Instrumental Variable Adaptive Array Processing

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An instrumental variable (IV) approach is presented for estimating the weights of an adaptive antenna array. Theoretical analysis of the IV method shows that the antenna gain weights are independent of finitely correlated noise, so that unbiased estimation of signal arrival angles is possible. Only matrix inversions are required to compute the weight estimates. In this sense the IV method provides performance comparable with eigenvector techniques but with lower computational burden.

Both minimal and overdetermined IV estimators are derived. The overdetermined estimators give the same theoretical array weights as minimal estimators do, but yield more accurate weight estimates in real data situations. Simulation results are presented to compare these IV methods with one another and with conventional matrix inversion weight estimators.

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I. INTRODUCTION

Adaptive antenna arrays have become extremely important in a wide range of engineering problems. These antennas adjust their patterns to attenuate unknown or time-varying interference. This property makes them especially useful for protecting communication and radar systems from interference.

An adaptive array forms its output signal as a weighted sum of the individual antenna signals. The main signal processing task in an adaptive antenna system is the determination and updating of the weights used in this weighted sum. Much effort has focused on this problem, and several weight control algorithms have been developed [1–9]. Nearly all of these algorithms are based on the information in the spatial covariance matrix of the antenna signals.

When an adaptive array is used to protect a communication or radar signal from interference, the weights in the array are often chosen to maximize the signal-to-interference-plus-noise ratio (SINR) at the array output. This criterion is generally appropriate because the performance of a signal detector at the array output will ultimately depend on this ratio. Well-known adaptive array weight control techniques, such as those due to Applebaum [17] and Widrow, et al. [20], maximize this ratio.

Adaptive array concepts are also useful in another problem context: estimating the arrival angles of signals. Since an adaptive array forms nulls on interfering signals, the directions of these nulls can be used to estimate signal arrival angles. Angle estimation techniques are of interest in a number of important applications, such as surveillance, remote sensing, sonar, etc.

When an adaptive array is used for angle estimation, maximizing array output SINR is no longer the relevant goal. (In fact, in angle estimation problems, there may not even be a desired signal. There is simply a set of incoming signals, whose arrival angles we wish to know.) Instead, the goal is to obtain the best estimates of the unknown signal arrival angles. In this case one is more interested in quantities such as the bias and variance of the angle estimates.

It is well known that when the weights in an adaptive array are controlled by a conventional technique based on the inverse of the covariance matrix (such as the least mean square (LMS) algorithm), the array pattern that results does not necessarily have nulls pointed precisely at incoming signals. To maximize array output SINR, these algorithms compromise between nulling the interference and the thermal noise at the array output. As a result, nulls are usually not infinitely deep and often do not point exactly at the signals. In other words, angle estimates based on these null directions are biased.

To remove this bias, other techniques, not based simply on the inverse of the covariance matrix, have been of interest. For example, the so-called eigenvector-eigenvalue decompositions of the covariance matrix have been found to yield better performance. These techniques attempt to remove the bias in the angle estimate by eliminating the thermal noise contribution to the covariance matrix before computing the weights. Ideally, without thermal noise, the nulls would be infinitely deep and would point exactly in the correct directions. However, algorithms for obtaining eigenvector-eigenvalue decompositions are iterative in nature; thus, these methods can be computationally burdensome, although improvements have been demonstrated [23].

We present a different weight computation algorithm that may be used for angle estimation in adaptive arrays. The procedure we present is an alternative to the eigenvalue-eigenvector decomposition methods. It is computationally simpler than those methods and is related to the instrumental variable (IV) approach of parameter estimation. These algorithms take advantage not only of spatial correlations in the data, but temporal correlations as well. The use of this extra information can provide lower variances in the weight estimates when autocorrelations must be estimated from data.

The outline of the paper is as follows. Section II presents weight selection algorithms for the case that exact data autocorrelations are available. Section III treats the case where the autocorrelations must be estimated from the data, and derives numerically reliable estimation procedures. In Section IV we present simulation studies. Section V concludes the paper.

II. NARROWBAND PROCESSING GIVEN EXACT CORRELATIONS

The basic narrowband (NB) array processing configuration is shown in Fig. 1. For simplicity we

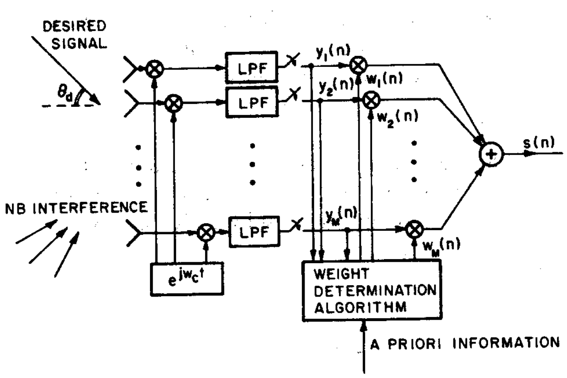

Fig. 1. Basic NB array processing configuration.

assume that the M array elements are collinear and equally spaced. The signals measured on the array may include p narrowband interference (NBI) terms incident at various angles θ_k $1 \leq k \leq p$, and a background noise term $\{v(t)\}$. In some applications there is also a desired signal $d(t)$ incident at an angle θ_d. After sampling and frequency shifting, the antenna data vector $y(n)$ is given by

$$\mathbf{y}(n) = [y_1(n) \ldots y_M(n)]' \tag{1a}$$

where

$$y_m(n) = d(n)e^{j\nu_d m} + \sum_{k=1}^{p} A_k e^{j[\varphi_k + (n\omega_k - m\nu_k)]} + v_m(n) \tag{1b}$$

with

$$\nu_k = \frac{2\pi D \sin\theta_k}{\lambda_k} \tag{1c}$$

and

p	Number of sinusoidal interference signals.
A_k	Amplitude of each interference signal.
φ_k	Initial phase of each interference signal.
ω_k	Temporal frequency of each interference signal.
λ_k	Wavelength of each interference signal.
ν_k	Spatial frequency of each interference signal.
θ_k	Incident angle of each interference signal.
$v_m(n)$	Sampled noise term measured on the mth antenna element.
D	Distance between array elements.

