AN AUTO-CALIBRATION METHOD FOR UNATTENDED GROUND SENSORS

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ABSTRACT

We present an algorithm for locating and orienting a set of sensor arrays that have been deployed in a scene at unknown locations and orientation angles. This self-calibration problem is solved using a number of source signals also deployed in the scene. We assume each array can estimate the time-of-arrival and direction-of-arrival of every source. From this information we compute the array locations and orientations. We consider four subproblems, in which the source signals or emission times are either known or unknown. We develop necessary conditions for solving the self-calibration problem and provide a maximum likelihood solution and corresponding location error estimate.

1. INTRODUCTION

Unattended Ground Sensors (UGSs) are becoming increasingly important for providing situational awareness in battlefield deployments [1]. The basic concept is to deploy a large number of low-cost, self-powered sensors that acquire and process data. The sensors typically consist of an array of microphones to detect, track, and classify acoustic signatures. Each sensor is equipped with a local processor and a low-power communication transceiver. The sensed data is processed locally, and the result is transmitted to a local Central Information Processor (CIP) through a low-power communication network. The CIP fuses sensor information and transmits it to a more distant command center.

In order to fuse sensor information at the CIP or command center, knowledge of each sensor's location and orientation is vital. Ground sensors are placed in the field by persons, air drop, or artillery launch. Except for careful hand placement, it is difficult or impossible to know accurately the location and orientation of each sensor. Adding a GPS and compass adds to the expense and power requirements and may increase susceptibility to jamming. Thus, there is interest in developing methods to self-calibrate the sensor array with a minimum of additional hardware or processing.

We consider an approach to array self-calibration using a number of signal sources deployed in the same region as the sensors. Each source emits a unique signature that is detected by the sensors. From the time-of-arrival (TOA) and direction-of-arrival (DOA) of each source signal, we compute the unknown locations and orientations of the sensors. We consider four related subproblems: (i) the source locations and emission times are known; (ii) source locations are known and emission times are unknown; (iii) the source locations are unknown and emission times are known; and (iv) the source locations and emission times are unknown.

Several researchers have considered the problem of array calibration, but less work is devoted to calibrating networks of sensors. A number of papers have considered calibration of both narrowband and broadband arrays of sensors to improve direction-of-arrival estimation accuracy [2,3]. A recent paper considers sensor self-calibration using a single acoustic source that travels in a straight line [4].

2. THE SELF-CALIBRATION PROBLEM

Assume we have a set of A sensors, each with unknown location $\{a_i = (x_i, y_i)\}_{i=1}^{A}$ and unknown orientation angle θ_i with respect to a reference direction (*e.g.*, North). We consider the two-dimensional problem in which the sensors lie in a plane and the unknown reference direction is azimuth; an extension to the three-dimensional case is possible using similar techniques. In the array field are also placed S point source signals at locations $\{s_j = (\tilde{x}_j, \tilde{y}_j)\}_{j=1}^{S}$. The source locations may be known or unknown. Each source emits a finite-length signal that begins at time t_j; the emission times may be known or unknown. We thus consider four related subproblems, depending on the prior knowledge of the source locations and their emission times.

We initially assume each emitted source signal is detected by all of the sensors in the field and that each sensor

This material is based in part upon work supported by the U.S. Army Research Office under Grant No. DAAH-96-C-0086 and Batelle Memorial Institute under Task Control No. 01092, and in part through collaborative participation in the Advanced Sensors Consortium sponsored by the U.S. Army Research Laboratory under the Federated Laboratory Program, Cooperative Agreement DAAL01-96-2-0001. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the U.S. Army Research Office, the Army Research Laboratory or the U.S. Government.

measures the TOA and DOA for that source. We denote the measured TOA of source j to sensor i as t_{ij} and the measured DOA as θ_{ij}.

The DOA measurements are made with respect to a local frame of reference. The times of arrival are measured with respect to a known, common time base which can be established either by using the electronic communication network linking the sensors or by synchronizing the sensor processor clocks before deployment.

The set of $2AS$ measurements are gathered in a vector

$$X = \begin{bmatrix} \text{vec}(T) \\ \text{vec}(\Theta) \end{bmatrix}^T \quad (2AS \times 1) \tag{1}$$

where $\text{vec}(M)$ stacks the elements of a matrix M columnwise, $T = [t_{ij}], \Theta = [\theta_{ij}]$, with $i = 1, ..., A; j = 1, ..., S$.

