.

## I. INTRODUCTION

ADAPTIVE arrays are used in a wide variety of signal processing applications to improve communication between a source and receiver by exploiting special knowledge of the time, frequency, or spacial characteristics of the transmitted signal. In an adaptive array, the received signal is a sum of the weighted outputs of a number of sensors.

It is well known that the set of array weights  $\mathbf{W}_{\text{LMS}}$  that minimizes the mean-square error between the output of an  $N$ -element array and a desired (reference) signal is given by  $\mathbf{W}_{\text{LMS}} = \Phi^{-1} \mathbf{S}$  where  $\Phi$  is the array covariance matrix and  $\mathbf{S}$  is the steering vector. In addition, for a CW<sup>1</sup> desired signal, the least mean square (LMS) weights maximize the signal-to-interference-plus-noise ratio (SINR) at the array output [1]. The sample matrix inverse (SMI) procedure [1] is a commonly used technique for setting the array weights. The weight estimate is found as  $\hat{\mathbf{W}}_{\text{LMS}} = \hat{\Phi}^{-1} \hat{\mathbf{S}}$  where  $\hat{\Phi}$  and  $\hat{\mathbf{S}}$  are sample estimates of the true covariance matrix and steering vector.

The performance characteristics of the SMI array can be varied by adding or subtracting a positive real number from the covariance matrix diagonal; we denote this as positive and negative diagonal loading, respectively. Positive diagonal loading provides faster convergence and decreased sensitivity to noise and clutter [2]–[4].

Negative diagonal loading increases the suppression of weak interference signals [5] at the expense of longer convergence times [6], [7]. Negative diagonal loading is applicable to slowly varying environments in which interference rejection to below the noise floor is needed. An example of this is geosynchronous satellite transmission of television signals [8]. In this application an interfer-

ence signal may be another television signal being transmitted by an adjacent satellite. The interference signal must be attenuated to  $\approx 10$  dB below the noise floor before the interference patterns on the television picture become undiscernable to the viewer [9]. This can be accomplished by using negative diagonal loading as we will show in the simulations.

A natural question that has been addressed in the literature asks how many snapshots are needed for “good” performance of the array. Reed *et al.* [1] characterized the performance of the standard SMI array for the case of no desired signal and zero-mean narrow-band Gaussian interference and noise. Miller [10], [11] and Monzingo and Miller [12] expanded this work to include a desired signal. Cases where, in addition, the steering vector is approximate or is estimated from the data have been addressed by Miller [10], [11], Monzingo and Miller [12], and Boroson [13].

This paper characterizes the performance of the modified SMI array (and the standard SMI array as a special case) based on  $K$ -snapshot estimates of the diagonally loaded covariance matrix. We give  $O(1/K)$  expressions for the bias and variance of the weight estimates, output powers, and output power ratios such as INR and SINR. These expressions are valid for wide-band signals. We compare the statistical expressions with Monte Carlo simulation studies. This paper extends the results of Ganz *et al.* [14] in which  $O(1/K)$  expressions for the bias and variance of the weights and the bias of the powers only were derived. Dilsavor and Gupta [8] have compared these expressions with the performance of an experimental array system.

## II. DIAGONALLY LOADED SMI

Let  $\mathbf{X}(k)$  be the vector of signals received at the  $N$  elements of an antenna array at time  $t_k$ :

$$\mathbf{X}(k) = \mathbf{X}_D(k) + \sum_{m=1}^M \mathbf{X}_{I_m}(k) + \mathbf{X}_w(k) \quad (1)$$

where the  $N \times 1$  vectors  $\mathbf{X}_D(k)$ ,  $\mathbf{X}_{I_m}(k)$ , and  $\mathbf{X}_w(k)$  are the desired,  $m$ th interference, and noise components of the received signal, respectively. We assume that  $\mathbf{X}_w(k)$  is a vector of zero mean, i.i.d. Gaussian random variables with variance  $\sigma^2$ . The array output signal at time  $t_k$  is given by  $s(k) = \mathbf{W}^H(k) \mathbf{X}(k)$  where  $\mathbf{W}(k)$  is the vector of complex

