Cyclic Wiener Filtering Based Multirate DS-CDMA Receivers

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Abstract— **Detection methods for multiple rate Direct Sequence Code Division Multiple Access (DS-CDMA) signalling are addressed. Attention is focussed on the development of receivers based on the minimum mean squared error (MMSE). Due to the cyclostationarity of the multirate signal, a representation of the multirate signal in terms of the Fourier basis is possible. This expansion facilitates construction of the MMSE receivers. Both variable spreading gain as well as variable chipping rate DS-CDMA access schemes are considered. For the multiple chipping rate system, the effect of front-end filter bandwidth on receiver performance is studied. Simulation results are provided to compare the performance of the proposed receivers.**

I. Introduction

The growth of wireless networks has resulted in systems offering heterogenous services, such as voice, video and data. Several new system architectures based on Direct Sequence Code Division Multiple Access (DS-CDMA) signalling have been proposed recently; for example [1]. These proposals include systems with multiple chipping rates [1], variable spreading gain with constant chip rate [1] and multicode CDMA [2]. Another instance of a multiple chipping rate system is where a wideband DS-CDMA system might be overlaid over an existing narrowband DS-CDMA system.

Receivers which truly exploit the nature of multi-rate signals have focussed on synchronous and pseudo-synchronous systems in non-multipath environments [3], [4], [2], [5], [6]. Designs which accomodate asynchronous or multipath channels [7] do not always take advantage of the multi-rate nature of the problem.

In this paper, we derive linear multiuser receivers based on the minimum mean squared error (MMSE) criterion, for multiuser systems where users with different symbol rates coexist. We first derive MMSE receivers without any causality constraints; this extends the work in [8], [9]. The non-causal structure proposed in this paper provides a basis for more practical causal realizations and also provides a baseline for comparison to other multi-rate receiver stuctures. It is shown that optimal linear MMSE receiver is, in general, time-varying. The same observation was also independently made in [6] for a dual rate synchronous DS-CDMA system; a non multipath environment with time-limited square pulses was assumed. The reciever structure developed herein is general enough to include the cases of arbitrary alphabet size, multipath channels and multiple bandwidth systems. As the work in [6] and the current work exploit the cyclostationarity of the multirate signals, there wil be similarity in some of our conclusions.

In systems with multiple chipping rates, different user classes occupy different bandwidths. The choice of bandwidth of the front-end filter for the smaller bandwidth users determines the performance of their receivers, which is also intimately tied to the total system capacity. The front-end filter bandwidth determines the sampling rate which is one of the factors determining receiver cost for users with lesser spectral resources.

This paper is organized as follows. In Section II, the system model is described. The time-varying MMSE receiver is developed in Section III. The effect of front-end filter bandwidth on the performance of smaller bandwidth users is studied in Section IV. The optimal time-invariant MMSE receiver is derived in Section V and its mean-squared error is compared to that of the time-varying MMSE receiver. Simulation results for a three class system are provided in Section VI; to gain insight, only a non-multipath environment is considered. The analytical predictions on relative performance and the effect of filter bandwidth are verified by the numerical results.

II. Problem Formulation

We consider multiuser multiple rate transmission. For simplicity of presentation, we assume that all users use the same carrier frequency. It is noted that for practical multiple bandwidth systems, some form of frequency planning and thus multiple carriers will improve the efficiency of the system [7]. The methods considered in the current work are easily extended to multiple carrier systems.

The received baseband signal is

$$x(t) = \sum_{k=1}^{C} \sum_{i=1}^{P_k} A_{ik} r_{ik}(t) + n(t) \qquad (1)$$

$$r_{ik}(t) = \sum_{l=-\infty}^{\infty} b_{ik}(l) s_{ik}(t - lT_k - \tau_{ik}) \qquad (2)$$