Define the parameter vectors

$$\begin{aligned} \alpha &= [\beta^T, \gamma^T]^T \quad (3(A+S) \times 1) && (2) \\ \beta &= [x_1, y_1, \theta_1, \ldots, x_A, y_A, \theta_A]^T \quad (3A \times 1) && (3) \\ \gamma &= [\tilde{x}_1, \tilde{y}_1, t_1, \ldots, \tilde{x}_S, \tilde{y}_S, t_S]^T \quad (3S \times 1) && (4) \end{aligned}$$

We denote the actual TOA and DOA of source signal j at sensor i as $\tau_{ij}(\alpha)$ and $\phi_{ij}(\alpha)$, respectively, and include their dependence on the parameter vector α; they are computed as:

$$\begin{aligned} \tau_{ij}(\alpha) &= t_j + \|a_i - s_j\|/c && (5) \\ \phi_{ij}(\alpha) &= \theta_i + \angle(a_i, s_j) && (6) \end{aligned}$$

where $\|\cdot\|$ is the Euclidean norm, $\angle(\xi, \eta)$ is the angle between the points $\xi, \eta \in \mathcal{R}^2$, and c is the signal propagation velocity.

Each element of X has measurement uncertainty; we model the uncertainty as

$$X = \mu(\alpha) + E \tag{7}$$

where $\mu(\alpha)$ is the noiseless measurement vector with elements given by equations (5) and (6) for values of i, j that correspond to the vector stacking operation in (1), and E is a random vector with known probability density function.

The self-calibration problem, then, is given the measurement X, estimate β. Note that none, some, or all of the parameters in γ may be known depending on the particular subproblem of interest.

3. EXISTENCE AND UNIQUENESS OF SOLUTIONS

In this section we address the existence and uniqueness of solutions to the self-calibration problem and establish the minimum number of sensors and sources needed to obtain a unique calibration solution. We give the self-calibration algorithms for the minimal cases that provide initial estimates to an iterative descent algorithm for the practical case of non-minimal, noisy measurements presented in Section 4.

Case 1: Known source locations and emission times. A unique solution for β can be found for any number of sensors as long as there are $S \geq 2$ sources. The location of the ith sensor, a_i, is found from the intersection of two circles with centers at the source locations and with radii $(t_{i1} - t_2)/c$ and $(t_{i2} - t_1)/c$. The intersection is in general two points; the correct location can be found using the sign of $\theta_{i2} - \theta_{i1}$. We note that the two circle intersections can be computed in closed-form. From the known source and sensor locations and the DOA measurements, the sensor orientations can also be uniquely found.

Case 2: Known source locations, unknown emission times. For $S \geq 3$ sources, the location and orientation of each sensor can be computed in closed form. Consider the pair of sources (s_1, s_2). Sensor i knows the angle $\theta_{i2} - \theta_{i1}$ between these two sources. The set of all possible locations for sensor i is an arc of a circle whose center and radius can be computed from the source locations. Similarly, a second circular arc is obtained from the source pair (s_1, s_3). The intersection of these two arcs is a unique point. Once the sensor location is known, its orientation is readily computed from any of the three DOA measurements.

A solution for Case 2 can also be found using $S = 2$ sources and $A = 2$ sensors. The solution requires a one-dimensional search of a parameter over an finite interval. The known location of s_1 and s_2 and the known angle $\theta_{11} - \theta_{12}$ means that array a_1 must lie on a known circular arc. At exactly one position along the arc, the emission times are consistent with the measurements from the second sensor.

Case 3: Unknown source locations, known emission times. In this case and in Case 4 below, the calibration problem can only be solved to within an unknown translation and rotation of the entire sensor-source scene since the t_{ij} and θij measurements do not change. To eliminate this ambiguity, we assume the location and orientation of the first sensor are known; without loss of generality we set $x_1 = y_1 = \theta_1 = 0$. We solve for the remaining $3(A-1)$ parameters in β.

A unique solution for β is computable in closed form for $S = 2$ and any $A \geq 2$ (the case $A = 1$ is trivial). From sensor a_1 the range to each source can be computed from $r_j = (t_{1j} - t_j)/c$, and its bearing is known, so the locations of the two sources can be found. The locations and orientations of the remaining sensors is then computed using the method of Case 1.

Case 4: Unknown source locations and emission times. For this case it can be shown that an infinite number of calibration solutions exists for $A = S = 2$, [1] but that a unique solution exists in almost all cases for either $A = 2$, $S = 3$ or $A = 3$, $S = 2$. In some degenerate cases, not all of the

[1] Note that for $A = S = 2$ there are 8 measurements and 9 unknown parameters. The set of possible solutions in general lies on a one-dimensional manifold in the 9-dimensional parameter space.