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<sup>1</sup>CW or “continuous wave” is synonymous with narrow band.

weights applied at time  $t_k$  and  $^H$  denotes Hermitian transpose. From (1) we can write  $s(k)$  in terms of its components as

$$s(k) = s_D(k) + \sum_{m=1}^M s_{Im}(k) + s_w(k).$$

The expected power  $P$  of the array output signal  $s(k)$  (assuming fixed weights) is

$$P = E[|s(k)|^2] = E[\mathbf{W}^H \mathbf{X}(k) \mathbf{X}(k)^H \mathbf{W}] = \mathbf{W}^H \mathbf{\Phi} \mathbf{W} \quad (2)$$

where

$$\mathbf{\Phi} \triangleq E[\mathbf{X}\mathbf{X}^H] \quad (3)$$

is the  $(N \times N)$  covariance matrix of the received signals and time dependence has been omitted for notational simplicity. Using (1) in (3) and assuming that the received signal components  $X_J$  (for  $J = D, I_m$ , or  $w$ ) are uncorrelated and zero mean, we can decompose the data covariance matrix as

$$\begin{aligned} \mathbf{\Phi} &= E[X_D X_D^H] + \left( \sum_{m=1}^M E[X_{Im} X_{Im}^H] \right) + E[X_w X_w^H] \\ &= \mathbf{\Phi}_D + \mathbf{\Phi}_I + \mathbf{\Phi}_w \quad \text{where } \mathbf{\Phi}_w = \sigma^2 \mathbf{I}. \end{aligned} \quad (4)$$

From (4) and (2), the output power  $P$  can be expressed as  $P = P_D + P_I + P_w$  where  $P_J = \mathbf{W}^H \mathbf{\Phi}_J \mathbf{W}$  for  $J = D, I$ , or  $w$ . The output signal-to-interference-plus-noise ratio (SINR) is

$$\text{SINR} \triangleq \frac{P_D}{P_I + P_w} = \frac{P_D}{P_I + \sigma^2 \mathbf{W}^H \mathbf{W}} \quad (5)$$

It is well known that for a CW desired signal, the SINR is maximized when the weight vector is a scalar multiple of

$$\mathbf{W}_{\text{LMS}} = \mathbf{\Phi}^{-1} \mathbf{S} = [\mathbf{\Phi}_D + \mathbf{\Phi}_I + \sigma^2 \mathbf{I}]^{-1} \mathbf{S} \quad (6)$$

where  $\mathbf{S}$  is the steering vector or correlation between the received signals and the desired signal.

The LMS weight vector can be modified by introducing diagonal loading into the covariance matrix, i.e., by replacing  $\mathbf{\Phi}$  with  $\mathbf{\Gamma} \triangleq \mathbf{\Phi} - F\sigma^2 \mathbf{I}$  for some loading factor  $F < 1$ . We have restricted  $F < 1$  to ensure that  $\mathbf{\Gamma}$  is positive definite and thus full rank. In this case, the modified weights are given by

$$\mathbf{W} = [\mathbf{\Gamma}]^{-1} \mathbf{S} = [\mathbf{\Phi}_D + \mathbf{\Phi}_I + (1 - F)\sigma^2 \mathbf{I}]^{-1} \mathbf{S} \quad (7)$$

When the desired signal is CW, the modified weights maximize a modified SINR (MSINR) given by

$$\text{MSINR} = \frac{P_D}{P_I + (1 - F)P_w} = \frac{P_D}{P_I + (1 - F)\sigma^2 \mathbf{W}^H \mathbf{W}} \quad (8)$$

Note that the modified SMI weights (7) reduce to the standard SMI weights (6) when  $F = 0$ ; When  $0 \leq F < 1$ , the covariance matrix undergoes negative diagonal

loading and this is the focus of the paper. However, the theory developed here holds for the positive diagonal loading ( $F < 0$ ) case as well.