Equations (1) may be written more compactly as

$$\mathbf{y}(n) = d(n)\,\mathbf{s}(\nu_d) + \sum_{k=1}^{p} A_k e^{j[\varphi_k + n\omega_k]}\,\mathbf{s}(\nu_k) + \mathbf{v}(n) \tag{2}$$

with

$$\mathbf{v}(n) = [v_1(n)\ v_2(n) \ldots v_M(n)]'$$

and

$$\mathbf{s}(\alpha) = [e^{-j\alpha} \ldots e^{-jM\alpha}]'.$$

It is necessary to make some assumptions about the noise statistics. We assume $\{v(n)\}$ to be stationary, and correlated for only a finite time, so that

$$R_{vv}(l) \triangleq E\{\mathbf{v}(n)\,\mathbf{v}^H(n-l)\} = 0, \qquad |l| > q. \tag{3}$$

Note that no assumption is made about the correlation of $\{v(n)\}$ for lags less than or equal to q; in particular, $v_i(n)$ and $v_j(n)$ may be correlated for $i \neq j$. Note also that the (more common) white noise assumption is a special case of (3) for q equal to zero. We further assume that the sinusoidal sources are uncorrelated with each other. Thus, we assume that the φ_k are mutually independent random variables distributed on $[-\pi, \pi]$.

The output of the antenna array of Fig. 1 is a weighted sum of the individual antenna signals. Thus,

$$s(n) = \mathbf{w}^H(n)\,\mathbf{y}(n) \tag{4a}$$

where $\mathbf{w}(n)$ is the $M \times 1$ (complex-valued) weight vector

$$\mathbf{w}(n) = [w_1(n), \ldots, w_M(n)]'. \tag{4b}$$

The array processing problem is to choose $\mathbf{w}(n)$ to realize some goal. If a desired signal is present, the goal usually is to separate that signal from the noise. In other applications one wishes to estimate the angles of arrival of the interference signals. We consider these cases separately.

A. No Desired Signal Present

We first consider the case in which no desired signal term is present, that is, when $d(n) = 0$ in (2). For this case, the goal is to choose $\mathbf{w}(n)$ to obtain unbiased estimates of the interference signal arrival angles. We consider this problem first by assuming that the second-order statistics of these signals are known exactly.

From the assumptions on φ_k and $v(n)$, the autocorrelation sequence $R_{yy}(l)$ of $\mathbf{y}(n)$ can be found. From (2) and (3) it follows that

$$R_{yy}(l) \triangleq E\{\mathbf{y}(n)\,\mathbf{y}^H(n-l)\} = R_{ii}(l) + R_{vv}(l) \tag{5a}$$

where

$$R_{ii}(l) = \sum_{k=1}^{p} |A_k|^2 e^{j\omega_k l}\,\mathbf{s}(\nu_k)\mathbf{s}^H(\nu_k). \tag{5b}$$

Let us now consider the selection of the weight vector. The classical solution is found by selecting $\mathbf{w}$ so as to minimize the output noise power, with the constraint that $\mathbf{w} \neq \mathbf{0}$, or

$$\min_{\substack{\mathbf{w}\\ \mathbf{h}^H\mathbf{w}=1}} \mathbf{w}^H R_{yy}(0)\mathbf{w} \tag{6}$$

where $\mathbf{h}$ is some $M \times 1$ constraint vector. Note that by stationarity the optimum weight is independent of the time index n. The solution is (6) is well known [1, 2]:

$$\mathbf{w} = -K[R_{yy}(0)]^{-1}\mathbf{h} \tag{7}$$

where K is a scalar chosen to ensure that $\mathbf{h}^H\mathbf{w} = 1$.

One major drawback of the weight vector in (7) is that, unless $v(n)$ is not present, these weights do not give unbiased estimates of the interference signal arrival angles. As a consequence, two (spatially) closely spaced interferences cannot be resolved. (See Section IV for an example of this phenomenon).

Most research effort at removing this bias has focussed on eigenvalue-eigenvector decompositions of $R_{yy}(0)$ to separate the noise from the NBI. However, such decompositions can be computationally inefficient, as they require iterative algorithms to realize solutions. Moreover, eigenvalue methods may not work well in low SNR or colored noise environments.

Another approach is to obtain the weights using the correlation sequence $R_{yy}(l)$ for $|l| > q$; in this way the noise component $R_{vv}(l)$ is zero in (5a), and only interference signal correlation terms are present. As a result, a weight vector chosen using $R_{yy}(l)$ for $|l| > q$ is not affected by the noise.

In particular, we still use the weight vector defined as any (nonzero) solution to

$$R_{yy}(l)\mathbf{w} = \mathbf{0}. \tag{8}$$

This weight choice is similar in philosophy to IV estimators in the time series literature [11, 12, 15], so we call the weight vector in (8) an IV weight vector. We claim the IV weight vector completely nulls the interference signals, even when noise is present. Moreover, this weight vector is the same as the classical one in (7) when there is no noise term ($v(n) = 0$) there. To show this, note that for any $|l| > q$, $R_{yy}(l) = R_{ii}(l)$, and it can be written as (cf. (5b))

$$R_{yy}(l) = S\,T(l)S^H \tag{9a}$$

where

$$S = \begin{bmatrix} 1 & \cdots & 1 \\ e^{-j\nu_1} & \cdots & e^{-j\nu_p} \\ \vdots & \vdots & \vdots \\ e^{-j(M-1)\nu_1} & \cdots & e^{-j(M-1)\nu_p} \end{bmatrix}_{M\times p} \times \begin{bmatrix} e^{-j\nu_1} & & 0 \\ & \ddots & \\ 0 & & e^{-j\nu_p} \end{bmatrix}_{p\times p} \tag{9b}$$

$$T(l) = \begin{bmatrix} |A_1|^2 e^{j\omega_1 l} & & 0 \\ & \ddots & \\ 0 & & |A_p|^2 e^{j\omega_p l} \end{bmatrix} \tag{9c}$$

The first matrix in (9b) is p columns of a Vandermonde matrix. It is well known that these columns are linearly independent (if the ν_i are distinct). The second matrix is nonsingular, so it follows that $\operatorname{rank}(S) = p$. Moreover, $T(l)$ in (9c) is nonsingular, so it follows that $R_{yy}(l)$ has rank p. Thus, $R_{yy}(l)$ has $M-p$ eigenvalues equal to zero.

Let $\mathbf{w}$ be a solution to (8). Then

$$S\,T(l)S^H\mathbf{w} = \mathbf{0}. \tag{10}$$

Since S has full column rank, it has a pseudoinverse $S^\dagger = (S^HS)^{-1}S^H$ such that $S^\dagger S = I_p$, the $p \times p$ identity matrix [22]. Premultiplying (10) by $T^{-1}(l)\,S^\dagger$ gives

$$S^H\mathbf{w} = \mathbf{0} \tag{11a}$$

or

$$\mathbf{s}^H(\nu_i)\mathbf{w} = 0, \qquad i = 1, 2, \ldots, p. \tag{11b}$$

Equation (11b) states that the response of the array at each interference arrival angle θ_i is zero.

