

SELF-CALIBRATION OF UNATTENDED GROUND SENSOR NETWORKS

R. Moses, R. Patterson, D. Krishnamurthy

Department of Electrical Engineering
The Ohio State University, Columbus, OH 43210

N. Srour, T. Pham

Army Research Laboratory
Adelphi, MD 20708

ABSTRACT

We present an algorithm for locating and orienting a set of acoustic arrays that have been deployed in a scene at unknown locations and orientation angles. This self-calibration problem is solved using a number of source signals also deployed in the scene. We assume each array can estimate the time-of-arrival and direction-of-arrival (with respect to the array's local orientation coordinates) of every source. From this information we compute the array locations and orientations. We consider four subproblems, in which the source signals or emission times are either known or unknown. We develop necessary conditions for solving the self-calibration problem, and provide a maximum likelihood solution and corresponding location error estimate.

1 INTRODUCTION

Unattended Ground Sensors (UGSs) are becoming increasingly important for providing today's Army with needed situational awareness in battlefield and MOUT deployments [1]. The basic concept is to deploy a large number of low-cost, self-powered sensors that acquire and process data. The sensors typically consist of an array of microphones to detect, track, and classify acoustic signatures. In addition, seismic and low-cost imaging sensors may also be present. Ground sensors are placed in the field by hand, by an air drop, or by artillery launch. Each sensor monitors its environment to detect, track, and characterize signatures; this information is sent to a central information processor (CIP) for subsequent data fusion with other sensors and report generation.

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In order to fuse sensor information, it is important to know the location and orientation of each sensor. In some deployments, such as careful hand placement, accurate location and orientation of the sensors can be assumed. However, for many sensor deployment situations, it is difficult or impossible to ensure accurate location and orientation. One could equip each sensor with a GPS and compass to obtain location and orientation information, but this adds to the expense and power requirements of the sensor, in conflict with the goals of low cost and long battery life. Thus, there is interest in developing methods to self-calibrate the sensor array with a minimum of additional hardware or processing. Recent work on blind beamforming considers a related problem of forming a maximum power beam to a source [2].

In this paper we consider an approach to array self-calibration using calibration sources in the field. A number of signal sources are deployed in the same region as the sensors (see Figure 1). Each source generates a unique signature that is acquired by the sensors. From the time-of-arrival (TOA) and direction-of-arrival (DOA) of each source signal, we compute the unknown locations and orientations of the sensors. We consider four related problems:

- (a) the source locations and emission times are known,
- (b) the source locations are known and emission times are unknown,
- (c) the source locations are unknown and emission times are known,
- (d) the source locations and emission times are unknown.

An outline of the paper is as follows. In Section 2 we present a statement of the problem and justify

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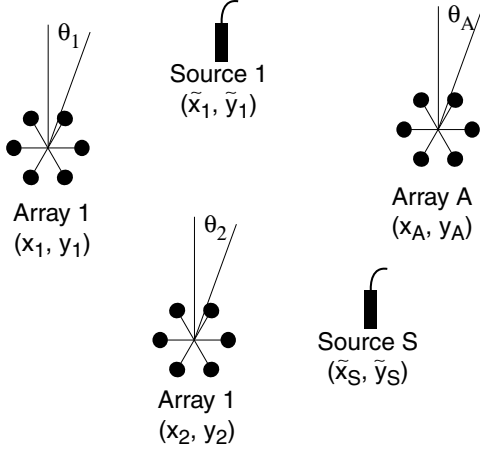


Figure 1: Array self-calibration scenario.

our assumptions. In Section 3 we first consider necessary conditions for a self-calibration solution and present methods for solving the self-calibration problem with a minimum number of sensors and sources. These methods provide initial estimates for a maximum likelihood self-calibration method we develop next. Expressions for the calibration uncertainty are also derived. Section 4 presents numerical examples to illustrate the approach, and Section 5 presents conclusions and directions for future work.