γ parameters can be uniquely determined, although we do not know of a case for which the β parameters cannot be uniquely found.

Calibration solutions for this case require a two-dimensional search. We outline a solution that works for either $A = 2$ and $S \geq 3$ or $S = 2$ and $A \geq 3$. Assume sensor a_1 is at location $(x_1, y_1) = (0, 0)$ with orientation $\theta_1 = 0$. If we knew the two source emission times t_1 and t_2, we can find the locations of sources s_1 and s_2 as in Case 3. All remaining sensor locations and orientations can then be found using the procedure in Case 1, and all remaining source locations can be found using triangulation. The solutions will be inconsistent except for the correct values of t_1 and t_2. The calibration procedure, then, is to iteratively adjust t_1 and t_2 to minimize the error between computed and measured time delays and arrival angles.

4. MAXIMUM LIKELIHOOD SELF-CALIBRATION

In this section we derive a maximum likelihood (ML) estimator for the unknown array location and orientation parameters. The algorithm involves the solution of a set of nonlinear equations for the unknown parameters (and the unknown nuisance parameters in γ). The solution is found by iterative minimization of a cost function; we use the methods in Section 3 to initialize the iterative descent. Also, we derive the Cramer-Rao Bound (CRB) for the variance of the unknown parameters in α; the CRB also gives high-SNR parameter variance of the ML parameter estimates.

4.1. The Maximum Likelihood Estimate

We assume the measurement uncertainty E in equation (7) is Gaussian with zero mean and known covariance Σ. In this case the likelihood function is

$$f(X;\alpha) = \frac{1}{(2\pi\sigma_t\sigma_\theta)^{AS}} \exp\left\{-\frac{1}{2}Q(X;\alpha)\right\} \quad (8)$$

$$Q(X;\alpha) = [X - \mu(\alpha)]^T \Sigma^{-1} [X - \mu(\alpha)] \quad (9)$$

A special case is when the measurement errors are uncorrelated and the TOA and DOA measurement errors are Gaussian with zero mean and variances σ_t^2 and σ_θ^2, respectively; then equation (9) becomes

$$Q(X;\alpha) = \sum_{i=1}^{A}\sum_{j=1}^{S}\left[\frac{(t_{ij} - \tau_{ij}(\alpha))^2}{\sigma_t^2} + \frac{(\theta_{ij} - \phi_{ij}(\alpha))^2}{\sigma_\theta^2}\right] \quad (10)$$

In the four cases considered in Section 3, some of the parameters in α are known. We denote α_1 to be the unknown parameters in α and α_2 to be the known parameters. Using this notation along with equation (8), the maximum likelihood estimate of α_1 is

$$\hat{\alpha}_{1,ML} = \arg\max_{\alpha_1} f(X, \alpha_2; \alpha) = \arg\min_{\alpha_1} Q(X;\alpha)$$

4.2. Nonlinear Least Squares Solution

The solution of (4.1) involves solving a nonlinear least squares problem. A standard iterative descent procedure can be used, initialized using one of the solutions in Section 3. In our implementation we used the Matlab function `lsqnonlin`.

The straightforward nonlinear least squares solution we adopted converged quickly (in several seconds for all examples tested) and displayed no symptoms of numerical instability; however, alternative methods for solving equation (4.1) may reduce computation [5, 6].

4.3. Estimation Accuracy

The Cramer-Rao Bound (CRB) gives a lower bound on the covariance of any unbiased estimate of α_1. It is a tight bound in the sense that $\hat{\alpha}_{1,ML}$ has parameter uncertainty given by the CRB for high signal-to-noise ratio; that is, as $\max_i \Sigma_{ii} \to 0$. Thus, the CRB is a useful tool for analyzing calibration uncertainty.

The CRB can be computed from the Fisher Information Matrix (FIM) of α_1. The FIM is given in [7]. The partial derivatives are readily computed from equations (8), (5), and (6); we find that

$$I_{\alpha_1} = [G'(\alpha_1)]^T \Sigma^{-1} [G'(\alpha_1)] \quad (11)$$

where $G'(\alpha_1)$ is the $2AS \times \dim(\alpha_1)$ matrix whose ijth element is $\partial\mu_i(\alpha_1)/\partial(\alpha_1)_j$.

For Cases 3 and 4, the FIM is rank deficient due to the translational and rotational ambiguity in the self-calibration solution in those two cases. In this situation, two approaches can be taken. First, one can assume some of the sensor parameters are known. Let $\tilde{\alpha}_1$ denote the vector obtained by removing these assumed known parameters from α_1. To compute the CRB matrix for $\tilde{\alpha}_1$, we first remove all rows and columns in I_{α_1} that correspond to the assumed known parameters, then invert the remaining matrix [7].

