TABLE VI
COMPARISON RESULTS FOR EXAMPLE (20). THE SEQUENCES HAD 80 SAMPLES

$p = 2, a_{12} = -1.37, a_{22} = 0.56, \sigma^2 = 1, N = 80$

k	PDC	PLAV	PLS	AIC	MDL	PMDL
0	0	6	36	0	0	10
1	7	121	238	0	1	92
2	973	839	713	700	946	849
3	19	31	13	112	44	41
4	1	3	0	56	5	3
5	0	0	0	48	2	2
6	0	0	0	35	2	2
7	0	0	0	24	0	1
8	0	0	0	25	0	0

TABLE VII
COMPARISON RESULTS FOR EXAMPLE (21)

$p = 4, a_{14} = -1.352, a_{24} = 1.338, a_{34} = -0.662$
$a_{44} = 0.240, \sigma^2 = 1, N = 100$

k	PDC	PLAV	PLS	AIC	MDL	PMDL
0	0	21	99	0	0	11
1	0	25	96	0	0	11
2	100	407	452	5	28	104
3	386	325	233	114	324	407
4	505	214	118	625	613	405
5	8	8	2	120	31	42
6	1	0	0	57	3	12
7	0	0	0	31	1	6
8	0	0	0	48	0	2

TABLE VIII
COMPARISON RESULTS FOR EXAMPLE (22)

$p = 4, a_{14} = -2.760, a_{24} = 3.809, a_{34} = -2.654$
$a_{44} = 0.924, \sigma^2 = 0.1, N = 20$

k	PDC	PLAV	PLS	AIC	MDL	PMDL
0	0	2	18	0	0	8
1	0	2	27	0	0	1
2	0	185	617	0	0	12
3	3	7	11	0	1	19
4	940	752	307	90	158	71
5	44	42	19	60	75	83
6	7	9	1	359	416	261
7	3	1	0	296	243	232
8	3	0	0	195	107	313

mance stems from the nature of the quasi-likelihood predictive density. This density does not precisely penalize for overparametrization since it does not take into account that the parameters of the model used to determine its form are not true, but rather are estimated from data.

As a final note we emphasize that the comparison between PLAV and PLS with the rest of the methods is not fair because the underlying assumptions for their use are different. To employ PLAV or PLS we do not require knowledge of the probability density function of the data, while for the other approaches this information is of fundamental importance.

V. CONCLUSION

A criterion for order selection of AR models has been derived based on Bayesian predictive densities. Its performance was assessed by extensive simulations and compared to other methods. The simulations show that this approach often yields better results than its competitors. Also, as an alternative to the PLS approach, the PLAV criterion was introduced. In the numerical simulations it systematically outperformed PLS.

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The Bispectrum of Complex Signals: Definitions and Properties

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***Abstract*—This correspondence is concerned with the definition and properties of the bispectrum of complex-valued signals. The symmetry**

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properties and the relationship between all forms of third-order cumulants of complex signals are investigated. It is shown that all cumulants (for different position of the complex conjugate) are related by simple transformations. This article also investigates autoregressive modeling of complex-valued signals using third-order cumulants. It is shown that modeling of complex-valued signals requires a different approach from modeling of real-valued signals.

I. Introduction

Bispectral analysis of real-valued data has been an active research area in the last few years. The applications of the bispectrum extend over several disciplines. These applications include ARMA modeling, analysis of bilinear models, detection of phase coupling, signal reconstruction, image processing, and so on [1]. These applications have been mainly limited to real-valued signals; consequently, development of bispectral analysis techniques for complex-valued signals has not received as much attention. Bispectral processing of complex signals is of interest in a number of areas, including radar signal processing [8].

It has been suggested by Brillinger and Rosenblatt [2] that for complex-valued signals different bispectra can be obtained dependent on the placement of the complex conjugate in the third-order cumulant. In this correspondence we investigate the definitions and properties of these bispectra and we establish the relationship between the different bispectra. We also study the aspect of parametric modeling of complex-valued signals using third-order cumulants. We show that modeling of complex-valued signals requires a different approach from modeling of real-valued signals.