2 THE SELF-CALIBRATION PROBLEM

Assume we have a set of A sensor arrays, each with unknown location $\{a_i = (x_i, y_i)\}_{i=1}^A$ and unknown orientation angle θ_i with respect to a reference direction (*e.g.*, North). We consider the two-dimensional problem in which the sensors lie in a plane and the unknown reference direction is azimuth; an extension to the three-dimensional case is straightforward.

In the array field are also placed S source signals at locations $\{s_j = (\tilde{x}_j, \tilde{y}_j)\}_{j=1}^S$. The source locations may be known or unknown. Each source emits an acoustic signal with emission time t_j ; the emission times may be known or unknown. We thus consider four related subproblems, depending on the prior knowledge of the source locations and emission times.

We assume each emitted source signal is detected by every sensor in the field, and that each sensor forms

an estimate of the TOA and DOA for that source. Specifically, array i estimates a time-of-arrival t_{ij} and direction-of-arrival θ_{ij} from the signal emitted by source j . The times of arrival are assumed to be with respect to a known, common time base; this time base needs to be accurate to within $\approx 1\text{--}5$ msec, as we show in Section 4. A common time reference to this accuracy could be obtained by synchronizing the sensor processor clocks before deployment, or by electronically synchronizing their clocks using the array's electronic communication system. The directions of arrival are relative to each array's local orientation reference; absolute directions of arrival are not available because the orientation angle of each array is unknown.

The set of $2AS$ measurements are gathered in matrix form as:

$$T = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{1S} \\ t_{21} & t_{22} & \dots & t_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ t_{A1} & t_{A2} & \dots & t_{AS} \end{bmatrix} \quad (1)$$

$$\Theta = \begin{bmatrix} \theta_{11} & \theta_{12} & \dots & \theta_{1S} \\ \theta_{21} & \theta_{22} & \dots & \theta_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{A1} & \theta_{A2} & \dots & \theta_{AS} \end{bmatrix} \quad (2)$$

Each element of T and Θ has estimation uncertainty; we assume the uncertainty to be Gaussian with known variance σ_θ^2 or σ_t^2 , and the measurement errors are uncorrelated. Extensions to non-Gaussian or dependent errors are straightforward.

Each array transmits its TOA and DOA estimates to a central information processor, and these $2AS$ measurements form the data with which to self-calibrate the array.

Define the parameter vectors

$$\beta = [x_1, y_1, \theta_1, \dots, x_A, y_A, \theta_A]^T \quad (3)$$

$$\gamma = [\tilde{x}_1, \tilde{y}_1, t_1, \dots, \tilde{x}_S, \tilde{y}_S, t_S]^T \quad (4)$$

The self-calibration problem, then, is given the measurement $\{T, \Theta\}$, estimate β . Note that none, some,

or all of the parameters in γ may be known, depending on the particular subproblem of interest.

3 SELF-CALIBRATION SOLUTION

3.1 Uniqueness and Minimal Solutions

We first address the question of when unique solutions exist, and we determine the minimum number of sources and sensors needed for a unique solution. Algorithms for finding β in the minimal case are outlined for each of the four cases described above. These algorithms provide initial estimates for the iterative descent algorithm discussed in the next subsection.

In all four cases the number of measurements is $2AS$, and determination of β involves solving a nonlinear set of equations for its $3A$ unknowns. Depending on the case considered, we may also need to estimate the unknown nuisance parameters in γ ; in fact, in all cases, one obtains estimates for the γ parameters for free.

Case (a): Known source locations and emission times. In this case a unique solution for β can be found for any number of sensors, as long as there are $S \geq 2$ sources. In fact, the location and orientation of each sensor can be computed independently of other sensor measurements. Briefly, the array location of the i th sensor is found from the intersection of two circles with centers at the source locations and with radii $(t_{i1} - t_2)/c$ and $(t_{i2} - t_1)/c$, where c is the propagation velocity. The intersection is in general two points; the correct location can be found using the sign of $\theta_{i2} - \theta_{i1}$. We note that the circle intersections admit a closed-form solution.

