Electronics Letters Online No: 19940994
13 June 1994

F. Adachi (*R&D Department, NTT Mobile Communications Network, Inc., 1-2356 Take, Yokosuka-shi, Kanagawa-ken, 238 Japan*)

References

1 KAM, P.K.: 'Maximum-likelihood digital data sequence estimation over the Gaussian channel with unknown carrier phase', *IEEE Trans.*, 1987, **COM-35**, pp. 764–767

2 KAM, P.K., and HO, H.C.: 'Viterbi detection with simultaneous suboptimal maximum likelihood carrier phase estimation', *IEEE Trans.*, 1988, **COM-36**, pp. 1327–1330

3 KAM, P.K., SINHA, P., and KAN, A.M.C.: 'Adaptive digital coherent receiver for MPSK', *Electron. Lett.*, 1992, **28**, pp. 2099–2101

4 KAM, P.K., and TAY, S.Y.: 'Coherent detection of MPSK via efficient block estimation', *Electron. Lett.*, 1994, **30**, pp. 184–185

5 D'ANDREA, A.N., MENGALI, U., and VITETTA, G.M.: 'Approximate ML decoding of coded PSK with no explicit carrier phase reference', *IEEE Trans.*, 1994, **COM-42**, pp. 1033–1039

Bispectra of modulated stationary signals

I.I. Jouny and R.L. Moses

Indexing terms: Random processes, Spectral analysis, Statistics for communications

Complex modulation affects the bispectral signatures of a random process and complicates the interpretation of the bispectrum. The impact of complex modulation on bispectral signature analysis and on the detection performance of multitone signals is examined.

Introduction: Bispectral analysis has recently been a topic of extensive research. The applications of the bispectrum include detection of quadratic phase coupling, signal reconstruction, ARMA modelling of non-Gaussian signals, detection of signals in noise, and tests for deviation from normality and nonlinearity [1, 3, 4]. These applications are based on the assumption that the signal under consideration is third order stationary and cycloergodic.

The bispectrum provides valuable information about coupling between the spectral components of third order stationary random processes. This feature has been used in radar signature analysis [5, 6] for identifying scattering mechanisms, and target identification. However, radar signals are susceptible to modulation, a feature that has complicated the interpretation of the bispectral signatures of radar targets and affected the process of target identification [5, 6]. The purpose of this Letter is to alert the readers who are employing bispectral analysis in practical applications to the effects of complex modulation on the identification of phase coupling and the detection of multitone signals.

Detection of phase coupling: The third order cumulant of a time series $\{x(k)\}$ is defined as [1, 2]

$$R^x(k;m,n) = E\{x^*(k)x(k+m)x(k+n)\} \qquad (1)$$

A random signal is 'third-order stationary' when [1]

$$R^x(k;m,n) = R^x(k+j;m,n) \ \forall \ j,m,n$$

If the signal is third order stationary we drop the first argument of the third order cumulant as follows:

$$R^x(m,n) = E\{x^*(k)x(k+m)x(k+n)\} \qquad (2)$$

If $\{x(k)\}$ is third order stationary then its bispectrum is defined as [1]

$$B^x(\omega_1,\omega_2) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} R^x(m,n)e^{-j(\omega_1 m+\omega_2 n)} \qquad (3)$$

Notice that the third order cumulant preserves phase information. Therefore, modulating a third order stationary signal $\{x(k)\}$ by $e^{\pm j\omega_0 k}$ results in a time dependent third order cumulant. Consequently, whereas the bispectrum of $\{x(k)\}$ is defined as given in eqn. 3, the bispectrum of $\{x(k)e^{\pm j\omega_0 k}\}$ may not be defined. Also, using the same argument, if the bispectrum of a random process $\{x(k)\}$ is not defined then due to modulation the bispectrum of $\{x(k)e^{\pm j\omega_0 k}\}$ may be well defined.