II. The Bispectrum of Complex Signals

Let $\{x(k)\}$ represent a complex-valued stationary random process. To obtain the third joint cumulant, the complex conjugate can be placed either on one or two entries of the triple product in $E\{x(k)x(k+m)x(k+n)\}$ [2]. For each choice, only one symmetry relation is valid. In fact, placing the conjugate in a particular position defines which one of the six symmetry relations given in [1] remains valid. In general, the bispectrum of complex signals has twofold symmetry about an axis S in the ω_1, ω_2 plane, where S is one of three defined as $S \in \{\omega_1 = \omega_2, \omega_2 = -(1/2)\omega_1, \omega_1 = -(1/2)\omega_2\}$. Table I gives the definitions of the bispectrum of complex-valued signals for each of the six possible positions of the complex conjugate, where $\{X(\omega)\}$ denotes the Fourier transform of the signal $\{x(k)\}$ (see [7]), and $\langle , \rangle$ represents the ensemble average.

Note that it is possible to obtain one bispectral response from other bispectral responses using cumulant transformations. For example, if

$$C_1(m, n) = E\{x^*(k)x(k+m)x(k+n)\},$$

$$C_2(m, n) = E\{x(k)x^*(k+m)x(k+n)\}$$

Then $C_1(m, n) = C_2(-m, n-m)$. Similarly, if $C_3(m, n) = E\{x(k)x(k+m)x^*(k+n)\}$ then $C_1(m, n) = C_3(m-n, -n)$. Other relations can be obtained similarly.

The bispectra in Table I are interrelated also, and knowing one bispectrum is sufficient to derive the others. For example, if $B_1(\omega_1, \omega_2) = \langle X(\omega_1)X(\omega_2)X^*(\omega_1+\omega_2)\rangle$ and $B_2(\omega_1, \omega_2) = \langle X^*(-\omega_1)X(\omega_2)X(-\omega_1-\omega_2)\rangle$ then $B_1(\omega_2, -\omega_1-\omega_2) = B_2(\omega_1, \omega_2)$. Similar relations hold for the other bispectra defined above.

Example: Let $h(k)$ be a complex-valued waveform given as

$$h(k) = e^{j(kw_a+\theta_a)} + e^{j(kw_b+\theta_b)} + e^{j(k(\omega_a+\omega_b)+\theta_a+\theta_b)} \quad (1)$$

where θ_a and θ_b are uniformly distributed random variables over $[0, 2\pi]$. Notice the quadratic phase coupling at (ω_a, ω_b) which results in two bispectral peaks at (ω_a, ω_b) and (ω_b, ω_a). The bispectrum of $h(k)$ is shown in Fig. 1 and it depends on the placement of the complex conjugate in the third-order cumulant. Notice that the symmetry axis of the bispectral response changes according to the format of the third-order cumulant.

The hexagonal shape of the bispectrum using all of the above definitions remains valid because in each of these definitions the term $X(\omega_1+\omega_2)$ or $X(-\omega_1-\omega_2)$ is present. Note, however, that aliasing in the bispectrum of complex-valued signals is slightly different from aliasing in bispectrum of real-valued signals. For example, if $B(\omega_1, \omega_2) = \langle X(\omega_1)X(\omega_2)X^*(\omega_1+\omega_2)\rangle$ then the aliasing triangle has twofold symmetry with respect to $\omega_1 = \omega_2$ with the bispectral signatures that belong to the hexagon centered at (0, 0) different from those that belong to the hexagons centered at $(0, 2\pi)$ and $(2\pi, 0)$, respectively. The triangles that belong to the hexagons centered at $(0, 2\pi)$ and $(2\pi, 0)$ are symmetric with respect to $\omega_1 = \omega_2$ (see Fig. 2). Furthermore, for this case, it is impossible to have a response in the region *ebgd* since the symmetry relation $B(\omega_1, -\omega_2-\omega_1) = B(\omega_1, \omega_2)$ is no longer valid (and therefore the Nyquist sampling criterion is completely valid [7]). Other definitions of the bispectrum have different aliasing characteristics dependent on the type of symmetry involved.