It immediately follows from (11) that the array output $s(n)$ has no interference component. From (2)–(4) we have

$$\begin{aligned} s(n) &= \mathbf{w}^H\left\{\sum_{k=1}^{p}\mathbf{s}(\nu_k)A_k e^{j(\varphi_k+n\omega_k)} + \mathbf{v}(n)\right\} \\ &= \mathbf{w}^H\mathbf{v}(n). \end{aligned} \tag{12}$$

Thus, no interference component is present at the output of the array.

It turns out that the IV weight vector from (8) (even with noise present) is the same solution as that of the classical minimization problem when no noise is present. If $R_{vv}(0) = 0$, then

$$R_{yy}(0) = S \begin{bmatrix} |A_1|^2 & & 0 \\ & \ddots & \\ 0 & & |A_p|^2 \end{bmatrix} S^H. \tag{13}$$

This matrix has $M-p$ zero eigenvalues, so any weight vector $\mathbf{w}$ that minimizes (6) is an eigenvector corresponding to a zero eigenvalue of (13). Thus, the set of weight vector solutions is an $(M-p)$–dimensional subspace. Since any IV weight vector has the property (11), it follows from (11) and (13) that

$$R_{yy}(0)\,\mathbf{w} = \mathbf{0}$$

for any IV weight vector. Thus, the IV weight vector (obtained for nonzero noise) is the same subspace as the classical weight vector for the noiseless case. (Note that rank $(R_{yy}(l)) = p$, so the solution to (8) is not unique; the set of solutions is the same $M-p$–dimensional subspace as in the noiseless classical case.) As the noise is increased, the IV null locations do not change, but the classical ones do. This effect is shown in Fig. 2. It can

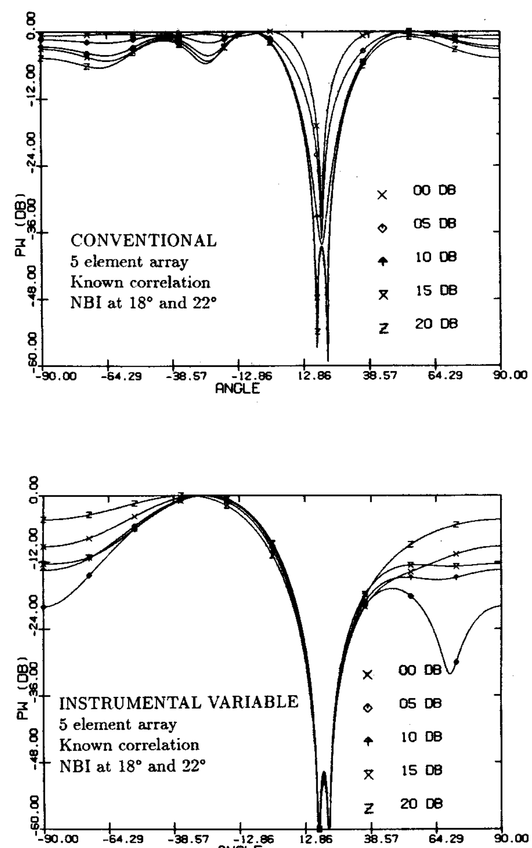

Fig. 2. Conventional and IV array gains parameterized on INR.

be seen in this figure that as the noise level increases, the classical weight vector is not able to resolve the two closely spaced interference signals; however, the IV weights continue to provide perfect nulling.

Note that the above derivation remains valid for any choice of l satisfying $|l| > q$. Therefore, perfect interference nulling is obtained if $\mathbf{w}$ is a solution to the weighted concatenation of $R_{yy}(l)$ matrices as

$$\bar{R}\mathbf{w} = \mathbf{0} \tag{14a}$$

where

$$\bar{R} \triangleq \begin{bmatrix} \alpha_{q+1} R_{yy}(q+1) \\ \alpha_{q+2} R_{yy}(q+2) \\ \vdots \\ \alpha_L R_{yy}(L) \end{bmatrix} \tag{14b}$$

and where $\alpha_l \geq 0$. The proof of this follows as before; $\bar{R}$ can be written as

$$\bar{R} = \begin{bmatrix} ST(q+1)S^H \\ \vdots \\ ST(L)S^H \end{bmatrix}$$

Premultiplying by the matrix $\text{diag}[T^{-1}(q+1)S^{\dagger}, \ldots, T^{-1}(L)S^{\dagger}]$ gives $S^H\mathbf{w} = \mathbf{0}$ as before.

To summarize, if $\mathbf{w}$ is any solution to (8) or (14), then the array output $s(n)$ has no interference component, as shown in (12). Moreover, this weight vector (which is obtained in the presence of noise) is also a solution to the classical minimization problem for the noiseless case. Thus, this IV weight vector effectively eliminates the bias due to noise, and provides perfect nulling of interference signals.

Equation (14) can also be interpreted as a weight selection which whitens the array output $s(n)$. Recall that the conventional weight selection of (7) renders $s(n)$ as white as possible [1, 5]. We can attempt to whiten $s(n)$ by ensuring that

$$r_{ss}(k) \triangleq E\{s(n)\,s^*(n-k)\} = 0, \qquad k > 0.$$

In particular, we can effect this by solving

$$\min_{\substack{\mathbf{w} \\ \mathbf{h}^H\mathbf{w}=1}} \{\alpha_1 |r_{ss}(1)|^2 + \cdots + \alpha_L |r_{ss}(L)|^2\}$$

for some given nonnegative weighting parameters $\alpha_1, \ldots, \alpha_L$. If $\mathbf{v}(n)$ is white noise (i.e., $q=0$ in (3)) then from (3)–(5) we have

$$r_{ss}(l) = \mathbf{w}^H R_{yy}(l)\mathbf{w}, \qquad l > 0$$
$$= \mathbf{w}^H R_{ii}(l)\mathbf{w}, \qquad l > 0.$$

Therefore, the above minimization becomes

$$\min_{\substack{\mathbf{w} \\ \mathbf{h}^H\mathbf{w}=1}} \left\{ \sum_{l=1}^{L} |\mathbf{w}^H \alpha_l R_{ii}(l)\mathbf{w}|^2 \right\}.$$

This minimum is zero, and is found for any **w** satisfying (14).