### III. STATISTICS OF MODIFIED SMI WITH SAMPLE COVARIANCE

This section presents a theoretical analysis of the statistical properties of the modified SMI algorithm. We develop asymptotic expressions for the bias and variance of the estimated weights, of the array output powers, and of output power ratios such as INR and SINR. The expressions are functions of the number of snapshots  $K$ , the amount of diagonal loading  $F$ , and the signal scenario. Much of this analysis is independent of signal model and thus is applicable to wide-band signals. Only at the end of the analysis do we specialize to a CW scenario consisting of one desired signal and  $M$  interference signals arriving from arbitrary directions at an equispaced linear array of elements of arbitrary gain.

#### A. Weight Estimates

In an adaptive array system, the true covariance matrix is unknown. In practice, it is estimated by averaging the outer products of  $K$  snapshots  $\mathbf{X}(k) \triangleq \mathbf{X}(t = t_k)$ :

$$\hat{\mathbf{\Phi}}_K = \frac{1}{K} \sum_{k=1}^K \mathbf{X}(k) \mathbf{X}(k)^H. \quad (9)$$

The  $K$ -snapshot-based estimate  $\hat{\mathbf{W}}_K$  of the modified SMI weight vector  $\mathbf{W}$  is given by

$$\hat{\mathbf{W}}_K = [\hat{\mathbf{\Phi}}_K - F\sigma^2 \mathbf{I}]^{-1} \mathbf{S} = [\hat{\mathbf{\Gamma}}_K]^{-1} \mathbf{S} \quad (10)$$

where it is assumed that a good estimate of the noise power  $\sigma^2$  is available. The noise power may be estimated from the noise eigenvalues of the sample covariance matrix. For the signal scenario of Section IV we found that the minimum eigenvalue of the sample covariance matrix was within 2.5% of the noise power  $\sigma^2 = 1$  after 10 thousand snapshots.

We define the weight error  $\tilde{\mathbf{W}}_K$  by

$$\tilde{\mathbf{W}}_K \triangleq \mathbf{W} - \hat{\mathbf{W}}_K = [\mathbf{\Gamma}^{-1} - \hat{\mathbf{\Gamma}}_K^{-1}] \mathbf{S} \quad (11)$$

and the modified covariance error by

$$\tilde{\mathbf{\Gamma}}_K \triangleq \mathbf{\Gamma} - \hat{\mathbf{\Gamma}}_K = \mathbf{\Phi} - \hat{\mathbf{\Phi}}_K = \tilde{\mathbf{\Phi}}_K. \quad (12)$$

Note that the error in the modified and unmodified covariance is the same. The expected value and variance of the  $K$ -snapshot-based weights can be expressed as

$$E[\hat{\mathbf{W}}] = \mathbf{W} - E[\tilde{\mathbf{W}}] \quad (13)$$

$$\text{var}[\hat{\mathbf{W}}] = \text{var}[\tilde{\mathbf{W}}] = E[\tilde{\mathbf{W}} \tilde{\mathbf{W}}^H] - E[\tilde{\mathbf{W}}] E[\tilde{\mathbf{W}}]^H \quad (14)$$

where the time subscripts have been omitted for convenience. The statistics of  $\tilde{\mathbf{W}}$  needed above can be expressed in terms of the covariance matrix error as follows. First, we manipulate  $\hat{\mathbf{\Gamma}}^{-1}$  using (12)

$$\begin{aligned} \hat{\mathbf{\Gamma}}^{-1} &= [\mathbf{\Gamma} - \tilde{\mathbf{\Gamma}}]^{-1} = [(\mathbf{I} - \tilde{\mathbf{\Gamma}} \mathbf{\Gamma}^{-1}) \mathbf{\Gamma}]^{-1} \\ &= \mathbf{\Gamma}^{-1} (\mathbf{I} - \tilde{\mathbf{\Gamma}} \mathbf{\Gamma}^{-1})^{-1}. \end{aligned} \quad (15)$$