The second approach is to compute the CRB of the parameter vector α_1 subject to knowledge of the translation and rotation. We compute an eigenvalue decomposition of I_{α_1}:

$$I_{\alpha_1} = [U_1 U_2] \begin{bmatrix} \Lambda_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_1^T \\ U_2^T \end{bmatrix} \quad (12)$$

Except in degenerate cases, it can be shown that U_2 has 3 columns and that its columns span the subspace corresponding to overall scene translation and rotation. Then the constrained CRB of the parameter vector α_1 subject to

knowledge of the translation and rotation is given by the pseudoinverse of I_{α_1} [8].

4.4. Partial Measurements

In this section we assume each emitted source signal is detected by a subset of the sensors in the field and that a sensor that detects a source may measure the TOA and/or DOA for that source. We denote the availability of a measurement using two indicator functions I^t_{ij} and I^θ_{ij}. If sensor i measures the TOA (DOA) for source j, then $I^t_{ij} = 1$ ($I^\theta_{ij} = 1$); otherwise, the indicator function is set to zero. Furthermore, let L denote the $2AS \times 1$ vector whose kth element is 1 if X_k is measured and is 0 if X_k is not measured; L is thus obtained by forming $A \times S$ matrices I^t and I^θ and stacking their column into a vector as in equation (1). Finally, define $\tilde{X}$ to be the vector formed from elements of X for which measurements are available, so X_k is in $\tilde{X}$ if $L_k = 1$.

The maximum likelihood estimator for the partial measurement case is similar to equation (4.1) but uses only those elements of X for which the corresponding element of L is one. Thus (assuming uncorrelated measurement errors as in equation (10)),

$$\hat{\alpha}_{1,ML} = \arg\min_{\alpha_1} \tilde{Q}(\tilde{X};\alpha) \tag{13}$$

$$\tilde{Q}(\tilde{X};\alpha) = \sum_{i=1}^{A}\sum_{j=1}^{S}\left[\frac{(t_{ij} - \tau_{ij}(\alpha))^2}{\sigma_t^2} I^t_{ij} + \frac{(\theta_{ij} - \phi_{ij}(\alpha))^2}{\sigma_\theta^2} I^\theta_{ij}\right] \tag{14}$$

The FIM for this case is similar to equation (11), but includes only information from available measurements; thus

$$\tilde{I}_{\alpha_1} = [\tilde{G}'(\alpha_1)]^T \Sigma^{-1} [\tilde{G}'(\alpha_1)] \tag{15}$$

$$\left[\tilde{G}'(\alpha_1)\right]_{ij} = L_i \cdot \frac{\partial \mu_i(\alpha_1)}{\partial(\alpha_1)_j} \tag{16}$$

5. NUMERICAL RESULTS

We present numerical examples of the self-calibration procedure. Ten sensor arrays and eleven sources are randomly placed in a 2 km×2 km region with random sensor orientations and source emission. We assume every sensor measures the TOA and DOA of each source. The measurement uncertainties are Gaussian with standard deviations of $\sigma_t = 1$ msec for the TOAs and $\sigma_\theta = 3°$ for the DOAs.

Setting the locations of sensor arrays $A1$ and $A2$ as known, we compute for each sensor the equivalent 2σ uncertainty radius, defined as the geometric mean of the major and minor axis lengths of the 2σ uncertainty ellipse. For this example the average of the 2σ uncertainty radii for all ten sensors is 0.80 m. In a second case we assume that both the location and orientation of sensor $A1$ is known. We get much larger uncertainty ellipses for the sensors. The average 2σ uncertainty radius is 3.28 m. The high tangential uncertainty is due to the DOA measurement uncertainty. We see that it is more desirable to know the locations of two sensors than to know the location and orientation of one sensor. For these examples, an average 2σ uncertainty radius of 1–3 meters is obtained when more than five signal sources are used for calibration.

6. CONCLUSIONS

We have presented a procedure for calibrating the locations and orientations of a network of sensors using source signals that are placed in the scene. We present maximum likelihood solutions to four variations on this problem. We also discuss existence and uniqueness of solutions and algorithms for initializing the nonlinear minimization step in the maximum likelihood estimation. An analytical expression for the sensor location and orientation error covariance matrix is also presented. A maximum likelihood calibration algorithm for the case of partial calibration measurements was also developed. The algorithms require minimal communications from the sensors to a CIP, and computation of the calibration solution takes about ten seconds using Matlab on a personal computer for examples considered.

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