Finally, note that we can combine the IV and the conventional estimate simply by appending the first $q + 1$ autocorrelation terms to $\bar{R}$ in (14) as

$$R = \begin{bmatrix} \alpha_0 R_{yy}(0) \\ \alpha_1 R_{yy}(1) \\ \vdots \\ \alpha_L R_{yy}(L) \end{bmatrix}. \tag{15}$$

We can in general not find a **w** so that $R\mathbf{w} = \mathbf{0}$, so we choose to find instead the minimum mean square error solution. Thus, the generalized minimization problem is

$$\min_{\substack{\mathbf{w} \\ \mathbf{h}^H\mathbf{w}=1}} \; [R\mathbf{w}]^H[R\mathbf{w}]. \tag{16}$$

If $\alpha_l = 0$ for $l \geq 1$, (16) reduces to the classical weight selection of (6). If $\alpha_l = 0$ for $0 \leq l \leq q$, then (16) yields the pure IV estimate. Choices of the $\{\alpha_l\}$ coefficients between these two extremes are compromises that retain a desired balance between the properties of each, such as a tradeoff between perfect nulls and minimum output power for example.

The solution to (16) is straightforward. Assume the first element of **h** is nonzero. (Since $\mathbf{h} \neq \mathbf{0}$, the columns of R and the elements of **h** and **w** can be compatibly rearranged so that the first element of **h** is nonzero). Now partition R and **w** as

$$R = [\underbrace{\boldsymbol{\rho}}_{1} \quad \underbrace{P}_{M-1}] \tag{17a}$$

$$\mathbf{w} = [\underbrace{1}_{1} \quad \underbrace{\tilde{\mathbf{w}}}_{M-1}]' \, w_1 \tag{17b}$$

then (16) can be solved by

$$\tilde{\mathbf{w}} = -[P^H P]^{\#} P^H \boldsymbol{\rho} \tag{17c}$$

where # denotes a generalized inverse. Then w_1 is chosen to make $\mathbf{h}^H\mathbf{w} = 1$.

B. Desired Signal Present

There are many array processing problems in which a desired signal is present. The processing goal is to recover this signal from the noise and interference. Generally, this goal is realized by choosing the weights in the array to maximize the SINR at the output. This criterion is appropriate in most cases, although it is not always the "best" criterion. For example, in digital communications the objective is to minimize the bit error probability at the array output; for some types of detectors, maximizing the SINR does not minimize the bit error probability. In HF communication systems, postprocessing to remove channel fading and dispersion effects is often required, but this processing is very sensitive to NB interference [10]. Finally, the weights that maximize SINR are derived under the assumption of signal stationarity; in time-varying environments (such as in the presence of pulsed jammers), these algorithms may no longer maximize the SINR. For these and other situations, it may be more desirable to select the array weights to completely attenuate the interference signals [21]. Thus, it is useful to generalize the IV method of the previous section to the case that $d(n)$ is present.

When a desired signal is present, two standard types of information are generally available: either the signal angle θ_d is known, or the desired signal itself is assumed known. These cases are discussed below.

1) *Known Signal Angle*: If θ_d is known, then the weight vector **w** must be constrained so that the antenna gain is some constant (say 1) at the incident angle θ_d. This can be accomplished by using the constraint vector

$$\mathbf{h} = [1 \; e^{-j\nu_d} \ldots e^{-j(M-1)\nu_d}]'$$

in (16) where ν_d is related to θ_d by (1c). Thus, the weight vector for this case is also given by (17).

2) *Known Desired Signal*: If some estimate $d(n)$ of the desired signal is known, then it may be incorporated into the weight selection procedure. In this case $d(n)$ is subtracted from the array output $s(n)$ to generate an error signal $e(n)$:

$$e(n) = s(n) - d(n) = \mathbf{w}^H\mathbf{y}(n) - d(n).$$

The optimal IV weight is the one that minimizes

$$\min_{\mathbf{w}} \left[\sum_{l=q+1}^{L} \alpha_l |E\{e(n)\, e^*(n-l)\}|^2 \right]$$

$$= \min_{\mathbf{w}} \left[\sum_{l=q+1}^{L} \alpha_l |\mathbf{w}^H R_{yy}(l)\mathbf{w} - \mathbf{r}_{yd}^H(-l)\mathbf{w} - \mathbf{w}^H\mathbf{r}_{yd}(l) + r_{dd}(l)|^2 \right]$$

where

$$\mathbf{r}_{yd}(l) \triangleq E\{\mathbf{y}(n)\, d^*(n-l)\} \quad (M \times 1).$$

This minimum is zero, and realized for **w** satisfying

$$\bar{R}\mathbf{w} = \bar{\mathbf{r}} \tag{18a}$$

where $\bar{R}$ is given in (14) and

$$\bar{\mathbf{r}} = \begin{bmatrix} \alpha_{q+1}\, \mathbf{r}_{yd}(q+1) \\ \vdots \\ \alpha_L \mathbf{r}_{yd}(L) \end{bmatrix}. \tag{18b}$$

As before, we can extend this method to include the first $q + 1$ autocorrelations to get

$$\min_{\mathbf{w}} [R\mathbf{w} - \mathbf{r}]^H [R\mathbf{w} - \mathbf{r}] \tag{19a}$$

where R is given by (15) and

$$\mathbf{r} = \begin{bmatrix} \alpha_0 \mathbf{r}_{yd}(0) \\ \vdots \\ \alpha_L \mathbf{r}_{yd}(L) \end{bmatrix}. \tag{19b}$$

Equation (19) is solved by

$$\mathbf{w} = [R^H R]^* R^H \mathbf{r}. \tag{20}$$

III. WEIGHT ESTIMATION FROM DATA

In practice, the array data autocorrelation are not known, but must be estimated from data. This section discusses autocorrelation estimators, and derives update formulas for the corresponding weight vector computations.

A. Some Autocorrelation Estimators

Consider first the estimation of the $M \times M$ autocorrelation matrix $R(l)$. A standard estimate is given by

$$R_N^1(l) = \sum_{n=l+1}^{N} \lambda^{N-n}\, \mathbf{y}(n)\mathbf{y}^H(n-l). \tag{21}$$

The subscript N is used to indicate an estimate based on $\{\mathbf{y}(1), \ldots, \mathbf{y}(N)\}$. An exponential forgetting factor λ $(0 < \lambda \leq 1)$ is included in (21) to enable the tracking of time variations in the data. A larger value of λ yields autocorrelation estimates with lower variances, but also causes time variations to be tracked more slowly.