III. On Autoregressive Modeling of Complex Signals Using Third-Order Cumulants

Modeling complex-valued signals requires an approach that is different from modeling real-valued signals. The following theorems explain the modeling aspect of complex-valued signals and the model order needed. We use complex exponentials as an example of complex-valued signals. These theorems show a fundamental difference between modeling real-valued signals and complex-valued signals.

We first review the AR modeling of real signals by restating the following result from [6]. The AR parameters can be obtained from the following set of linear equations:

$$\sum_{n=0}^{P} a_n C(n-k, n-l) = 0 \quad (2)$$

where $C(.,.)$ is the third-order cumulant and $\{a_n\}_{n=1}^{P}$ is the set of AR parameters. It is shown in [6] that the AR parameters of a real sinusoidal signal with I implicit couplings can be obtained from (2) where the AR model order $P = 6I$. In the following we restate a theorem from [6] concerned with AR modeling of real sinusoids with implicit couplings.

Theorem [6]: Let h(k) be of the following form:

$$h(k) = \sum_{i=1}^{I} [\cos(\omega_{ai}k + \theta_{ai}) + \cos(\omega_{bi}k + \theta_{bi}) + \cos((\omega_{ai}+\omega_{bi})k + \theta_{ai} + \theta_{bi})] \quad (3)$$

where θ_{ai} and θ_{bi} are uniformly distributed random variables over $[0, 2\pi]$. Then there is a unique set of real AR coefficients $\{a_n\}_{n=0}^{6I}$, with $a_0 = 1$, such that

$$\sum_{n=0}^{6I} a_n C_h(n-k, n-l) = 0. \quad (4)$$

Thus, for real-valued signals, the bispectrum can be obtained from the AR parameters found by the prediction equation on the cumulants. For complex data, this is not in general the case, as we show below.

TABLE I
EXPRESSIONS FOR THE BISPECTRUM AND THE ASSOCIATED SYMMETRY RELATIONS DEPENDENT ON THE PLACEMENT OF THE COMPLEX CONJUGATE IN THE THIRD-ORDER CUMULANT

Third-Order Cumulant	Bispectrum	Symmetry Relation
$E\{x^*(k)x(k+m)x(k+n)\}$	$\langle X(\omega_1)X(\omega_2)X^*(\omega_1+\omega_2)\rangle$	$B(\omega_1, \omega_2) = B(\omega_2, \omega_1)$
$E\{x(k)x^*(k+m)x(k+n)\}$	$\langle X^*(-\omega_1)X(\omega_2)X(-\omega_1-\omega_2)\rangle$	$B(\omega_1, \omega_2) = B(\omega_1, -\omega_1-\omega_2)$
$E\{x(k)x(k+m)x^*(k+n)\}$	$\langle X(\omega_1)X^*(-\omega_2)X(-\omega_1-\omega_2)\rangle$	$B(\omega_1, \omega_2) = B(-\omega_1-\omega_2, \omega_2)$
$E\{x(k)x^*(k+m)x^*(k+n)\}$	$\langle X^*(-\omega_1)X^*(-\omega_2)X(-\omega_1-\omega_2)\rangle$	$B(\omega_1, \omega_2) = B(\omega_2, \omega_1)$
$E\{x^*(k)x^*(k+m)x(k+n)\}$	$\langle X^*(-\omega_1)X(\omega_2)X^*(\omega_1+\omega_2)\rangle$	$B(\omega_1, \omega_2) = B(-\omega_2-\omega_1, \omega_2)$
$E\{x^*(k)x(k+m)x^*(k+n)\}$	$\langle X(\omega_1)X^*(-\omega_2)X^*(\omega_1+\omega_2)\rangle$	$B(\omega_1, \omega_2) = B(\omega_1, -\omega_1-\omega_2)$

