BAYESIAN POINT SET MATCHING OF SCATTERING FEATURES WITH APPLICATION TO OBJECT RECOGNITION

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ABSTRACT

We present a statistical decision approach for the point set matching of unordered feature sets. Both feature sets have associated uncertainties, and the number of elements in each set may be different. Computation of the match likelihood requires a correspondence between feature sets; we solve the correspondence problem in polynomial time using the Hungarian algorithm. We also consider the problem of matching when there is an unknown translation between the point sets. We present Bayes match solutions for both the deterministic and the random translation cases. Finally, we apply this matching method to the problem of synthetic aperture radar target classification from scattering center feature sets.

1. INTRODUCTION

In this paper we develop a statistical approach to the point set matching problem that is encountered frequently in signal and image processing applications. Example applications include image-based object classification problems, where a set of features (such as locations of local maxima in an image along with attributes that characterize, say, the shape or amplitude of each maximum) is used as a low-dimensional surrogate to the original image for use in likelihood computation.

We consider the following problem. A feature set Y is estimated from a test image and compared to a catalog of feature sets X^k corresponding to hypotheses H_k. We assume uncertainty in both the estimated feature set Y and each hypothesis feature set X^k; the uncertainty is encoded using a statistical model. The signal processing objective is to compute the posterior likelihood of each hypothesis H_k by comparing the feature sets X^k and Y. In the sequel we drop the script k on X^k and H_k and consider a generic hypothesis H with corresponding point set X.

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A particular application for point set matching is radar target classification from scattering features. Each feature set Y or X^k is an unordered set of scattering centers; typically the X^k are obtained via training or synthetic scattering prediction methods, and Y is obtained by applying a feature extractor to a measured SAR image chip that has passed a prescreening stage. Each element of X^k or Y is a vector of scattering center feature attributes whose elements typically characterize the location of the scattering center in the image plane, its amplitude, and possibly also polarization or shape attributes [1]. We use the term "set" to denote an unordered list of features and "vector" to denote an ordered list of elements. Thus, we have unordered sets of scattering features, but the attributes characterizing the feature (location, amplitude, etc.) are ordered.

Matching of inexact feature sets, and the related inexact graph matching problem, have been considered in [2]–[9]. Boyer and Kak [3] develop a structural matching technique that includes a conditional information measure to penalize attribute deviations in the match. Bayesian structural matching techniques [6, 7] provide subgraph matches and by considering graph edit distances. Matching based on mutual information is considered in [8], and matching using a Hausdorff distance metric under rigid body transformations is addressed in [9]. Here, we consider a problem in which there is no structural relationship between the nodes and seek Bayes optimal matches that incorporate prior probabilities of node deletions and extraneous nodes. This paper extends Chiang, *et. al.* [10] by (1) reducing the computational complexity of the correspondence search from $O((m+n)^3)$ to $O(\max(m,n)^3)$, and (2) developing a Bayes match algorithm that accommodates an unknown translation between the pair of point sets being matched.

We make use of the Hungarian algorithm to solve the matching problem, which enables us to find a Bayes optimal match in $O(max(m,n)^3)$ time, where m and n are the number of features in X and Y, respectively.

We also consider the problem of point set matching when there is an unknown translation between the point sets. We present an iterative algorithm to maximize the match likelihood for unknown, deterministic shifts. In addition, we consider a Bayesian solution under the assumption that the translation is random with a known prior probability density. The algorithm is guaranteed to converge to a local maximum of the match likelihood.

The paper is organized as follows. In Section 2 we state the classification problem and discuss the Bayesian match likelihood evaluation. The polynomial time solution to the matching problem is presented in Section 3. Section 4 presents an algorithm to find the match likelihood for the case of unknown translation between the point sets. Section 5 applies the matching techniques to target classification from X-band SAR imagery, and Section 6 concludes the paper.