Another standard autocorrelation estimator is the sliding window estimate given by

$$R_N^2(l) = \sum_{n=N-W+1}^{N} \mathbf{y}(n)\mathbf{y}^H(n-l). \tag{22}$$

Again, W is chosen in a particular application to provide a balance between lower variance (with larger W) and better tracking ability (with smaller W). Time updates for these two estimators are readily found to be

$$R_N^1(l) = \lambda R_{N-1}^1(l) + \mathbf{y}(N)\mathbf{y}^H(N-l) \tag{23}$$

$$R_N^2(l) = R_{N-1}^2(l) + \mathbf{y}(N)\mathbf{y}^H(N-l) - \mathbf{y}(N-W)\mathbf{y}^H(N-W-l). \tag{24}$$

In the case of collinear, equally spaced antennas, the autocorrelation estimates can be improved. In this case, the elements along any diagonal of $R(l)$ are equal. The coresonding elements along a diagonal of $R_N^1(l)$ or $R_N^2(l)$ are different estimates of the same quantity. An improved estimate (i.e., one with an asymptotically lower variance) results by averaging these estimates. To this end, define the Toeplitz matrix

$$R_N^K(l) = \begin{bmatrix} r_N^K(l,0) & \cdots & r_N^K(l,-M+1) \\ \vdots & \ddots & \vdots \\ r_N^K(l,M-1) & \cdots & r_N^K(l,0) \end{bmatrix} \tag{25}$$

where $K = 3$ or 4. The element $r_N^3(l, s)$ is estimated by averaging the corresponding diagonal elements in $R_N^1(l)$. Similarly, $r_N^4(l, s)$ is obtained from $R_N^2(l)$. Thus, we have

$$\begin{aligned} r_N^3(l,s) &= \sum_{n=l+1}^{N} \lambda^{N-n} \left[\frac{1}{M-s}\sum_{i=1}^{M-s} y_{i+s}(n)y_i^*(n-l)\right], \quad s \geq 0 \\ &= \sum_{n=l+1}^{N} \lambda^{N-n} \left[\frac{1}{M-|s|}\sum_{i=|s|+1}^{M} y_{i+s}(n)y_i^*(n-l)\right], \quad s < 0 \end{aligned} \tag{26}$$

$$\begin{aligned} r_N^4(l,s) &= \sum_{n=N-W+1}^{N} \left[\frac{1}{M-s}\sum_{i=1}^{M-s} y_{i+s}(n)y_i^*(n-l)\right], \quad s \geq 0 \\ &= \sum_{n=N-W+1}^{N} \left[\frac{1}{M-|s|}\sum_{i=s+1}^{M} y_{i+s}(n)y_i^*(n-l)\right], \quad s < 0. \end{aligned} \tag{27}$$

Thus, we have given four estimates of $R_{yy}(l)$. Two estimates give Toeplitz matrices, while the other two do not. Because the elements of the Toeplitz matrix estimates have lower asymptotic variances than do the non-Toeplitz ones, they are preferred. Of course, in applications where the array elements are not equally spaced and collinear, the Toeplitz estimates cannot be used. All four estimates incorporate a parameter to enable tracking of time-varying statistics. This parameter is chosen to trade off between tracking ability and variance of the autocorrelation estimates.

For applications in which a desired signal is given, an estimate of $\mathbf{r}$ (or estimates of $\mathbf{r}_{yd}(l)$) in (19b) is needed. Two possible choices are the exponentially weighted and finite window estimates of $\mathbf{r}_{yd}(l)$, given respectively by

$$\mathbf{r}_{yd}(l) = \sum_{n=l+1}^{N} \lambda^{N-n}\, \mathbf{y}(n)d^*(n-l), \quad l \geq 0 \tag{28a}$$

and

$$\mathbf{r}_{yd}(l) = \sum_{n=N-W+1}^{N} \mathbf{y}(n)d^*(n-l), \quad l \geq 0. \tag{28b}$$

B. Weight Estimation

Estimating the array weights from data in principle consists of first estimating the autocorrelations from the data and then substituting these sample correlations into (17) or (20). However, numerical difficulties can arise. For example, If $\alpha_0 = \cdots = \alpha_q = 0$ in (17) or (20), then R has rank p. Even if $\alpha_0, \ldots, \alpha_q$ are not all zero, $P^H P$ or $R^H R$ can be ill conditioned. Estimates of these matrices will also be ill conditioned, and care must be taken in inverting them.

One way to avoid these problems is to solve for the weights recursively in order. To this end consider the solution of (16), and define

$$R = [\underbrace{\boldsymbol{\rho}_1}_{1} \mid \underbrace{P_m}_{m} \mid \underbrace{Q_m}_{M-m-1}] \tag{29a}$$

$$\mathbf{w}_m = [1 \mid \tilde{\mathbf{w}} \mid 0]' \cdot w_{lm} \tag{29b}$$

$$\tilde{\mathbf{w}}_m = -[P_m^H P_m]^{-1} P_m^H \boldsymbol{\rho}_1. \tag{29c}$$

Equation (29c) can be updated in order by noting that

$$[P_{m+1}^H P_{m+1}] = \begin{bmatrix} P_m^H P_m & P_m^H \boldsymbol{\rho}_{m+1} \\ \boldsymbol{\rho}_{m+1}^H P_m & \boldsymbol{\rho}_{m+1}^H \boldsymbol{\rho}_{m+1} \end{bmatrix} \tag{30a}$$

$$P_{m+1}^H \boldsymbol{\rho}_1 = [P_m^H \boldsymbol{\rho}_1 \quad \boldsymbol{\rho}_{m+1}^H \boldsymbol{\rho}_1] \tag{30b}$$

where $\boldsymbol{\rho}_{m+1}$ is the $(m+1)$st column of R. The inverse of (30a) can be directly updated. Define

$$[P_{m+1}^H P_{m+1}]^{-1} = \begin{bmatrix} A & \mathbf{b} \\ \mathbf{b}^H & d \end{bmatrix}. \tag{31}$$

Then the components of the inverse matrix are given by [13]

$$d = \{\boldsymbol{\rho}_{m+1}^H \boldsymbol{\rho}_{m+1} - \boldsymbol{\rho}_{m+1}^H P_m [P_m^H P_m]^{-1} P_m^H \boldsymbol{\rho}_{m+1}\}^{-1} \tag{32a}$$

$$\mathbf{b} = -\{[P_m^H P_m]^{-1} P_m^H \boldsymbol{\rho}_{m+1}\} d \tag{32b}$$

$$A = [P_m^H P_m]^{-1} + \mathbf{b}\,\mathbf{b}^H d. \tag{32c}$$

Singularity or near singularity can be detected by the size of d. If P has rank m, then $\mathbf{w}_m$ is a solution to (17). If P becomes nearly singular, then $\mathbf{w}_m$ can be used as a numerically stable approximant of $\mathbf{w}$.