The effect of complex modulation can be further explained by considering another definition of the bispectrum. Assuming that $\{x(k)\} \overset{F}{\leftrightarrow} \{X(\omega)\}$ (where $\leftrightarrow$ denotes a Fourier transform pair) the bispectrum is defined as [1]

$$B^x(\omega_1,\omega_2) = E\{X(\omega_1)X(\omega_2)X^*(\omega_1+\omega_2)\} \qquad (4)$$

This definition of the bispectrum is a consequence of stationarity and it indicates that a nonzero bispectral response exists at the frequency pair (ω_a, ω_b) if the random frequency responses $X(\omega_a)$, $X(\omega_b)$, and $X^*(\omega_a + \omega_b)$ are correlated. It is important to note that the third frequency is the sum of the first two. If a peak is detected in the bispectrum at (ω_a, ω_b), modulating the signal $\{x(k)\}$ by $e^{(\pm j\omega_0 k)}$ will in general eliminate the bispectral peak at (ω_a, ω_b). The reason is that modulating the signal translates the spectral peak at $\omega_a + \omega_b$ to $\omega_a + \omega_b + \omega_0$ and not to $\omega_a + \omega_b + 2\omega_0$. Therefore, the detection of bicorrelations using the bispectrum is dependent on whether the signal is modulated, and on the modulating frequency as well. The above observation suggests that bispectral analysis may be used to detect bicorrelation between three spectral components that are not necessarily harmonically related.

Example: Assume that the time series $\{x(k)\}$ is defined as

$$x(k) = A_1e^{-j(\lambda_1 k+\phi_1)} + A_2e^{-j(\lambda_2 k+\phi_2)} + A_3e^{-j(\lambda_3 k+\phi_1+\phi_2)} \qquad (5)$$

where ϕ_1, ϕ_2 are independent and uniformly distributed over $[0, 2\pi]$. It can be shown that $\{x(k)\}$ is wide-sense (second order) stationary. Also, the third order cumulant of $\{x(k)\}$ is

$$R^x(k;m,n) = \frac{A_1A_2A_3}{4}\left[e^{-j((\lambda_3-\lambda_2-\lambda_1)k-\lambda_2 n-\lambda_1 m)} + e^{-j((\lambda_3-\lambda_1-\lambda_2)k-\lambda_2 m-\lambda_1 n)}\right] \qquad (6)$$

The bispectrum of $\{x(k)\}$, however, is not defined because $\lambda_3 \neq \lambda_1 + \lambda_2$. Modulating the signal $x(k)$ by $e^{(j\lambda_0 k)}$ where $\lambda_0 = \lambda_3 - \lambda_1 - \lambda_2$ introduces a bispectral response at $(\lambda_1 + \lambda_0, \lambda_2 + \lambda_0)$.

In the previous example and throughout this Letter we assume that the phase angles are coupled such that $\phi_1 + \phi_2 = \phi_3$. The case where $N\phi_1 + M\phi_2 = P\phi_3$ where M, N, and P are integers is not addressed in this Letter.

Detection of multitone signals: A detection scheme based on the bispectrum of multitone signals with random phase and implicit phase couplings is proposed in [3]. The detector uses the bispectrum to extract phase coupling information and thus improve the detection performance of multitone signals over conventional energy detectors. The improvement in detection performance, however, is on the basis that phase coupling occurs at frequencies ω_a, ω_b, and ω_c where $\omega_c = \omega_a + \omega_b$. A more general multitone signal detection scheme that recovers phase coupling even when $\omega_c \neq \omega_a + \omega_b$ can be devised by modulating the received signal. The decision criterion for detection of coupled multitone signals, derived in [3], is

$$\sum_{i=1}^{3} R_i^2 + \frac{2}{N_0}G > \frac{N_0}{4}\ln\eta + \frac{N_0E_s}{4} \qquad (7)$$