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where C is the number of service classes. There are P_k users in class k; each user in a particular class transmits at the same rate, $\frac{1}{T_k}$, and employs the spreading gain of L_{ik}. Users are indexed by two variables: k indicates the rate/class and i indicates the user number within rate k. The received signal for user ik is denoted by $r_{ik}(t)$. The corresponding received amplitude is A_{ik}. Each user ik is received after a propagation delay of τ_{ik}. The additive noise process, $n(t)$, is assumed to be white and Gaussian with zero mean and power $\frac{N_o}{2}$. The information stream for user ik is denoted by $b_{ik}(l)$. For simplicity of presentation, BPSK modulation is assumed. The information bits are independent from user to user and in time. The spreading waveform is denoted by $s_{ik}(t)$ and is formed by modulating a pseudo-random noise sequence by the pulse shape $\phi_k(t)$

$$s_{ik}(t) = \sum_{n=1}^{L_{ik}} a_{ik}(n)\phi_k\left(t - nT_{c_k}\right)$$

where $a_{ik}(n)$ is the pseudo-random code for user ik. Arbitrary pulse shapes are considered herein. In particular, we wish to consider bandwidth efficient pulses. Such bandlimitation is essential to our examination of the effect of the bandwidth of the front-end filter in smaller bandwidth user performance in Section IV [1].

To motivate the methods employed, we first investigate the second order statistics of a single spread spectrum signal in the absence of the noise. Consider a CDMA signal with symbol period T and spreading gain L, $r(t) = A\sum_l b(l)s(t-lT)$, where $s(t)$ is the symbol waveform; we assume periodic modulation. The covariance function of $r(t)$, $R_r(t,u)$, is

$$\begin{aligned}
&\mathbb{E}\left\{A\sum_{l=-\infty}^{\infty} b(l)s(t-lT)A\sum_{k=-\infty}^{\infty} b(k)s(u-kT)\right\}\\
&= A^2\sum_{l=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\mathbb{E}\left\{b(l)b(k)\right\}s(t-lT)s(u-kT)\\
&= A^2\sum_{m=-\infty}^{\infty} R_b(mT)\sum_{l=-\infty}^{\infty} s(t-lT)s(u-lT-mT)
\end{aligned}$$

where $R_b(mT)$ is the correlation of the stationary data sequence. For uncoded data, $R_b(mT) = \delta_m$, where δ_m is the Dirac delta. The summation in l is periodic with period T. If the support of $s(t)$ is time limited to $[0, T']$, then $R(t,u)$ is zero for $|t-u| > T'$. Since, $R(t,u)$ is a periodic function with period T, its Fourier series expansion exists with fundamental frequency as $1/T$. For the raised cosine pulse, $\phi(t)$, the non-zero *cycle frequencies* of $r(t)$ are $0, 1/T, \ldots, L/T$ [10].

[1]It is noted that rectangular pulse shapes and thus very wide bandwidth front end filters are assumed in [6].

Given the prior discussion, we can determine the autocorrelation of the received multirate signal, $x(t)$ in (1)

$$R_x(t,u) = \sum_{k=1}^{C}\sum_{i=1}^{P_k} A_{ik}^2 R_{ik}(t,u) + R_n(t-u). \quad (3)$$

The periodicity of $R_x(t,u)$ depends on the ratio of individual symbol periods, T_k. If any ratio $\frac{T_1}{T_k}$ is an irrational number, the function $R_x(t,u)$ is not periodic. For the rest of the paper, we will confine our attention to the case where all ratios $\frac{T_1}{T_k}$ are rational. This implies that $R_x(t,u)$ is periodic with period pT_1 where p is the smallest integer such that $\frac{pT_1}{T_k} \in \mathbb{Z}^+$ for all $1 < k \leq C$.

Based on the results in [11], [9] for single rate systems, the optimal linear MMSE receiver will be periodic with period pT_1. This observation was also made in [6]. This is because the cyclostationary process $x(t)$ can be converted into a vector stationary process (for bandlimited systems), with vector lengths propotional to pT_1. Thus for symbol-by-symbol MMSE detection, the linear filter will be time-varying.

III. MMSE Receiver

Without loss of generality, assume that the user of interest has symbol rate T_1. Further, we assume that the covariance is periodic with period pT_1 where p is the smallest integer such that $\frac{pT_1}{T_k} \in \mathbb{Z}^+$ for all $1 < k \leq C$. Since the symbols for each user are uncorrelated, the signal of interest can be decomposed into p virtual users, each with symbol rate pT_1. Similarly, each of the interfering users can be decomposed into $\frac{pT_1}{T_k}$ virtual users. This effectively maps the original multirate problem onto a single rate problem, and we need to find p MMSE receivers, one for each virtual user of interest. We use the results in [9] to derive the MMSE receivers for the multirate case as follows. The desired linear receiver structure is shown in Figure 1, where $h(t)$ is the front-end filter.