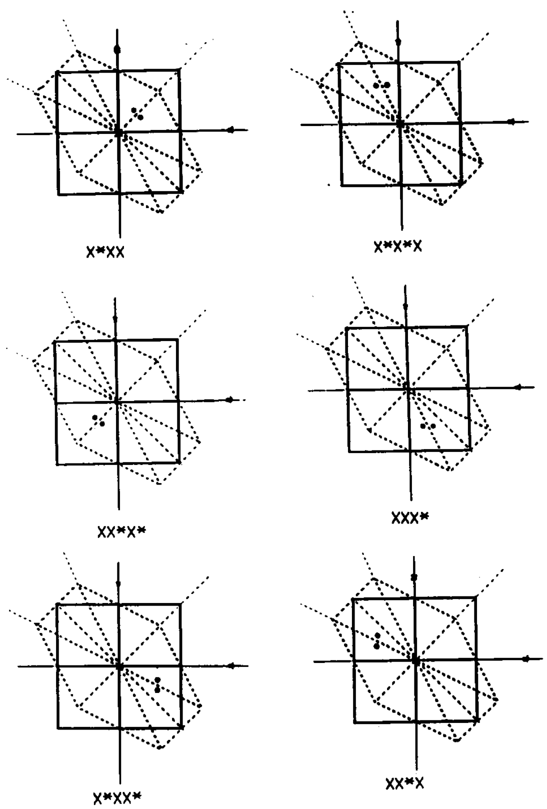

Fig. 1. Example showing the dependence of the bispectrum of a complex-valued signal on the placement of the complex conjugate in the third-order cumulant.

Theorem 1: Let h (k) be of the following form:

$$h(k) = \sum_{i=1}^{I} [e^{j(k\omega_{ai} + \theta_{ai})} + e^{j(k\omega_{bi} + \theta_{bi})} + e^{j(k(\omega_{ai} + \omega_{bi}) + \theta_{ai} + \theta_{bi})}] \tag{5}$$

where θ_{ai}, θ_{bi} are uniformly distributed random variables over $[0, 2\pi]$. Then there is a unique set of complex AR coefficients $\{a_n\}_{n=0}^{I}$, with $a_0 = 1$, such that

$$\sum_{n=0}^{I} a_n C_h(n-k, n-l) = 0. \tag{6}$$

However, when this set of AR coefficient is used to compute the bispectrum, the result is in general not the true bispectrum.

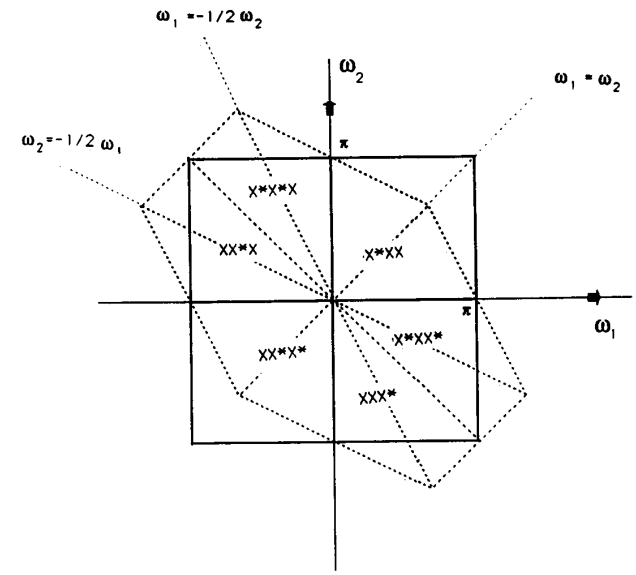

Fig. 2. Rectangular display of the bispectral hexagon in the (ω_1, ω_2) plane.