The order update recursions are equally valid when R is replaced by an estimate $\hat{R}$. Moreover, we note that when R or $\hat{R}$ is Toeplitz, the matrix (30a) has displacement rank 4, so that computationally efficient Levinson-like algorithms can be used to (recursively in order) invert it [18].

An important special case of this estimator is obtained by setting $\alpha_{q+1} = 1$ and all other $\alpha_i = 0$. In this case (16) becomes

$$R(q+1)\mathbf{w} = \mathbf{0} \tag{33}$$

If we partition $R(q+1)$ as

$$R(q+1) = \begin{bmatrix} \overbrace{x}^{1} & \overbrace{x}^{m-1} & \overbrace{x}^{M-m} \\ \mathbf{r}_m & R_m & x \\ x & x & x \end{bmatrix} \begin{matrix} \}\,1 \\ \}\,m-1 \\ \}\,M-m \end{matrix} \tag{34}$$

and again partition $\mathbf{w}$ as in (29b), then $\tilde{\mathbf{w}}_m$ can be found by

$$\tilde{\mathbf{w}}_m = -R_m^{-1}\,\mathbf{r}_m \tag{35}$$

If R_m is Toeplitz (which is the case if a Toeplitz estimate is used for R), then (35) can be efficiently solved, recursively in order, by using the Levinson algorithm. If R_m is not Toeplitz, then R_m^{-1} can still be recursively inverted. Note that R_m is of the form

$$R_m = \begin{bmatrix} R_{m-1} & \mathbf{b} \\ \mathbf{b}^H & c \end{bmatrix} \tag{36}$$

where $\mathbf{b}$ and c are $(m-2) \times 1$ and 1×1 vectors, respectively. If we define

$$R_m^{-1} = \begin{bmatrix} W & \mathbf{x} \\ \mathbf{x}^H & z \end{bmatrix} \tag{37a}$$

then

$$z = \{c - \mathbf{b}^H R_{m-1}^{-1} \mathbf{b}\}^{-1} \tag{37b}$$

$$\mathbf{x} = -R_{m-1}^{-1}\,\mathbf{b}\,z \tag{37c}$$

$$W = R_{m-1}^{-1} + \mathbf{x}\,\mathbf{x}^H z \tag{37d}$$

To summarize (29) and (35) are the two sets of order-recursive weight estimation equations. For the known-desired-signal case, these equations can be modified in an obvious way.

Finally, the desired weight estimates are obtained from either (29) or (35) with estimated autocorrelations replacing exact ones. We refer to estimates obtained from (29) as the overdetermined instrumental variable (OIV) estimates, and those obtained from (33) as the minimal instrumental variable (MIV) estimates.

It can be seen that (35) requires fewer computations to implement than (29) does. Moreover, in the ideal (known autocorrelation) case, the weight vector obtained from (35) is the same as that obtained from (29). It is natural to ask why (29) should be used at all. The answer is that for a given finite number of data points, or for $\lambda < 1$ in the autocorrelation estimates, more accurate weight vector estimates are often realized by using an overdetermined system of equations. Heuristic justification is that more information than noise is added to the weight estimate equation by including $R(q+2), \ldots, R(L)$, so the estimates are more accurate. This phenomenon has been both experimentally and theoretically substantiated in the time series literature [12, 15, 16]. The improvement in weight accuracy is most pronounced when $|R(k)|$ decreases slowly with k (as is the case here). Thus, there is a tradeoff between the computationally simpler MIV estimator (35) and the more accurate OIV estimator (29).

IV. EXAMPLES

This section presents numerical simulations that illustrate the performance of IV weight estimation methods, when compared with conventional weight estimators. We consider both the purely conventional estimators in which only $\alpha(0)$ is nonzero and equal 1 and purely IV estimators in which $\alpha(0) = 0$ and $\alpha(1) = \cdots$

$= \alpha(L) = 1$. Mixed estimates have properties somewhere between these two.

For these examples, the antenna signals $y_i(n)$ were generated using equation (1b) with the following parameters: $d(n) = 0$, $p = 2$, $\omega_1 = 0$, $\omega_2 = 0.05\,\pi$, $\theta_1 = 18°$, $\theta_2 = 22°$, $D = \lambda/2$. The interference amplitudes were equal, and chosen to give interference-to-noise ratio (INR) values as shown on the graphs. The frequencies ω_1 and ω_2 are slightly different, in order to decorrelate the interference signals in the simulations. in addition, each noise signal $v_m(n)$ was zero mean, Gaussian white noise; thus $R_{vv}(l) = I_m\delta(l)$ and $q = 0$ for this case.

Fig. 2 shows ideal antenna patterns for five-element arrays using the conventional weight estimator and the IV estimator. Patterns are shown for varying INR. Note that the conventional weight estimator is not able to resolve the two sources at all INR levels; the IV method not only resolves the interference sources, it also completely attenuates them. (Fig. 2 is on page 195.)

For Figs. 3–5, patterns are obtained from estimated weight vectors. The correlation matrix R was estimated using either (21) for the non-Toeplitz estimates or (25)–(26) for the Toeplitz estimates.

Fig. 3 compares array gain patterns estimated from data. To facilitate comparison with the theoretical limit,

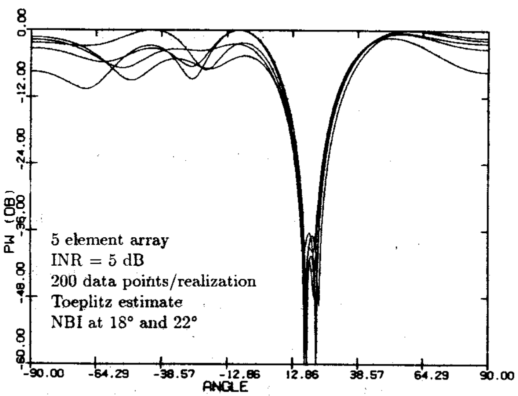

Fig. 3(a). OIV array gains for Toeplitz correlation estimate, $L = 10$.