Let $T = pT_1$ and define $v_k = \frac{T}{T_k}$; each user in class k can be split into v_k virtual users. The received signal for user ik is written as

$$r_{ik}(t) = \sum_{z=0}^{v_k-1}\sum_{l=-\infty}^{\infty} b(lv_k+z)s_{ik}(t - zT_k - lT - \tau_{ik}). \quad (4)$$

Note that the original multirate problem has $\sum_{k=1}^{C} P_k$ users and is equivalent to a single rate problem with $P = \sum_{k=1}^{C} P_k v_k$ virtual users. The only difference is that the symbol period of each virtual user is longer than its original symbol period. Define

$$\mathcal{S}_{ik}(f) = \begin{bmatrix} H(f)S_{ik}(f) \\ H(f)S_{ik}(f)e^{-j2\pi f T_k} \\ H(f)S_{ik}(f)e^{-j2\pi f 2T_k} \\ \vdots \\ H(f)S_{ik}(f)e^{-j2\pi f(v_k-1)T_k} \end{bmatrix}$$

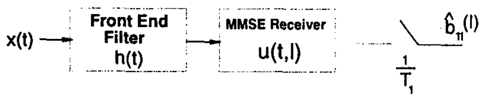

Fig. 1. Receiver structure.

where $S_{ik}(f)$ is the Fourier transform of the spreading waveform $s_{ik}(t-\tau_{ik})$ and $H(f)$ is the Fourier transform of the front end filter $h(t)$. Further define

$$\begin{aligned} \mathcal{Q}_k^T(f) &= \left[\mathcal{S}_{1k}^T(f) \quad \cdots \quad \mathcal{S}_{P_k k}^T(f)\right] \\ \mathcal{Q}^T(f) &= \left[\mathcal{Q}_1^T(f) \quad \cdots \quad \mathcal{Q}_C^T(f)\right] \end{aligned}$$

and the normalized matrix

$$\mathbf{R}(f) = \frac{1}{T}\mathbf{A} \sum_{n=-\infty}^{\infty} \frac{\mathcal{Q}\left(f-\frac{n}{T}\right)\mathcal{Q}^H\left(f-\frac{n}{T}\right)}{|H\left(f-\frac{n}{T}\right)|^2 N_o/2} + \mathbf{I}$$

where $\mathbf{A}$ is a diagonal matrix with powers of each virtual user and $\mathbf{I}$ is an identity matrix. For each of the p virtual users of interest, we define a column vector $\mathcal{B}_z$ with only one non-zero entry at the z^{th} location, equal to the power of the z^{th} virtual user, $\frac{A_{ik}^2}{p}$. With the above notation, the time-invariant optimal linear filter for each of the z virtual users is given by

$$U_z(f) = \mathcal{Q}^H(f)\frac{\mathcal{G}_z(f)}{|H(f)|^2 N_o/2} \qquad (5)$$

where $\mathcal{G}_z(f) = \mathbf{R}^{-1}(f)\mathcal{B}_z$ and we have assumed that all P virtual users have linearly independent signaling waveforms. Further simplification of (5) yields the structure of the optimal linear receiver for z^{th} virtual user in Figure 2, and the complete time-varying optimal solution is seen in Figure 3. The filter $K_{ik}(f)$ is defined as follows,

$$K_{z,ik}(f) = \sum_{l=0}^{v_k-1} \mathcal{G}_{z,M+l}(f)e^{-j2\pi f l T_k} \qquad (6)$$

where $M = (i-1)P_k + \sum_{h=0}^{k-1} P_h v_h$. Thus, the receiver for virtual user z is comprised of the front end matched filter, followed by a bank of filters matched to each users' spreading waveform, a bank of interference suppresion filters and then combining.