Case (b): Known source locations, unknown emission times. Here we require $S \geq 3$ sources, but again the location of each array can be computed in closed form and independently of other array measurements. The solution procedure is as follows. Consider the pair of sources (s_1, s_2) . Sensor array i knows the angle $\theta = \theta_{i2} - \theta_{i1}$ between the sources. The set of all possible locations for array i is an arc of a circle whose center and radius can be computed from the source locations (see Figure 2). Similarly, a second circular arc is obtained from the source pair (s_1, s_3) . The intersection of these two arcs is a unique point, and can be computed in closed form.

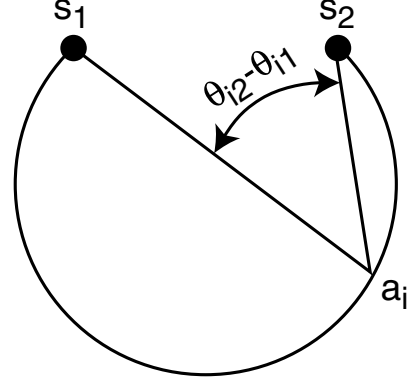


Figure 2: A circular arc is the locus of possible array locations whose angle between two known points is constant.

A solution for Case (b) can also be found using $S = 2$ sources and $A = 2$ sensors. The solution requires a one-dimensional search of a parameter over an interval.

Case (c): Unknown source locations, known emission times. In this case and the next, the calibration problem can only be solved to within an unknown translation and rotation of the entire array-source scene. Without loss of generality, we assume the location and orientation of the first array is known; thus, $x_1 = y_1 = \theta_1 = 0$. We solve for the remaining $3(A - 1)$ parameters in β .

For the case of unknown source locations, a unique solution for β is available for $A = S = 2$. From Sensor 1 the range to each source can be computed from $r_j = (t_{1j} - t_j)/c$, and its bearing is known, so the locations of all the sources is known. The locations and orientations of the remaining sensors is then computed using the method of Case (a).

Case (d): Unknown source locations and emission times. In this problem a unique solution exists for $A = 2, S = 3$ or $A = 3, S = 2$. For each case a closed-form solution is not known, but an iterative solution exists that requires only a two-dimensional search over a rectangle in \mathcal{R}^2 .

There are a number of ways to solve the self-calibration problem in this case. We outline one such method for $A = 2$ and $S = 3$. Assume the second array is at

known location (x_2, y_2) . This information and the time differences of arrival $t_{2j} - t_{1j}$ define a hyperbola on which each source s_j must lie; the intersection with the hyperbola and the (known) direction of each source with respect to Array #1 gives the source locations; these can be computed in closed-form. The orientation angle θ_2 for Array #2 is then found from the source locations. The algorithm requires iteration over (x_2, y_2) ; an equivalent polar parameterization is (r_2, ϕ_2) , where $0 < r_2 < \infty$ and $0 \leq \phi_2 < 2\pi$. But r_2 can be lower bounded by R_{min} which is computed from the maximum time difference of arrival; reparameterizing $\tilde{r}_2 = 1/r_2$ gives a compact interval $\tilde{r}_2 \in [0, 1/R_{min}]$ over which to search for \tilde{r}_2 . This interval, combined with the compact interval for ϕ_2 , yields a solution search over a rectangle in \mathcal{R}^2 .

Table 1 summarizes the properties of the minimal self-calibration solution techniques for the four cases discussed above.

3.2 MAXIMUM LIKELIHOOD SOLUTION

In this section we derive a maximum likelihood (ML) estimator for the unknown array location and orientation parameters. The algorithm involves the solution of a set of nonlinear equations for the unknown parameters (and the unknown nuisance parameters in γ). The solution is found by iterative minimization of a cost function; we use estimates obtained in Section 3.1 to seed initialize the iterative descent. In addition, we compute the Cramér-Rao Bound (CRB) for the variance of the parameters in β ; the CRB also gives high-SNR parameter variance of the ML parameter estimates.