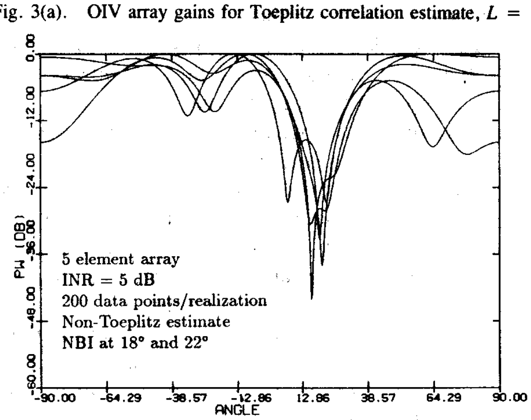

Fig. 3(b). OIV array gains for non-Toeplitz correlation estimate, $L = 10$.

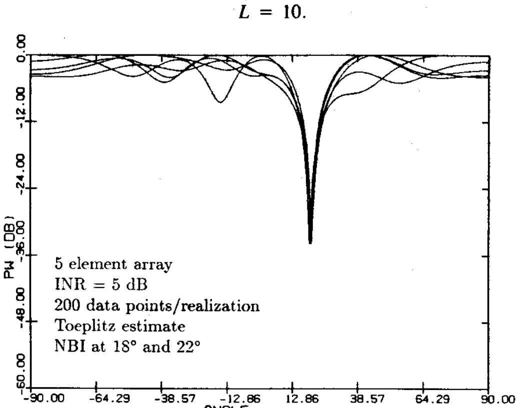

Fig. 3(c). Conventional array gains for Toeplitz estimate.

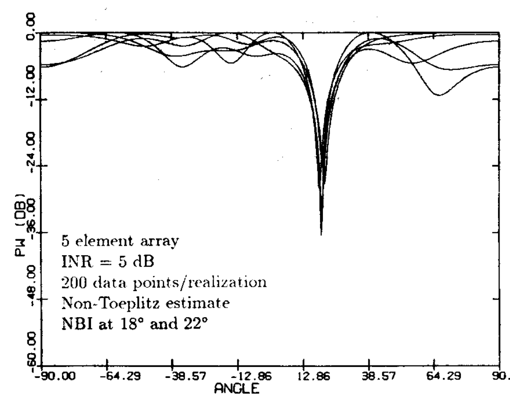

Fig. 3(d). Conventional array gains for non-Toeplitz estimate.

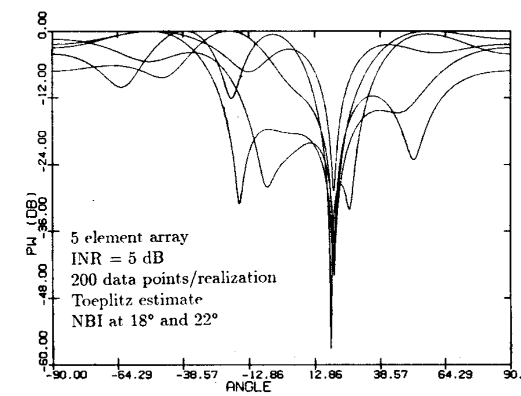

Fig. 3(e). MIV array gains for Toeplitz correlation estimate, $L = 1$.

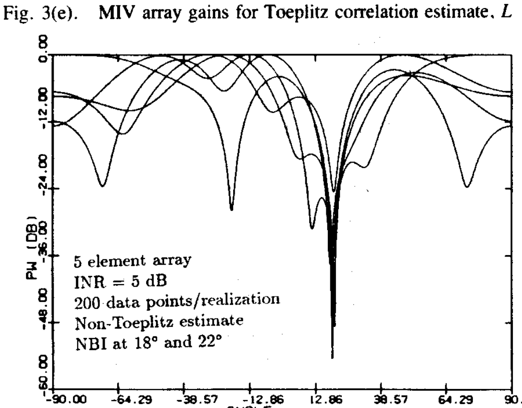

Fig. 3(f). MIV array gains for non-Toeplitz correlation estimate, $L = 1$.

we use the above example, i.e., two interference sources, one at 18° and the other at 22°. To provide some idea of the statistical variation of these patterns, five independent estimates are superimposed in the figure. Each estimate was obtained using 200 data vectors, and in all cases the exponentially weighted autocorrelation estimates were used with $\lambda = 1$ (i.e., no forgetting). Fig. 3 shows the estimates obtained for a five-element array. We see that the OIV method with L equal to 10 exhibits lower variance than the MIV or conventional methods. In fact the conventional estimates do not resolve the interference sources at all; this is expected even for an infinite number of data points (i.e., known correlations) as shown in Fig. 2. Note that the conventional method is actually a minimal method in that the number of equations is equal to the number of unknowns. Moreover, lower variances result from using Toeplitz autocorrelation estimates rather than non-Toeplitz ones. These results are in keeping with the heuristic arguments provided earlier.

Fig. 4 shows array patterns for the OIV method for array lengths of 3, 5, and 7 based on single realizations,

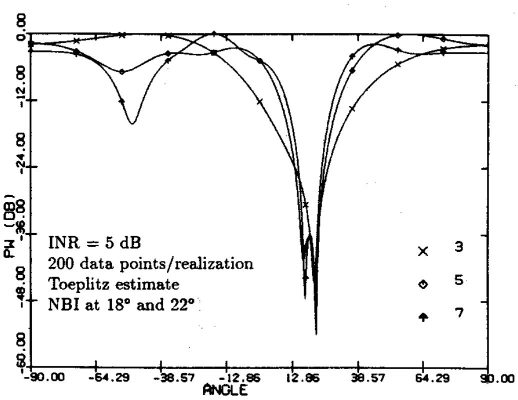

Fig. 4. OIV array gains for 3, 5, and 7 element arrays, $L = 10$.

and Toeplitz autocorrelation estimates. This experiment shows that more accurate pattern estimates are sometimes obtained by increasing the number of array elements. However, such an increase is undesirable because of increase in hardware, computational burden, and spurious attenuation peaks. This phenomenon has been noted also in the context of spectral estimation [12, 19], but the reason for this behavior is not well understood.

For 200 data points, 5 dB INRs, and very closely spaced interference angles, we are operating close to the limits of performance. A marked increase in resolving power consistency is shown in Fig. 5, as compared with Fig. 3(a), due to an increase in INR from 5 to 10 dB. We note that the MIV and OIV methods perform more alike when INRs and/or data record lengths are increased.