IV. Effect of Front-End Filter Bandwidth

In a system with multiple chipping rates, users with smaller bandwidth will potentially be provided with low complexity (*i.e.* low cost) receivers. This motivates us to investigate the effect of front-end filter bandwidth on the receiver performance, since the sampling rate is an important factor in the cost of the receiver. Furthermore, the same analysis will also indicate the effect of the front-end filter on system capacity in terms of the number of

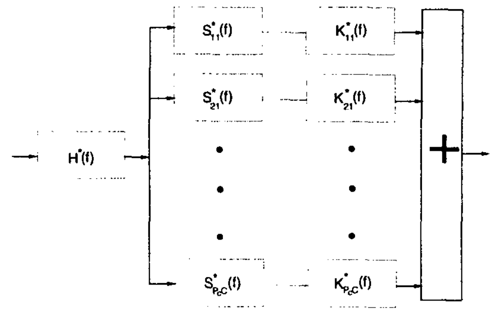

Fig. 2. MMSE receiver for each virtual user.

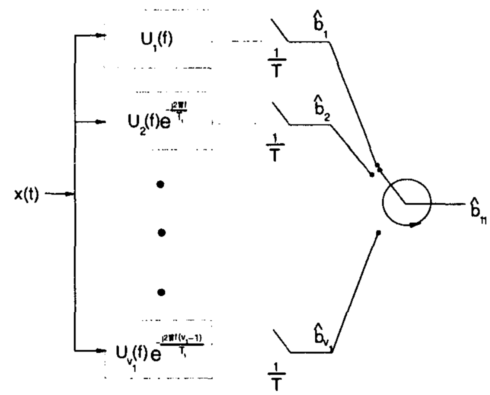

Fig. 3. Complete MMSE receiver for a user in the presence of multirate interference.

active users. Note that the subsequent analysis allows us to draw conclusions without having to resort to any assumptions about the correlation between user codes; for the same reason, the conclusions only provide trends.

Consider a multirate system with two user classes with symbol periods, $T_1 > T_2$, chipping rates, $T_{c_1} > T_{c_2}$ and spreading gains, L_1 and L_2. Let the number of users in each class be P_1 and P_2, respectively.

First consider detection of users in class 2. The observation interval is T_2 and bandwidth of the front end filter, $B = \frac{1}{2T_{c_2}}$. The approximate number of dimensions for the low rate users is $L_1' = \left\lceil L_1 \frac{T_2}{T_1} \right\rceil$ [12] and for the high rate users is $2\frac{1}{2T_{c_2}}T = L_2$ [12]. Thus, the number of users in the system is governed by

$$\frac{P_2}{L_2} + \frac{P_1}{L_1'} \leq 1 \qquad (7)$$

For demodulating the low rate users, the observation interval is T_1 and B is such that $\frac{1}{2T_{c_1}} \leq B \leq \frac{1}{2T_{c_2}}$. The approximate number of dimensions for each low rate user is $2\frac{1}{2T_{c_1}}T = L_1$. Each high rate user contributes at most $g = \left\lceil \frac{T_1}{T_2} \right\rceil + 1$ virtual users and each virtual user has approximately $L_2' = \lfloor L_2 2BT_{c_2} \rfloor$ dimensions. Thus,

$$\frac{P_1}{L_1} + \frac{gP_2}{L_2'} \leq 1 \quad (8)$$

Therefore, the total number of dimensions increase linearly with bandwidth. This implies that the interference suppression capabilities for the low rate users increases linearly with bandwidth, B.

V. Comparison with Time-invariant MMSE Receiver

In this section, we derive and study the performance of the time-invariant MMSE (TI-MMSE) receiver for the user of interest. As shown in Section III, the optimal linear MMSE receiver may be time-varying for the multirate DS-CDMA signal (1). For the cases where the optimal linear receiver is time-varying, the time-varying receiver has higher computational complexity compared to a time-invariant structure. Further, if a wideband DS-CDMA system is being overlaid over an existing narrowband system, then the analysis of the time-invariant receivers is required to quantify the loss in performance for the existing users employing time-invariant MMSE receivers[2].