Now expanding  $[I - \tilde{\Gamma}\Gamma^{-1}]^{-1}$  as a power series [15] and using (11) gives

$$\hat{W} = \Gamma^{-1} [I - \{I + \tilde{\Gamma}\Gamma^{-1} + (\tilde{\Gamma}\Gamma^{-1})^2 + \dots\}] S. \quad (16)$$

The power series expansion is valid if  $\|\tilde{\Gamma}\Gamma^{-1}\| < 1$  where  $\|\cdot\|$  is any norm satisfying the submultiplicative property  $\|AB\| \leq \|A\|\|B\|$ . From (16) we compute the expectations

$$E[\hat{W}] \approx -\Gamma^{-1} E[\tilde{\Gamma}\Gamma^{-1}\tilde{\Gamma}] W, \quad (17)$$

$$E[\hat{W}\hat{W}^H] \approx \Gamma^{-1} E[(\tilde{\Gamma}W)(\tilde{\Gamma}W)^H](\Gamma^{-1})^H \quad (18)$$

$$E[\hat{W}\hat{W}^T] \approx \Gamma^{-1} E[(\tilde{\Gamma}W)(\tilde{\Gamma}W)^T](\Gamma^{-1})^T \quad (19)$$

where  $E[\tilde{\Gamma}] = \mathbf{0}$  and  $W = \Gamma^{-1}S$  have been used. The approximations are the result of neglecting  $(\tilde{\Gamma}\Gamma^{-1})^i$  for  $i > 2$  in (16). Using (12), one may express (17)–(19) in terms of  $E[\Phi_{il}\Phi_{sl}^*]$ , the expected value of the product of the  $i$ th and  $s$ th elements of the  $K$ -snapshot-based covariance error matrix.

### B. Output Signal Power Statistics

The output signal powers of an array given a set of  $K$ -snapshot-based weights are

$$\hat{P}_J \triangleq E[|\hat{s}_J(k)|^2] = \hat{W}^H E[X_J X_J^H] \hat{W} = \hat{W}^H \Phi_J \hat{W} \quad (20)$$

where  $J = D, Im$ , or  $w$ , for desired,  $m$ th interference, or noise power, respectively. Now substitute  $\hat{W} \triangleq W - \tilde{W}$  in (20) to express the expected value and variance of the output signal powers as

$$\begin{aligned} E[\hat{P}_J] &= E[(W - \tilde{W})^H \Phi_J (W - \tilde{W})] \\ &= P_J - 2 \operatorname{Re} \{W^H \Phi_J E[\tilde{W}]\} \\ &\quad + E[\tilde{W}^H \Phi_J \tilde{W}] \end{aligned} \quad (21)$$

$$\begin{aligned} \operatorname{var} [\hat{P}_J] &\triangleq E[(\hat{P}_J - E[\hat{P}_J])^2] \\ &\approx E[(2 \operatorname{Re} \{W^H \Phi_J \tilde{W}\})^2] \quad \text{for large } K \end{aligned} \quad (22)$$

where  $\operatorname{Re}$  denotes real part and  $J$  is as before. The approximation (22) is  $O(1/K)$  as we will show.

### C. Output Power Ratio Statistics

The next step is to develop expressions for the expected value and variance of the output power ratios  $\widehat{\text{SINR}}$  and  $\widehat{\text{INR}}$ . To do so, define the generalized power ratio

$$R(X, Y, \alpha, Z) \triangleq \frac{\hat{P}_X}{\hat{P}_Y + \alpha \hat{P}_Z} = \frac{\hat{P}_X}{\hat{P}_B} = \frac{\hat{W}^H \Phi_X \hat{W}}{\hat{W}^H \Phi_B \hat{W}} \quad (23)$$

where  $\hat{P}_B \triangleq \hat{P}_Y + \alpha \hat{P}_Z$  and  $\Phi_B \triangleq \Phi_Y + \alpha \Phi_Z$ . Note that  $\widehat{\text{SINR}} = R(D, I, 1, w)$  and  $\widehat{\text{INR}} = R(I, w, 0, Z)$  where, for example,  $\widehat{\text{SINR}}$  represents the output SINR of an array whose weights are estimated using  $K$  snapshots.