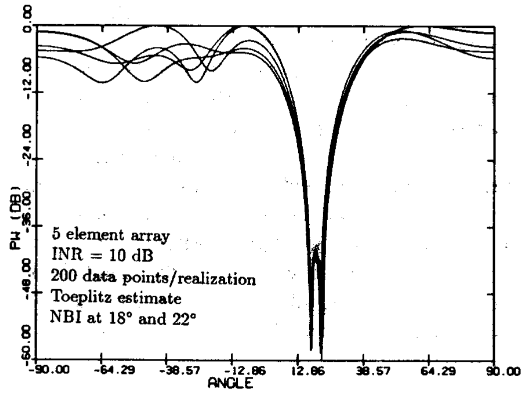

Fig. 5. OIV array gains for Toeplitz estimate and 10 dB INR, $L = 10$.

V. CONCLUSIONS

We have presented a class of algorithms for estimating the weights in an adaptive antenna array system. This class is based on the well known IV method of parameter estimation in time series analysis. The ideal weights obtained using this method are not affected by finitely correlated noise in the antenna signals, and are thus comparable to ideal weights obtained by eigendecomposition methods. On the other hand, the IV methods require only a matrix inversion, so they are computationally more appealing.

We have also derived overdetermined versions of the IV algorithms that, although giving the same ideal weights, can provide weight estimates with lower variance than their corresponding minimal algorithms. These overdetermined algorithms require more computation than the minimal ones, but still do not need iterative solution procedures. They should prove useful in environments in which it is essential to obtain accurate weight estimates from a small data sample, as is the case in raidly time-varying signal environments.

The conventional, MIV, and OIV methods were then combined into a single general estimator. Selection of certain coefficients can effect a particular algorithm with a desired blend of these three methods.

Finally, we have presented some simulation examples to test the effectiveness of these algorithms. In these examples it is seen that IV methods are able to resolve closely spaced interference sources when conventional matrix inversion techniques cannot. It is also shown that overdetermined methods are capable of providing weight estimates with lower variances than those of minimal methods.

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REFERENCES

[1] Monzingo, R.A. and Miller, T.W. (1980)
Introduction to Adaptive Arrays.
New York: Wiley, 1980.

[2] Hudson, J.E. (1981)
Adaptive Array Principles.
London: Peter Peregrinus, 1981.

[3] Cadzow, J.A., and Bronez, T.P. (1981)
An algebraic approach to super-resolution adaptive array processing.
In *Proceedings of the 1981 International Conference on Acoustics, Speech, and Signal Processing*, Atlanta, Ga., Mar. 1981, pp. 302–305.

[4] Moses, R.L. (1983)
Combined time-space filtering for HF antenna array systems.
Final Report, SCEEE Summer Faculty Research Program, Sept. 1983.

[5] Gabriel, W.F. (1980)
Spectral analysis and adaptive array superresolution techniques.
Proceedings of the IEEE, 68, 6 (June 1980), 654–666.

[6] Gabriel, W.F. (1976)
Adaptive arrays—An introduction.
Proceedings of the IEEE, 64, 2 (Feb. 1976), 259–272.

[7] Sharman, K.C., and Durrani, T.S. (1983)
Spatial lattice filter for high-resolution spectral analysis of array data.
Proceedings of the IEE, 130, Pt. F, 3 (Apr. 1983), 279–287.

[8] Johnson, D.H., and DeGraaf, S.R. (1982)
Improving the resolution of bearing in passive sonar arrays by eigenvalue analysis.
IEEE Proceedings of Acoustics, Speech, and Signal Processing, ASSP-30, 4 (Aug. 1982), 638–647.

[9] Gatlin, B.J. (1983)
Superresolution of multiple noise sources in antenna beam.
IEEE Transactions on Antennas and Propagation, AP-31, 3 (May 1983), 456–462.

[10] Sylvania Systems Group (1981)
HF Wideband Modem Program.
Technical Report CDRL-A008, Rome Air Development Center, Griffiss Air Force Base, Rome, N.Y., Nov. 1981.

[11] Eykhoff, P. (1974)
System Identification.
New York: Wiley, 1974.

[12] Moses, R.L. (1984)
Design and analysis of fast recursive ARMA spectral estimators.
Ph.D. dissertation, Virginia Polytechnic Institute and State University, Blacksburg, Va., June 1984.

[13] Kailath, T. (1980)
Linear Systems.
Englewood Cliffs, N.J.: Prentice-Hall, 1980.

[14] Markel, J.D., and Gray, A.H. (1974)
Linear Prediction of Speech.
New York: Springer Verlag, 1974.

[15] Söderström, T. and Stoica, P. (1983)
Instrumental Varible Methods for System Identification.
New York: Springer Verlag, 1983.

[16] Stoica, P., and Söderström, T. (1983)
Optimal instrumental variable estimation and approximate implementation.
IEEE Transactions on Automatic Control, AC-28, 7 (July 1983), 757–772.

[17] Applebaum, S.P. (1976)
Adaptive arrays.
IEEE Transactions on Antennas and Propagation, AP-24, 9 (Sept. 1976), 585–598.

[18] Friedlander, B., et al (1979)
New inversion formula for matrices classified in terms of their distance from Toeplitz matrices.
Linear Algebra and Applications, 27 (1979), 31–60.

[19] Cadzow, J.A., and Moses, R.L. (1981)
A superresolution methods of ARMA spectral estimation.
In *Proceedings of the 1981 International Conference on Speech and Signal Processing*, Atlanta, Ga., Mar. 1981, pp. 484–487.

[20] Widrow, B., Mantey, P.E., Griffiths, L.J., and Goode, B.B. (1967)
Adaptive antenna systems.
Proceedings of the IEEE, 55, 12 (Dec. 1967), 2143–2159.

[21] Compton, R.T., Jr. (1982)
The effect of a pulsed interference signal on an adaptive array.
IEEE Transactions on Aerospace and Electronic Systems, AES-18, 3 (May 1982), 297–309.

[22] Luenberger, D. (1969)
Optimization by Vector Space Methods.
New York: Wiley, 1969.

[23] Beex, A.A. (1986)
Fast recursive/iterative Toeplitz eigenspace decomposition.
In *Proceedings of the European Signal Processing Conference* (EUSIPCO-86), The Hague, The Netherlands, Sept. 2–5, 1986, pp. 1001–1004.

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