By inspection of (4), it is clear the optimal linear receiver will be time-invariant for class k if and only if $v_k = 1$. For the general case, the time-invariant receiver can be found as follows[3]. Again consider demodulation of user 11. Since the received signal is wide-sense cyclostationary, the symbols $b_{11}(lv_1)$ and $b_{11}(lv_1 + k)$ for a fixed $k \in [0, \ldots, v_1]$, encounter the same interference. Since the receiver is time-invariant, it has to suppress the interference encountered by each of the v_1 virtual users of interest. In other words, even though the interference environment is periodically time-varying, a time-invariant processing requires simultaneous suppression of interference encountered by all virtual users. Hence the effective symbol periods of each interfering virtual user is T_1 instead of T. Equivalently, we have a single rate problem with covariance of the received signal as $\overline{R}(t,u) = \sum_{i=0}^{v_1-1} R(t + iT_1, u + iT_1)$, which is periodic with period T_1. Define

$$\overline{\mathcal{Q}}_1^T(f) = \left[S_{11}(f) \quad \cdots \quad S_{P_1 1}(f)\right] \quad (9)$$

$$\overline{\mathcal{Q}}^T(f) = \left[\overline{\mathcal{Q}}_1^T(f) \quad \mathcal{Q}_2^T(f) \quad \cdots \quad \mathcal{Q}_k^T(f)\right] \quad (10)$$

The TI-MMSE can be found as

$$\overline{U}(f) = \overline{\mathcal{Q}}^H(f) \frac{\overline{\mathcal{G}}(f)}{|H(f)|^2 N_o/2} \quad (11)$$

[2]Note that the need for time-varying receivers can be completely obviated by appropriate spectral shaping of the user pulses to resemble an FDMA system [7].

[3]Due to lack of space, we will only sketch the derivation of the TI-MMSE receiver.

where $\overline{\mathcal{G}}(f) = \overline{\mathbf{R}}^{-1}(f)\overline{\mathcal{B}}_1$, where the first entry $\overline{\mathcal{B}}_1$ is the power of user 11 and the rest are all zero. The matrix $\overline{\mathbf{R}}(f)$ is obtained by replacing T with T_1 and $\mathcal{Q}(f)$ with $\overline{\mathcal{Q}}(f)$ in the expression for $\mathbf{R}(f)$. The mean-squared error (MSE) for the two receivers, assuming $\mathbb{E}\{|b_{11}|^2\} = 1$, is given by

$$MSE_{TV} = 1 - \sum_{z=1}^{v_1} \int_{-\infty}^{\infty} \frac{\mathcal{G}_z^T(f)\mathcal{Q}(f)\mathcal{Q}^H(f)\mathcal{B}_z}{|H(f)|^2 N_o/2} \quad (12)$$

$$MSE_{TI} = 1 - \int_{-\infty}^{\infty} \frac{\overline{\mathcal{G}}^T(f)\overline{\mathcal{Q}}(f)\overline{\mathcal{Q}}^H(f)\overline{\mathcal{B}}_1}{|H(f)|^2 N_o/2} \quad (13)$$

Note that MMSE receiver designed for the z^{th} virtual user has the lowest MSE among all linear receivers, *i.e.*, $\frac{1}{v_1} MSE_{TI} \geq MSE_{TV}^z$ for all $z = 1, \ldots, v_1$. This implies, that $MSE_{TI} \geq \sum_{z=1}^{v_1} MSE_{TV}^z = MSE_{TV}$. Essentially, the TI-MMSE is over-designed to suppress more users than the TV-MMSE, and hence achieves a higher MSE.

VI. Simulation Results

First, we study the performance of the proposed receivers for the following three class system. The symbol periods of the three classes is assumed to be $T_k = k$, $k = 1, 2, 3$. All users in all three classes employ the same spreading gain, $L = 16$. There are three users in class 1 and four each in classes 2 and 3, *i.e.*, $P_1 = 3, P_2 = 4, P_3 = 4$. Further, $A_{1k} = 1$ for $k = 1, 2, 3$ and $A_{ik} = 3$ for $i = 2, \ldots, P_k$, $k = 1, 2, 3$. The codes for all users were chosen at random and a half-cosine pulse was chosen as $\phi_k(t)$ for simplicity. Finally, a pseudo-synchronous situation is assumed, *i.e.*, $\tau_{ik} = 0$ for i, k.

The probability of error as a function of SNR for the matched filter, time-invariant MMSE (TI-MMSE) and the proposed time-varying MMSE (TV-MMSE) receivers is studied. In Figures 4, 5, 6, the above mentioned three receivers are compared for users 11, 12, 13 respectively; single user bound is also shown. It can be seen that for probability of error equal to 10^{-3}, the loss incurred by the TI-MMSE receiver as compared to the TV-MMSE varies from 1 dB (user 13) to 4 dB (user 12). Finally, the user with smallest bandwidth, user 13, has the poorest performance as compared to the single user bound. These trends were also observed in [7].