Exact expressions for the probability density function, expected value, and variance of ratios similar to (23) have been found by Reed *et al.* [1] and Miller [10] under certain *a priori* assumptions as discussed in the introduction.

These exact expressions assume standard SMI ( $F = 0$ ) and a narrow-band desired signal. In this paper, we obtain asymptotic expressions for a more general case by using the Taylor expansion of  $R$ .

The second-order Taylor series approximation of  $R(\hat{W})$  about the optimal weight vector  $W$  is given by

$$R(\hat{W}) \approx R(W) + (\nabla R|_{z=Z}) \tilde{Z} + \frac{1}{2} \tilde{Z}^T (\nabla^2 R|_{z=Z}) \tilde{Z} \quad (24)$$

where  $\nabla = \partial/\partial z^T$  is the gradient operator,  $\nabla^2 = \partial^2/\partial z \partial z^T$  is the Hessian operator,  $Z \triangleq [\operatorname{Re}\{W\}^T \operatorname{Im}\{W\}^T]^T$ , and  $\tilde{Z} = \hat{Z} - Z$ . By substituting  $P_X/P_B$  for  $R$  we obtain

$$\nabla R = \frac{1}{P_B} (\nabla P_X - R \nabla P_B) \quad (25)$$

$$\begin{aligned} \nabla^2 R &= -\nabla R^T \left( \frac{\nabla P_B}{P_B} \right) + \frac{1}{P_B} (\nabla^2 P_X \\ &\quad - \nabla P_B^T \nabla R - R \nabla^2 P_B). \end{aligned} \quad (26)$$

By expanding (20) in terms of real and imaginary parts and performing some straightforward calculations, we obtain

$$\begin{aligned} R(\hat{W}) &\approx R(W) - \frac{1}{P_B^2} \left[ 2 \operatorname{Re} \{W^H \Phi_Z \tilde{W}\} - \frac{1}{2} \tilde{W}^H \Phi_Z \tilde{W} \right] \\ &\quad - \frac{2}{P_B^3} \operatorname{Re} \{W^H \Phi_B \tilde{W}\} \operatorname{Re} \{\tilde{W}^H \Phi_Z W\} \end{aligned} \quad (27)$$

where  $\Phi_Z \triangleq P_B \Phi_X - P_X \Phi_B$ .

The statistics of  $R(\hat{W})$  follow from the Taylor approximation (27):

$$\begin{aligned} E[R(\hat{W})] &\approx R(W) - \frac{1}{P_B^2} \left( 2 \operatorname{Re} \{W^H \Phi_Z E[\tilde{W}]\} \right. \\ &\quad \left. - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \Phi_{Zij} E[\tilde{W}_i^* \tilde{W}_j] \right) \\ &\quad - \frac{1}{P_B^3} \operatorname{Re} \{W^H \Phi_B E[\tilde{W} \tilde{W}^H] \Phi_Z W \\ &\quad + W^H \Phi_B E[\tilde{W} \tilde{W}^T] \Phi_Z^* W^*\} \end{aligned} \quad (28)$$

$$\begin{aligned} \operatorname{var} [R(\hat{W})] &\approx E \left[ \left\{ \left( \nabla R|_{z=Z} \right) \tilde{Z} \right. \right. \\ &\quad \left. \left. + \frac{1}{2} \tilde{Z}^T \left( \nabla^2 R|_{z=Z} \right) \tilde{Z} \right\}^2 \right] \\ &\quad - \left\{ E \left[ \left( \nabla R|_{z=Z} \right) \tilde{Z} \right. \right. \\ &\quad \left. \left. + \frac{1}{2} \tilde{Z}^T \left( \nabla^2 R|_{z=Z} \right) \tilde{Z} \right] \right\}^2 \end{aligned} \quad (29)$$

$$\approx E[\{(\nabla R|_{z=Z}) \tilde{Z}\}^2] \quad \text{for large } K \quad (30)$$

$$= \frac{1}{P_B^4} E[(2 \operatorname{Re} \{W^H \Phi_Z \tilde{W}\})^2]. \quad (31)$$

The approximations in (28) and (29) are obtained by neglecting  $O(K^{-s})$  terms for  $s \geq 3/2$ , thus the approximations are valid for large  $K$ .