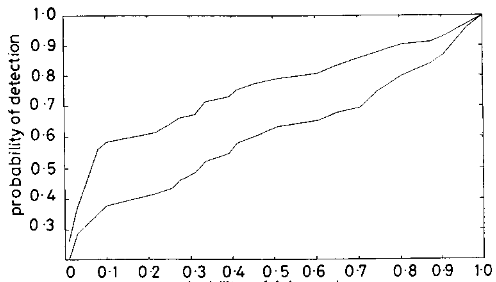
where G is a function of the amplitudes of the coupled sinusoidal tones ($G = 0$ indicates 'no coupling' where the detector reduces to an energy detector), E_s represents the energy of the multitone signal, η denotes the *a priori* probability of occurrence of the multitone signal, N_0 is the variance of the additive noise and R_i is dependent on the received signal. Clearly, the detection decision is directly dependent on the amplitude of the coupled tones G.

To examine the significance of modulation in the detection of phase coupled multitone signals consider the following example. Let

$$x(k) = e^{-j(\omega_1 k+\phi_1)} + e^{-j(\omega_2 k+\phi_2)} + 0.005e^{-j((\omega_1+\omega_2)k+\phi_1+\phi_2)} + e^{-j(\omega_3 k+\theta_1)} + e^{-j(\omega_4 k+\theta_2)} + 10.0e^{-j(\omega_5 k+\theta_1+\theta_2)} \qquad (8)$$

where ϕ_1, ϕ_2, θ_1, θ_2 are independent and uniformly distributed over

$[0, 2\pi]$ and $\omega_5 \neq \omega_1 + \omega_2$. If detection is performed without modulating $\{x(k)\}$ then the contribution of phase coupling information (extracted by the bispectrum) is proportional to $0.005/N_0$, where $N_0/2$ represents the noise spectral density [3]. Modulating $\{x(k)\}$ by $e^{(j(\omega_5-\omega_3-\omega_4)k)}$ will increase the likelihood of the multitone signal by a factor proportional to $10.0/N_0$ without affecting its energy. Fig. 1 shows the probability of detection against the probability of a false alarm (receiver operating characteristics, ROC) for a bispectral based detector of $x(k)$ and of $x(k)e^{(j(\omega_5-\omega_3-\omega_4)k)}$. Hence, in this example, modulating the received signal increased its probability of detection.



[6797]

Fig. 1 *Probablility of detection against probablility of false alarm (ROC) for bispectrum based detectors of modulated and unmodulated signals*

Top curve : modulated
Bottom curve : unmodulated

Conclusion: The detection of coupling between spectral components of a random process may be enhanced via complex modulation prior to bispectral analysis. Complex modulation may also improve the performance of bispectral based multitone detection systems.

Acknowledgment: This work was sponsored by the Department of Navy, Office and Naval Research, under contract N00014-80K-0202.

28 June 1994

Electronics Letters Online No: 19940995

I.I. Jouny (*Department of Electrical Engineering, Lafayette College, Easton, PA 18042, USA*)

R.L. Moses (*Department of Electrical Engineering, The Ohio State University, Columbus, OH 43210, USA*)

References

1 NIKIAS, C.L., and RAGHUVEER, M.R.: 'Bispectrum estimation: A digital signal processing framework', *Proc. IEEE*, 1987, **75**, (7), pp. 869–891

2 JOUNY, I., and MOSES, R.L.: 'The bispectrum of complex signals: Definitions and properties', *IEEE Trans. Signal Processing*, 1992, pp. 2833–2836

3 KLETTER, D., and MESSER, H.: 'Optimal detection of a random multitone signal and its relation to bispectral analysis'. Proc. IEEE 1990 ICASSP, Albuquerque, New Mexico, April 1990, pp. 2391–2394

4 DANDAWATE, A.V., and GIANNAKIS, G.B.: 'Polyspectral analysis of non-stationary signals: system identification, classification and ambiguity functions'. Int. Workshop on Higher Order Statistics, Grenoble, France, July 1991, pp. 147–150

5 ANDERSON, S.J., MAHONEY, A.R., and ZOLLO, A.O.: 'Applications of higher-order statistics signal processing to radar'. Int. Symp. on Signal Processing and its Applications (ISSPA), Queensland, Australia, August 1992