2. PROBLEM STATEMENT

The Bayes matching problem we consider is given as follows. We are given a set of n features

$$Y = \{Y_1, Y_2, \ldots, Y_n\} \tag{1}$$

extracted from a measurement. Each feature Y_i is an $\ell \times 1$ vector of ordered attributes. The attributes may characterize location, amplitude, pose, or other properties of the feature. Corresponding to each candidate hypothesis H we have a set of m features

$$X = \{X_1, X_2, \ldots, X_m\} \tag{2}$$

Both feature sets have uncertainty in the feature attributes. We assume a probability model for the uncertainty of the attributes for any feature, and for the feature attributes if Y_i is a false-alarm feature (*i.e.*, Y_i does not correspond to a feature in X). We also assume a probability model for the appearance or absence of a given feature in Y.

We seek to compute the posterior likelihood of the hypothesis, that is to find $P(H|Y)$. By Bayes' rule

$$P(H|Y) = \frac{f(Y|H)P(H)}{f(Y)} = \frac{f(Y|X)P(H)}{f(Y)} \tag{3}$$

Assuming $P(H)$ is known, computation of the posterior likelihood $P(H|Y)$ requires finding $f(Y|X)$ under H, where some elements of X may be missing in Y and conversely, and where attributes in X_i and Y_j have uncertainties.

3. COMPUTATION OF THE MATCH LIKELIHOOD

Computation of the likelihood $f(Y|X)$ requires a correspondence map Γ between features in X and Y. The correspondence map is a nuisance parameter that arises because feature sets are not ordered with respect to each other.

Let Γ denote the correspondence between X and Y, with $\Gamma_j = i$ denoting the match $X_i \leftrightarrow Y_j$. We set $\Gamma_j = 0$ if Y_j has no match in X, and we call Y_j a "false alarm". Similarly if no $\Gamma_j = i, 1 \leq j \leq n$ then X_i has no match in Y, and we say X_i is "missed". Given a correspondence Γ, and assuming the feature uncertainties are conditionally independent given H, then

$$f(Y|X) = \arg\max_{\Gamma} f(Y|X, \Gamma) \tag{4}$$

where

$$\begin{aligned} f(Y|X,\Gamma) \quad = \quad & \left\{ P(n_f \text{ false alarms}) \prod_{\{j:\Gamma_j=0\}} f_{FA}(Y_j) \right\} \\ & \cdot \left\{ \prod_{\{j:\Gamma_j=i>0\}} P_i \cdot f(Y_j|X_i) \right\} \\ & \cdot \left\{ \prod_{\{i:\Gamma_j\neq i,\forall j\}} [1-P_i] \right\} \end{aligned} \tag{5}$$

where P_i is the detection probability of X_i and $f_{FA}(\cdot)$ is the pdf of a false alarm rate. The first braced term in (5) models the likelihood of false alarm features, the second term contains the likelihood of each $X_i \leftrightarrow Y_j$ match, and the third term penalizes the missed X_i features.

Computation of $f(Y|X)$ involves maximizing the right hand side of (5) over all correspondence maps Γ. Since the number of maps is exponential in m and n, it at first appears that finding the best match has exponential complexity. Surprisingly, the problem can be solved in $O(\max(m,n)^3)$ time if the false alarm probability obeys an exponential rule

$$P(n_f \text{ false alarms}) = ce^{\beta n_f} \tag{6}$$

for some constants c and β.

The Hungarian algorithm finds, in $O(k^3)$ computations, the one-to-one correspondence between the elements of the $k \times 1$ vectors $[x_1, \ldots, x_k]^T$ and $[y_1, \ldots, y_k]^T$ that minimizes the cost of the correspondence [11], where the cost of corresponding x_i with y_j is given by the ijth entry of the $k \times k$ matrix C. The correspondence is equivalent to selecting exactly one element from each row and column of the array such that the sum of the selected entries is minimized.

The Hungarian algorithm can be modified to find the optimal correspondence between X and Y that includes both insertions and deletions in the correspondence. From (5) and (6) we observe

$$\begin{aligned} \log f(Y|X,\Gamma) \quad = \text{const} \quad & + \sum_{\{j:\Gamma_j=0\}} \beta + \log[f_{FA}(Y_j)] \\ & + \sum_{\{j:\Gamma_j=i>0\}} \log[P_i \cdot f(Y_j|X_i)] \\ & + \sum_{\{i:\Gamma_j\neq i,\forall j\}} \log[1-P_i] \end{aligned} \tag{7}$$