#### D. Covariance Statistics for a Particular Scenario

The array performance measures have been expressed in terms of the weight error statistics which, in turn, depend on  $E[\tilde{\Phi}_{il}\tilde{\Phi}_{st}^*]$ . The derivations were independent of signal scenario. Only now, for the calculation of  $E[\tilde{\Phi}_{il}\tilde{\Phi}_{st}^*]$ , must a signal scenario be assumed.

Consider a narrow-band scenario consisting of one desired signal, and  $M$  interference signals arriving from arbitrary directions at an  $N$ -element linear array with element spacing  $\Delta$  and with complex Gaussian noise,  $N(0, \sigma^2)$ , at each element. The complex envelope of the desired and interference signal components in the  $i$ th antenna element are

$$x_{\alpha i}(k) = A_{\alpha i} \exp \{j[-(i-1)\phi_{\alpha} + \Theta_{\alpha}(k)]\} \quad (32)$$

where  $A_{\alpha i}$  is the amplitude of signal  $\alpha$  in the  $i$ th element,  $\alpha = 0$  corresponds to the desired signal, and  $\alpha \in [1, M]$  corresponds to the interference signals.  $\phi_{\alpha} = (2\pi/\lambda)\Delta \sin \theta_{\alpha}$  is the interelement phase shift of signal  $\alpha$ , and  $\Theta_{\alpha}(k)$  are uniformly distributed  $U(0, 2\pi)$  random phases of signal  $\alpha$ . Under this scenario the desired expectation is given by [14]

$$E[\tilde{\Phi}_{il}\tilde{\Phi}_{st}^*] = \frac{1}{K} \left[ \sigma^4 \delta_{il} \delta_{is} + \sum_{\alpha=0}^M \sum_{\substack{\beta=0 \\ \alpha \neq \beta}}^M A_{\alpha i} A_{\alpha s} \cdot e^{-j(i-s)\phi_{\alpha}} A_{\beta l} A_{\beta t} e^{-j(t-l)\phi_{\beta}} + \sum_{\alpha=0}^M \sigma^2 \{ \delta_{il} A_{\alpha i} A_{\alpha s} e^{-j(i-s)\phi_{\alpha}} + \delta_{is} A_{\alpha i} A_{\alpha t} e^{-j(t-l)\phi_{\alpha}} \} \right] \quad (33)$$

where  $\delta_{il}$  is the Kronecker delta.

#### IV. SIMULATION COMPARISONS

In this section, Monte Carlo simulations of a modified SMI array are compared with the theoretical expressions for bias and variance derived above. In the simulations, a snapshot is formed by sampling the array element signals whose components are given by (32) plus i.i.d. complex Gaussian noise,  $N(0, \sigma^2)$ , at each element. A new random phase  $\Theta_{\alpha}(k)$  is generated for each signal component in each snapshot, thus the snapshots are uncorrelated. The scenario consists of a linear array with a high-gain main element and four auxiliary elements with half-wavelength spacing. A desired signal is incident from broadside while an interference signal arrives  $30^\circ$  from broadside. The SNR of the desired signal is 14.6 dB in the main antenna while it is  $-10$  dB in the auxiliaries. The INR is  $-5$  dB in the main antenna and is  $-3$  dB in the auxiliaries. This is a typical example of weak interference since the interference power is about 20 dB beneath the desired signal