Under the assumption of independent Gaussian measurement errors for the elements in T and Θ , the likelihood function is

$$f(T, \Theta; \alpha) = \frac{1}{(2\pi\sigma_t\sigma_\theta)^{AS}} \cdot \exp \left\{ -\frac{1}{2} Q(T, \theta; \alpha) \right\} \quad (5)$$

where

$$Q(T, \theta; \alpha) = \sum_{i=1}^A \sum_{j=1}^S \left[\frac{(t_{ij} - \tau_{ij}(\alpha))^2}{\sigma_t^2} + \frac{(\theta_{ij} - \phi_{ij}(\alpha))^2}{\sigma_\theta^2} \right] \quad (6)$$

In the above equations, $\alpha = [\beta^T, \gamma^T]^T$ is the set of all sensor and source parameters, and

$$\tau_{ij}(\alpha) = t_j + \|a_i - s_j\|/c \quad (7)$$

$$\phi_{ij}(\alpha) = \theta_i + \angle(a_i, s_j) \quad (8)$$

where $\|\cdot\|$ is the standard Euclidean norm and $\angle(\xi, \eta)$ is the angle between the points $\xi, \eta \in \mathcal{R}^2$.

By stacking the elements of T and Θ into a single $2AS \times 1$ vector that we denote X , correspondingly stacking the elements of $\tau_{ij}(\alpha)$ and $\phi_{ij}(\alpha)$ into a $2AS \times 1$ vector function $g(\alpha)$, and putting σ_t^2 and σ_θ^2 into the appropriate diagonal elements of a $2AS \times 2AS$ diagonal matrix Σ we can write equation (6) as

$$Q(T, \theta; \alpha) = [X - g(\alpha)]^T \Sigma^{-1} [X - g(\alpha)] \quad (9)$$

In the four cases considered in Section 3, some of the parameters in α are known (or assumed to be zero to admit a unique solution), and the remaining are unknown. We denote α_1 to be the unknown parameters in α and α_2 to be the known parameters for the particular case of interest. Using this notation along with equation (5), the maximum likelihood estimate of α_1 is

$$\begin{aligned} \hat{\alpha}_{1,ML} &= \arg \max_{\alpha_1} f(T, \Theta; \alpha) \\ &= \arg \min_{\alpha_1} Q(T, \theta; \alpha) \end{aligned} \quad (10)$$

The solution of (10) involves solving a nonlinear least squares problem. A standard iterative descent procedure can be used, initialized using one of the solutions in Section 3.1.

The CRB can be computed from the Fisher Information Matrix of α . The Fisher Information Matrix is given by [3]

$$I_\alpha = E \left\{ [\nabla_\alpha \ln f(T, \Theta; \alpha)] [\nabla_\alpha \ln f(T, \Theta; \alpha)]^T \right\}$$

Table 1: Summary of Minimal Solutions for Array Self-Calibration Problem

Case	# Unknowns	Minimum A, S	Comments
Known Locations Known Times	$3A$	$A = 1, S = 2$	closed-form solution
Known Locations Unknown Times	$3A + S$	$A = 1, S = 3$	closed-form solution
	$3A + S$	$A = 2, S = 2$	1-D iterative solution
Unknown Locations Known Times	$3(A - 1) + 2S$	$A = 2, S = 2$	closed-form solution
Unknown Locations Unknown Times	$3(A + S - 1)$	$A = 2, S = 3$ or $A = 2, S = 3$	2-D iterative solution

The partial derivatives are readily computed from equations (5), (7), and (8); it can be shown that

$$I_\alpha = [G'(\alpha)]^T \Sigma^{-1} [G'(\alpha)] \quad (11)$$

where $G'(\alpha)$ is the $2AS \times \dim(\alpha)$ matrix whose ij th element is $\partial g_i(\alpha) / \partial \alpha_j$.