6 WALTON, E.K., and JOUNY, I.I.: 'Bispectrum of radar signatures and applications to target classification', *Radio Science*, 1990, **25**, (2), pp. 101–113

Burstiness of interrupted Bernoulli process

M.E. Woodward

Indexing terms: Asynchronous transfer mode, Markov processes, Stochastic processes

The author first rectifies an error that has been propagating through the literature concerning the squared coefficient of variation of an interrupted Bernoulli process; secondly, the author shows that under the practical constraint that the mean length of the idle period of an interrupted Bernoulli process is finite, the squared coefficient of variation (and hence the burstiness) has a finite maximum value. The relationship between the transition probabilities of the Markov chain to give this maximum is derived.

Introduction: The interrupted Bernoulli process (IBP) is used extensively to model the arrival of cells in asynchronous transfer mode (ATM) systems (see for example [1–3]). Specifically, the IBP is a doubly stochastic Bernoulli process, governed by a two state, discrete-time Markov chain, the two states commonly labelled 'active' and 'idle'. The transition probability matrix has the following form:

$$\begin{array}{c} \\ \text{active} \\ \text{idle} \end{array} \begin{array}{c} \begin{array}{cc} \text{active} & \text{idle} \end{array} \\ \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix} \end{array} \qquad (1)$$

When used to model cell arrivals in an ATM system, with the process in the idle state no cells can arrive. The process either remains in the idle state with probability q, or switches to the active state with probability $1-q$ in the next slot. In the active state a cell can arrive with probability α, or no cell arrives with probability $1-\alpha$. The process either remains active with probability p, or switches to idle with probability $1-p$ in the next slot.

Squared coefficient of variation: [1] is one of the most widely referenced works on the IBP. Here, the generating function of the cell interarrival time $A(z)$, has been calculated as follows:

$$A(z) = \frac{z\alpha[p + z(1-p-q)]}{(1-\alpha)(p+q-1)z^2 - [q + p(1-\alpha)]z + 1} \qquad (2)$$

The squared coefficient of variation (SCV), denoted by C^2, is generally accepted as a measure of burstiness. This is given in [1] as

$$C^2 = 1 + \alpha\left(\frac{(1-p)(3-q)}{(2-p-q)^2} - 2\right) + \alpha^2\frac{(1-q)^2}{(2-p-q)^2} \qquad (3)$$

The SCV of a random variable can be calculated from its generating function by

$$C^2 = \frac{A''(1) + A'(1)}{[A'(1)]^2} - 1 \qquad (4)$$

Here, $A'(1)$ and $A'(1)$ are, respectively, the first and second derivatives of $A(z)$ evaluated at $z = 1$. Carrying out this calculation using eqn. 2 for $A(z)$ results in the following:

$$C^2 = 1 - \alpha\frac{[2(1-p-q)(2-p-q) + (1-q)(p+q)]}{(2-p-q)^2} \qquad (5)$$

This is obviously very different from eqn. 3 which is a quadratic in α. The latter can be shown to be erroneous using a simple check, as follows. With $p = 1$, $q = 0$, the idle state becomes transient, and the IBP collapses to a simple Bernoulli process with parameter α. The cell interarrival time distribution then becomes geometric with parameter α. It is well known that the SCV of such a distribution is $1-\alpha$ (see [4]). Thus, setting $p = 1$, $q = 0$ in eqns. 3 and 5, the right hand side of both should reduce to $1-\alpha$. It is easy to verify that eqn. 5 gives the correct result, but eqn. 3 does not, suggesting the latter is erroneous.

Burstiness of IBP: Using eqn. 5, the following limiting values result for C^2:

$$\begin{array}{lll} p \to 0 & q \to 0 & C^2 \to 1-\alpha \\ p \to 1 & q \to 0 & C^2 \to 1-\alpha \\ p \to 0 & q \to 1 & C^2 \to 1 \\ p \to 1 & q \to 1 & C^2 \to \infty \end{array}$$