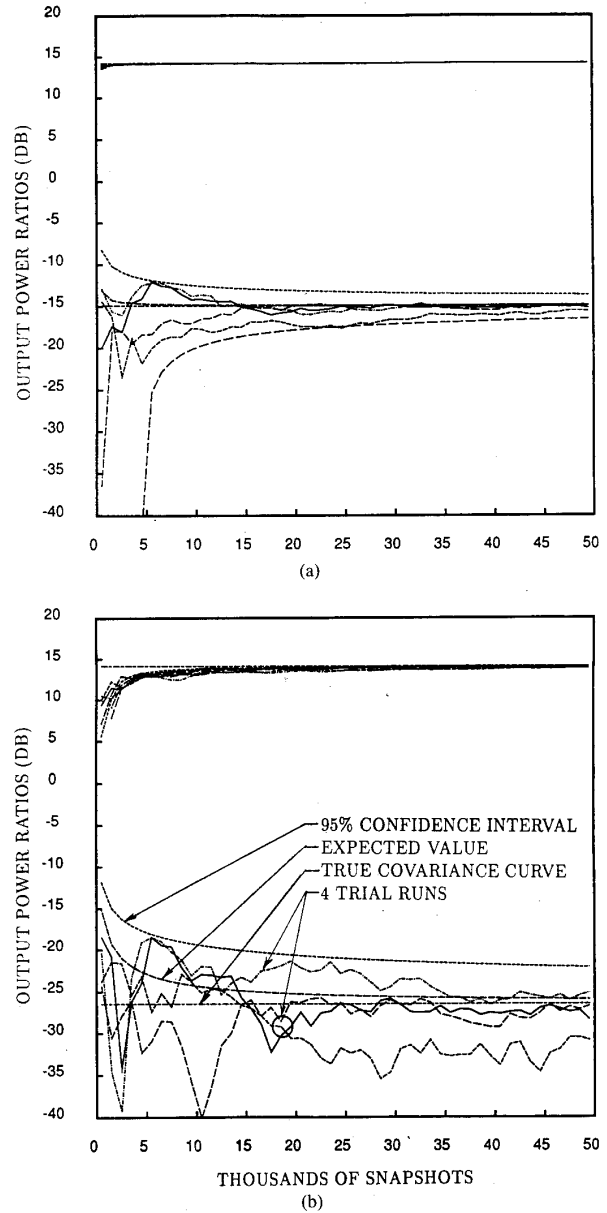


Fig. 1. Output INR and SINR versus number of snapshots  $K$  for (a)  $F = 0$  and (b)  $F = 0.8$ .

and is 5 dB below the noise in the main element. This scenario was considered by Gupta [5].

Fig. 1 shows the output INR and SINR for  $F = 0$  (standard SMI) and  $F = 0.8$ . Four trial runs were made for each value of  $F$ . The curves at the top of the two graphs (above 0 dB) are the SINR curves while those at the bottom (below 0 dB) are the INR curves. For both INR and SINR the figures show four trial runs which appear as jagged lines, a straight horizontal line for the value of the performance measure assuming that the true covariance is known, the expected value of the estimator which is a smooth curve that lies among the trial runs and asymptot-