We employ the Hungarian algorithm with $(m+n)\times(m+n)$ cost matrix C given by

$$C = \left[\begin{array}{ccc|ccc} c_{11} & \cdots & c_{1n} & M_1 & & \infty \\ \vdots & \ddots & \vdots & & \ddots & \\ c_{m1} & \cdots & c_{mn} & \infty & & M_m \\ \hline F_1 & & \infty & 0 & \cdots & 0 \\ & \ddots & & \vdots & \ddots & \vdots \\ \infty & & F_n & 0 & \cdots & 0 \end{array}\right] \tag{8}$$

where

$$\begin{aligned} c_{ij} &= -\log[P_i \cdot f(Y_j|X_i)] \\ M_i &= -\log[1-P_i] \\ F_j &= -\beta - log[f_{FA}(Y_j)] \end{aligned}$$

The elements on the right hand side of (7) appear in the cost matrix above. To see that the Hungarian algorithm with this cost matrix maximizes (7), consider $i \le m$. If $\Gamma_i = j$ for some $j \in [1,\ldots,n]$, then X_i corresponds to Y_j with cost c_{ij}. If $\Gamma_i > n$, then no Y_j corresponds to X_i. In this case $j = i+n$ and M_i is the log miss probability cost for X_i. Note that j cannot be any other integer greater than n, because the corresponding cost is infinity. Similarly, if $i > m$ and $\Gamma_i = j \in [1,\ldots,n]$, then $i = j+m$ with cost F_j. Here, feature Y_j corresponds to no X_i, and is thus labeled as a false alarm. Finally, correspondences $\Gamma_i = j$ for $i > m$ and $j > n$ incur zero cost. The Hungarian algorithm thus finds the correspondence that maximizes the log-likelihood score (7) in $O((m+n)^3)$ computations.

The special structure of the cost matrix in (8) admits an even simpler solution. From (8) we see that if the cost of matching feature Y_j with X_i is higher than that of declaring both Y_j a false alarm and X_i as a miss, then the latter will result in a lower score, and the correspondence will include it. Thus we may apply the Hungarian algorithm using a cost matrix $\overline{C}$, where, if $m \ge n$,

$$\overline{c_{ij}} = \begin{cases} \min(c_{ij}, m_i + f_j), & 1 \le j \le n \\ m_i, & n+1 \le j \le m \end{cases} \tag{9}$$

for $1 \le i \le m$. If $m < n$, we form $\overline{C}$ similarly, but by filling the last $n-m$ rows with f_j. Applying the Hungarian algorithm with cost matrix $\overline{C}$, we achieve a complexity of $O(\max(m,n)^3)$ instead of $O((m+n)^3)$. The reduction is significant in many applications because often $m \approx n$; when $m = n$, using $\overline{C}$ to compute the correspondence and likelihood reduces the computation by a factor of 8 compared to using C.

4. TRANSLATION UNCERTAINTY

We next turn to the problem of translation uncertainty in the match. Translation uncertainty may be caused by an unknown location shift between the test and hypothesized image features. In addition, by using the logarithm of amplitude, one can accommodate an unknown gain in image intensity. We consider the problem of finding the maximum match score $f(Y|X,\tilde{r})$ over an unknown translation vector $\tilde{r}$. Typically, translation uncertainty applies to only some of the attributes in each vector X_i or Y_j; thus, we partition $\tilde{r}$ as

$$\tilde{r} = \begin{bmatrix} r \\ 0 \end{bmatrix} \begin{matrix} \}d \\ \}\ell - d \end{matrix} \tag{10}$$

4.1. Deterministic Translation

We first consider the case in which r is an unknown, deterministic vector. We modify the match likelihood problem as one of finding

$$\max_r f(Y|X,r) \tag{11}$$

We assume the feature uncertainties are of the form

$$f(Y_j|X_i,r) = f_0(Y_j - X_i - r) \tag{12}$$

for some known pdf $f_0(\cdot)$, and that only the matched feature pairs depend on translation. Then from equations (11)–(12), the maximum likelihood estimate (MLE) of r for a given correspondence map Γ is

$$\hat{r}_{ML} = \arg\max_r \prod_{\Gamma_j = i > 0} f_0(Y_j - X_i - r) \tag{13}$$