Proof: The third-order cumulant of $h(k)$ is given as

$$C_h(k, l) = \sum_{i=1}^{I} [e^{j(k\omega_{ai} + l\omega_{bi})} + e^{j(k\omega_{bi} + l\omega_{ai})}] \tag{7}$$

and the bispectrum is

$$B_h(\omega_1, \omega_2) = \sum_{i=1}^{I} [\delta(\omega_1 - \omega_{ai}, \omega_2 - \omega_{bi}) + \delta(\omega_1 - \omega_{bi}, \omega_2 - \omega_{ai})]. \tag{8}$$

Then,

$$\sum_{n=0}^{I} a_n C_h(n-k, n-l) = \sum_{n=0}^{I} \sum_{i=1}^{I} a_n (e^{j[\omega_{ai}(n-k) + \omega_{bi}(n-l)]} + e^{j[\omega_{bi}(n-k) + \omega_{ai}(n-l)]}) = \sum_{i=1}^{I} \left[(e^{-j(\omega_{ai}k + \omega_{bi}l)} + e^{-j(\omega_{ai}l + \omega_{bi}k)}) \sum_{n=0}^{I} a_n e^{j(\omega_{ai} + \omega_{bi})n} \right].$$

This expression is zero if and only if the AR parameters $\{a_i\}$ satisfy

$$\sum_{n=1}^{I} a_n z^n = \prod_{i=1}^{I} (z - e^{j(\omega_{ai} + \omega_{bi})}). \tag{9}$$

Therefore, it is sufficient to choose a model of order I to model I responses in the bispectrum. The spectrum obtained using these AR parameters has I peaks at $\omega_{ai} + \omega_{bi}$, $i = 1, \cdots, I$, and is given as (using (5))

$$H(\omega) = \sum_{i=1}^{I} \frac{c_i}{e^{-j\omega} - e^{j(\omega_{ai} + \omega_{bi})}}. \tag{10}$$

The bispectrum $B(\omega_1, \omega_2)$ is obtained using $H(\omega_1)H(\omega_2)H^*(\omega_1 + \omega_2)$ as

$$B(\omega_1, \omega_2) = \left(\sum_{i=1}^{I} \frac{c_i}{e^{-j\omega_1} - e^{-j(\omega_{ai} + \omega_{bi})}}\right) \cdot \left(\sum_{i=1}^{I} \frac{c_i}{e^{-j\omega_2} - e^{-j(\omega_{ai} + \omega_{bi})}}\right) \cdot \left(\sum_{i=1}^{I} \frac{c_i^*}{e^{j(\omega_1 + \omega_2)} - e^{j(\omega_{ai} + \omega_{bi})}}\right).$$

In this case, the bispectrum peaks at (ω_j, ω_k), $\forall\, j, k$ when either ω_j, ω_k, or $\omega_j + \omega_k$ is equal to $\omega_{ai} + \omega_{bi}$. For example, if $H(\omega_j) = \alpha$, $H(\omega_k) = \beta$, (where $\alpha, \beta \neq 0$, for an all-pole model) and $H(\omega_j + \omega_k) = \gamma$ where $\lim_{\omega_j + \omega_k \to \omega_{ai} + \omega_{bi}} \{\gamma\} \to \infty$, and $(\omega_j + \omega_k = \omega_{ai} + \omega_{bi})$, $i = 1, \cdots, i$, then the bispectrum at (ω_j, ω_k) is such that

$$\lim_{\omega_j + \omega_k \to \omega_{ai} + \omega_{bi}} \{B(\omega_j, \omega_k)\} \to (\alpha)(\beta)(\infty). \tag{11}$$

Similarly, the bispectrum peaks at every ω_j, ω_k such that $\omega_j + \omega_k = \omega_{ai} + \omega_{bi}$, $\omega_j - \omega_k = \omega_{ai} + \omega_{bi}$, or $\omega_k - \omega_j = \omega_{ai} + \omega_{bi}$). Note that the true bispectral peak at $(\omega_{ai}, \omega_{bi})$ lies on the line defined by $\{\omega_1 + \omega_2 = \omega_{ai} + \omega_{bi}\}$; however, the magnitude of this peak is comparable to the other peaks along the same line and cannot be distinguished from the others. The only bispectrum that can be modeled correctly corresponds to the case where $\omega_{ai} = \omega_{bi} = 0$, since in this case $\lim_{\omega_1, \omega_2 \to 0} \{B(\omega_1, \omega_2)\} \to (\infty)(\infty)(\infty)$. ■

One way to circumvent this modeling problem is to compute the AR parameters in a different way, as shown below.