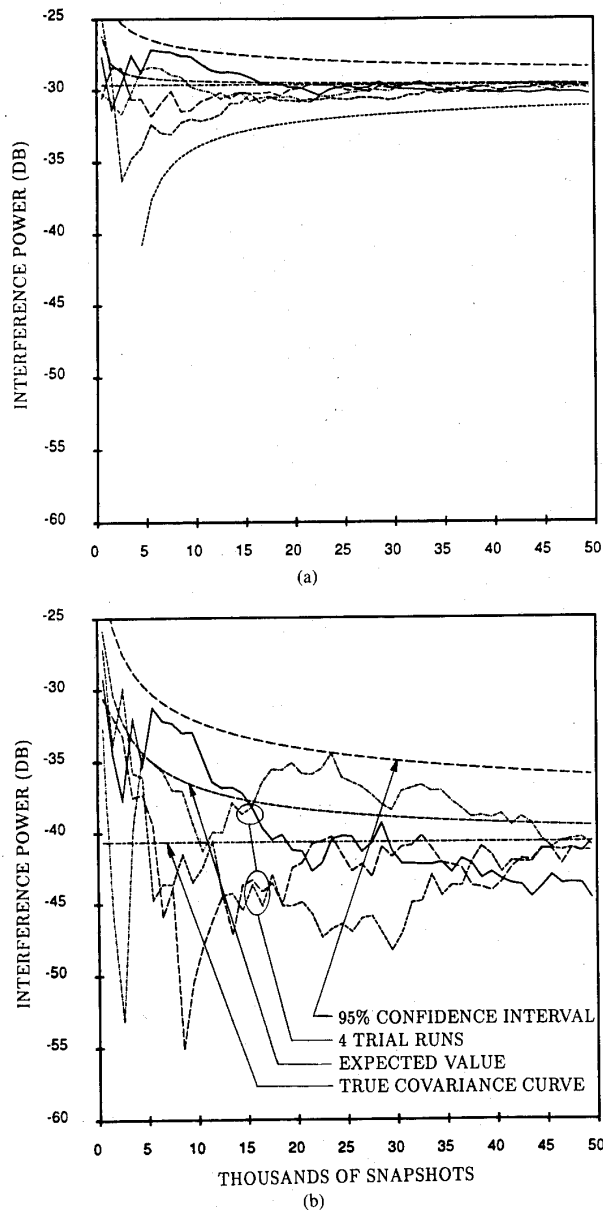


Fig. 2. Output interference signal power  $P_I$  versus number of snapshots  $K$  for (a)  $F = 0$  and (b)  $F = 0.8$ .

ically approaches the true covariance value, and two smooth curves (one above and one below the expected value curve) that represent a 95% confidence interval ( $\pm 2$  standard deviations) for the estimator. For a fully labeled graph see Fig. 1(b) or 2(b).

The plots in Fig. 1 show good agreement between theory and simulation. Note the minor degradation in SINR as  $F$  is increased and the dramatic 12 dB improvement in steady state INR. We also see that the increased interference suppression comes at the price of having to increase the number of snapshots in the covariance estimate to achieve that suppression. For example, comparing the

graphs in Fig. 1 we see that setting  $F = 0.8$  asymptotically increases interference suppression by about 12 dB relative to standard SMI but only 8 dB of that suppression is achieved after 10 000 snapshots. In a nearly stationary environment such as geosynchronous satellite communications such large values of  $K$  are practical. In fact, an experimental system has been built and tested in which 10 000 snapshots are taken in about 40 s. [8]. The geosynchronous satellite scenario is stationary for times on the order of an hour.

The dramatic difference in INR for  $F = 0$  versus  $F = 0.8$  is almost entirely due to an 11 dB drop in interference power. Fig. 2 shows the interference power  $P_I$  for  $F = 0$  and  $F = 0.8$ . The theoretical curves and the trial runs show good agreement. The explanation for the large variance in the interference power is intuitive from an array pattern viewpoint. Since the modified SMI algorithm is designed to maximize MSINR, it will form a pattern null in the interference signal direction. As a result, the gain of the pattern in the interference direction will be very sensitive. A small variance in the desired signal power would be seen since the desired signal arrives near a pattern maximum.

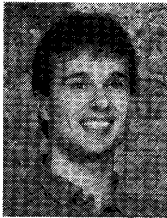
## V. CONCLUSIONS

The performance of the diagonally loaded SMI array has been described by deriving  $O(1/K)$  approximations of the statistics of the array weights, output powers, and output power ratios as a function of the amount of diagonal loading and the number of snapshots  $K$  used in the covariance matrix estimate. Much of the analysis is independent of signal scenario and as a result is applicable to wide-band scenarios including a wide-band desired signal. A narrow-band signal environment was used in Monte Carlo simulations to show that the statistical expressions correctly predict the increase in bias and variance of the diagonally loaded SMI array performance as the amount of negative diagonal loading is increased.

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