For the special case that $f_0 \sim \mathcal{N}(0,\Sigma)$, the MLE of r admits a closed-form solution

$$\hat{r}_{ML} = \frac{1}{p}S_1 + \frac{1}{p}\Sigma_{11}^{-1}\Sigma_{12}S_2 \tag{14}$$

where

$$\Sigma^{-1} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{bmatrix} \begin{matrix} \}d \\ \}\ell - d \end{matrix} \tag{15}$$

$$S = \sum_{\Gamma_j = i > 0} (Y_j - X_i) = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \begin{matrix} \}d \\ \}\ell - d \end{matrix} \tag{16}$$

$$p = \text{number of elements in } \{\Gamma_j = i > 0\} \tag{17}$$

4.2. Random Translation

If we instead assume the translation uncertainty is random and governed by a prior probability density function $f(r)$, then the MAP estimate of r is found using Bayes' rule:

$$f(r|Y,X) = \frac{f(Y|X,r)f(X|r)f(r)}{f(Y,X)} \tag{18}$$

so that

$$\hat{r}_{MAP} = \max_r f(Y|X,r)f(r) \tag{19}$$

For the case of Gaussian feature uncertainties and a Gaussian prior $f(r) \sim \mathcal{N}(0,\Sigma_R)$ we again obtain a closed-form solution

$$\hat{r}_{MAP} = (p\Sigma_{11} + \Sigma_R^{-1})^{-1}(\Sigma_{11}S_1 + \Sigma_{12}S_2) \tag{20}$$

4.3. Match Likelihood with Translation Uncertainty

Equations (14) and (20), coupled with the Hungarian algorithm match computation in Section 3, lead to a simple iterative algorithm for finding the match likelihood $f(Y|X)$ when translation uncertainty is present. Initially we find the correspondence map $\Gamma^{(0)}$, and using this correspondence, we compute an initial estimate $r^{(0)}$ of r using (14) or (20), along with $\Gamma^{(0)}$. We compute the match likelihood (and associated correspondence map $\Gamma^{(1)}$) using that translation estimate by applying the Hungarian algorithm, then update the estimate $r^{(1)}$. The algorithm iterates until convergence; it converges in finite time if two consecutive Hungarian algorithm applications produce the same correspondence mapping, *i.e.*, if $\Gamma^{(k+1)} = \Gamma^{(k)}$. In all examples tried to date the algorithm converges in finite time, usually within a few iterations for problems with m and n in the range of 10–30.

It can be shown that the match likelihood is nondecreasing for each step of the algorithm; thus, the algorithm converges to a local maximum of the likelihood function. The Hungarian algorithm is the most computationally expensive part of each iteration step, but the Hungarian algorithm has much lower computation if given an initial correspondence that is close to the final one; consequently, we found that initializing the Hungarian algorithm with the most recently found correspondence significantly reduces computation time.

5. RADAR TARGET CLASSIFICATION EXAMPLE

In this section we present an example of feature-based classification using point features extracted from X-band SAR image chips, using the 1ft×1ft MSTAR Public Targets data set. The data set contains SAR data chips of 10 targets at 15° and 17° depression angles. For each target, approximately 270 images are available. For each image chip we extract scattering center locations and amplitudes using a peak extractor that finds local maxima from the magnitude SAR image, and we keep the ten highest amplitude peaks. We use the 15° data to form the catalog feature sets (X^k) and the 17° data as the measurement feature sets (Y). Only the downrange and crossrange locations of the scattering centers are used as feature attributes. We note that higher correct classification rates are obtained if we use other scattering center attributes (such as amplitude), or if we use a larger number of scattering centers per image (see [10]); we have chosen the present example because it concisely illustrates the main points of the matching algorithm.