The effect of varying front-end filter bandwidth on probability of error for class 3 users is shown in Figure 7; a $\frac{T_{c_1}}{4}$ sampling of the input signal was used. From Figure 7, a linear decay in probability of error as a function of front-end filter bandwidth is observed. Furthermore, if the front-end filter for user 13 is matched to its own bandwidth, the loss in performance is approximately 8 dB for an error probability of 10^{-3}.

VII. Conclusions

In this paper, the optimal linear MMSE receiver and the optimal linear time-invariant receiver for multirate

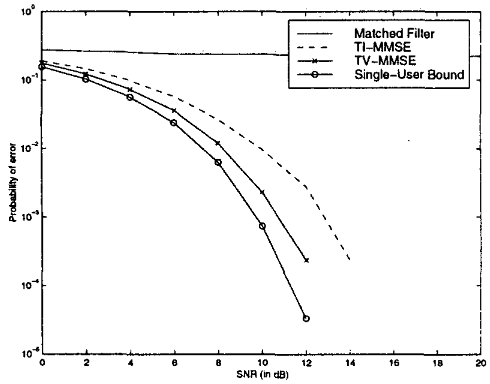

Fig. 4. Probability of error as a function of SNR for user 11.

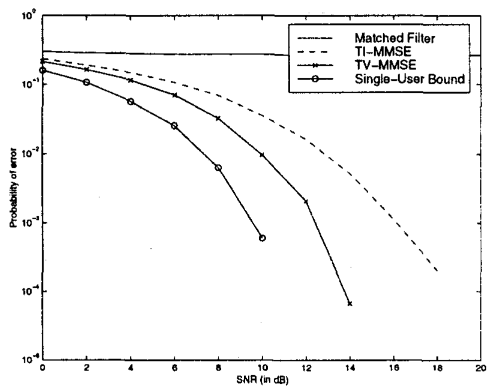

Fig. 5. Probability of error as a function of SNR for user 12.

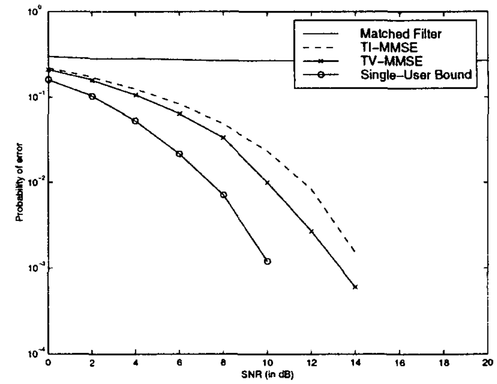

Fig. 6. Probability of error as a function of SNR for user 13.

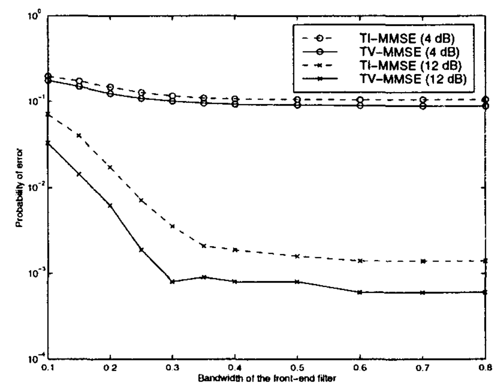

Fig. 7. Probability of error as a function of front-end filter bandwidth for user 13.

DS-CDMA systems were derived and compared. It was shown that the optimal linear MMSE filter is periodically time-varying and its period depends on the symbol periods of the interfering users. Using simulations, it was shown that time-invariant MMSE can incur a large loss in probability of error performance compared to the optimal time-varying MMSE receiver. Also, TV-MMSE receivers for users with more bandwidth had a lower probability of error as compared to the coexisting smaller bandwidth users.

The effect of the front-end bandwidth on smaller bandwidth user's receiver performance was studied both analytically and via simulations. It was observed that a loss in total system capacity and a degradation in probability of error is incurred, if the sampling frequency is less than the Nyquist frequency of any of the interfering users.

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