To compute the CRB matrix for α_1 , we first eliminate all rows and columns in I_α that correspond to elements in α_2 , then invert the remaining submatrix [3]:

$$C_{\alpha_1} = [I_{\alpha_1}]^{-1} \quad (12)$$

The nonlinear least squares estimate $\hat{\alpha}_{1,ML}$ has parameter uncertainty given by the CRB for high signal-to-noise ratio; that is, as $\max_i \Sigma_{ii} \rightarrow 0$.

4 NUMERICAL EXAMPLE

We present an example of the self-calibration procedure. Fifteen sensor arrays and ten sources are randomly placed in a 3 km \times 3 km region; the sensor orientations are also randomly chosen. Figure 3 shows the locations of the sensors and sources. We compute the 2σ location uncertainty ellipses for both the sources and sensors assuming only the location of array A1 is known. These ellipses are shown in the figure; most look like small diagonal lines. We find that the error is small compared to the distance between arrays; in this case, the average distance error is about 20.34 m. We note that the location uncertainty is highly correlated and seems to be due to a rotation uncertainty in the scene; for example, the

2σ uncertainty of array A6 is 0.2135 m in the radial direction and 15.3993 m in the tangential direction, with respect to a circle centered at (0,0). If the angle uncertainty σ_θ is reduced, the tangential uncertainty reduces accordingly.

If the source locations are assumed to be known, the CRB uncertainty ellipses around the arrays reduce dramatically in size, to under 0.5 m in all cases. For example, the maximum 2σ uncertainty for array A6 becomes 0.2710 m when the sensor locations are unknown.

5 CONCLUSIONS

We have presented a method for self-calibration of an array of unattended ground sensors. The calibration procedure involves placing a number of source signals in the scene, and computing array and source unknowns from estimated time-of-arrival and direction-of-arrival estimates obtained for each source-sensor pair. We present solutions to four variations on this problem, depending on whether the source locations are known or unknown and on whether the source signal emission times are known or unknown. The algorithm comprises an initialization step which is seeded using a minimal number of sources and sensors, followed by an iterative descent solution of a nonlinear system of equations. An analytical expression for the sensor location and orientation error covariance matrix is also presented for each of the four problem variations. The algorithm has low computational cost; calibration solutions require a few seconds of CPU time on a standard PC.

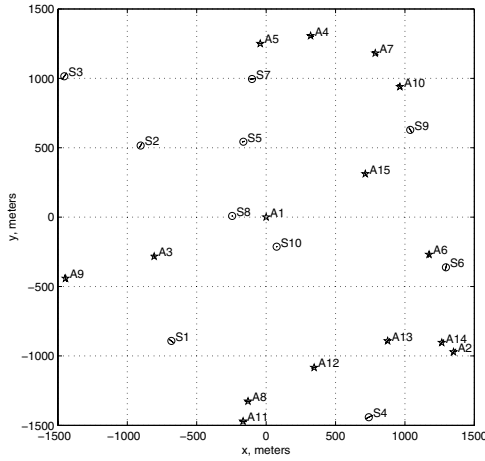


Figure 3: Array and source location and array orientation for the example considered. Arrays are denoted by stars and sources by circles.

Future work is directed toward generalizing the analysis to expand the set of practical applications addressed. Specific topics are: (1) extend the problem solution to a three-dimensional scenario; (2) include acoustic propagation models to develop more accurate estimates of source detection probability and of TOA and DOA estimation errors in practical deployments; and (3) develop principles for source placement that minimize the resulting array location uncertainties.

6 REFERENCES

- [1] N. Srour, "Unattended ground sensors a prospective for operational needs and requirements," tech. rep., Army Research Laboratory, October 1999.
- [2] K. Yao, R. E. Hudson, C. W. Reed, D. Chen, and F. Lorenzelli, "Blind beamforming on a randomly distributed sensor array system," *IEEE Journal on Selected Areas in Communications*, vol. 16, pp. 1555–1567, October 1998.
- [3] H. L. Van Trees, *Detection, Estimation, and Modulation Theory: Part I*. New York: Wiley, 1968.