For each of the 2747 target image chips, we find the five image chips in each of the ten target classes that have the highest SAR magnitude image correlation. The target classes and poses (pose is in this case azimuth angle) corresponding to these 50 image chips form the set of candidate hypotheses we test using the one-to-one likelihood function

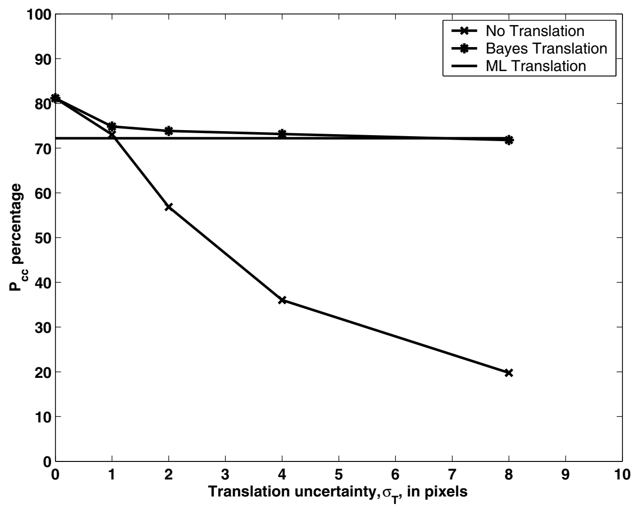

Figure 1: Probability of correct classification for feature-based matching of ten MSTAR public targets with feature translation uncertainty.

in equation (5). The uncertainty model we use is $\mathcal{N}(0, \sigma_p^2)$ for the location attributes for both measured and catalog feature sets, $P_d = 0.9$ and a 2D Poisson process with a rate of 3 clutter peaks per chip for the modeling of false alarms.

The image chips provided in the MSTAR data set are centered, and the translation to align similar image chips has a standard deviation of less than one pixel. To emulate higher translation uncertainty, we add a random Gaussian offset in range and crossrange to the extracted features from the measurement image; the offset has zero mean and variance σ_T^2. We then apply three classifiers: one that assumes no translation is necessary to align the features, one that uses the MLE translation, and one that uses the Bayes translation. In all cases we assume the feature uncertainty $f_0 \sim \mathcal{N}(0, \sigma_p^2 I)$ in equation (12) with $\sigma_p^2 = 1/2$. The classification results are shown in Figure 1.

We note from Figure 1 that classification performance without translation is slightly better than the ML-based classifier when no additional feature uncertainty is added (that is, when $\sigma_T = 0$ in Figure 1). This is not surprising because the MSTAR chips are already centered, leaving little translation uncertainty; if there is no translation uncertainty, an algorithm that incorporates this prior information should outperform an algorithm that does not. Thus, for applications in which a prior alignment procedure is applied, if the alignment uncertainty is on the order of or smaller than the feature uncertainty, one can obtain good classification performance by assuming no translation uncertainty and using a correspondingly simple (and computationally faster) match algorithm. When translation uncertainty is large compared to feature uncertainty (the feature uncertainty is $\approx \sigma_p = \sqrt{2}/2$ in this case), match algorithms that incorporate translation perform better than match algorithms that do not. The ML-based matcher is independent of translation uncertainty, and is therefore robust to translation as seen in Figure **??**. The Bayes-based classifier has the best performance of the three classifiers, and approaches the ML performance as translation uncertainty becomes large, as is theoretically predicted.

6. CONCLUSION

We have presented a Bayesian approach to the point set matching problem, in which uncertainties for the feature set attributes are included in the match metric. The match requires the correspondence between the two feature sets. We apply the Hungarian algorithm to provide the optimum correspondence in polynomial complexity, and show how the special structure of the match cost matrix can be used to reduce computations from $O((m+n)^3)$ to $O(\max(m,n)^3)$ when misses and false alarms are present.

We have also considered the point set matching problem in the presence of an unknown translation between the point sets. Both deterministic and random translation models are considered. An algorithm for providing the Bayes likelihood is found by iterating between finding the best match correspondence and finding the best translation. For the Gaussian case, a closed-form solution for the best translation is found. The convergence to a local maximum of the likelihood function is established.

We have applied the point set matching approach to features extracted from measured X-band SAR imagery. We showed that both the deterministic and random translation assumptions result in match algorithms that are robust to translation uncertainty. These algorithms provide better classification performance for cases in which the feature uncertainty is small compared to the translation uncertainty. On the other hand, for cases in which a prior alignment procedure is applied, if the alignment uncertainty is on the order of the feature uncertainty, one can obtain good classification performance by assuming no translation uncertainty and using a correspondingly simple (and computationally faster) match algorithm.

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