Theorem 2: Let h (k) be of the following form:

$$h(k) = \sum_{i=1}^{I} [e^{j(k\omega_{ai} + \theta_{ai})} + e^{j(k\omega_{bi} + \theta_{bi})} + e^{j(k(\omega_{ai} + \omega_{bi}) + \theta_{ai} + \theta_{bi})}] \tag{12}$$

where θ_{ai}, θ_{bi} are uniformly distributed random variables over $[0, 2\pi]$. *Then, there is a unique set of complex AR coefficients* $\{a_n\}_{n=0}^{3I}$, *with* $a_0 = 1$, *such that*

$$\sum_{n=0}^{3I} a_n C_h'(n - k, n - l) = 0 \tag{13}$$

where

$$C_h'(k, l) = E\{h^*(n)h(n + k)h(n + l) + h(n)h^*(n + k)h(n + l) + h(n)h(n + k)h^*(n + l)\}. \tag{14}$$

These parameters uniquely model the true bispectrum.

Proof: Using (14), the third-order cumulant of the signal in (12) is

$$C_h'(k, l) = \sum_{i=1}^{I} [e^{j(k\omega_{ai} + l\omega_{bi})} + e^{j(k\omega_{bi} + l\omega_{ai})} + e^{j[k(\omega_{ai} + \omega_{bi}) + l\omega_{bi}]} + e^{j(k\omega_{bi} + l[\omega_{ai} + \omega_{bi}]} + e^{j[k(\omega_{ai} + \omega_{bi}) + l\omega_{ai}]} + e^{j[k\omega_{ai} + l(\omega_{ai} + \omega_{bi})]}]. \tag{15}$$

The proof then follows similar to that in [6] for real sinusoidal signals with I couplings. The frequency response is then given as

$$H(\omega) = \sum_{i=1}^{I} \left[\frac{c_{i1}}{e^{-j\omega} - e^{-j(\omega_{ai} + \omega_{bi})}} + \frac{c_{i2}}{e^{-j\omega} - e^{-j\omega_{ai}}} + \frac{c_{i3}}{e^{-j\omega} - e^{-j\omega_{bi}}}\right] \tag{16}$$

where $c_{ij} \neq 0, j = 1, 2, 3$. The bispectrum is then obtained using $H(\omega_1)H(\omega_2)H^*(\omega_1 + \omega_2)$. Using this model, all responses at ω_{ai}, ω_{bi}, and $\omega_{ai} + \omega_{bi}$, $i = 1, \cdots, I$ are modeled in the impulse response and each set of $\{\omega_{ai}, \omega_{bi}, \omega_{ai} + \omega_{bi}\}$ in the frequency response produces a single peak in the bispectrum such that

$$\lim_{\omega_1 \omega_2 \to \omega_{ai}, \omega_{bi}} \{B(\omega_1, \omega_2)\} \to (\infty)(\infty)(\infty). \tag{17}$$

This peak can be easily distinguished from other peaks that are obtained as a result of the transformation defined by $H(\omega_1)H(\omega_2)H^*(\omega_1 + \omega_2)$. ■

We have shown in Theorem 1 (using AR modeling of complex exponentials) that although it is possible to obtain AR parameters that satisfy (2), the derived system does not necessarily model the true bispectrum. Theorem 2 presents an alternative approach to modeling complex exponentials. The AR parameters obtained as described in Theorem 2 uniquely model the true bispectrum of complex exponentials.

IV. Conclusions

There are three related forms for the bispectrum of complex-valued signals and three other forms that can be obtained by taking the complex conjugate of the first three. Aliasing in the case of complex-valued signals is different from the case of real-valued signals and depends on the type of symmetry available. It is also shown that a different approach to parametric modeling of complex signatures is needed and requires using more than a single form of the third-order cumulant.

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