

Frequency-based Rate Separation for Dual-rate CDMA Signals

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Abstract

Recently, there has been considerable research on developing receivers for multi-rate synchronous DS/CDMA systems. The previously proposed receivers operate on the complete received signal without separating users of different rates. Multi-rate users occupy different bandwidths, and this paper explores user separation by exploiting these bandwidth differences. Receiver structures which use filters to achieve frequency-based rate-separation and then apply single-rate detection schemes are considered for a dual-rate synchronous CDMA system. Such a scheme enables a tradeoff between complexity and performance. The probabilities of error for the proposed receivers are derived. Comparisons are made between the proposed receivers and single-rate matched filter and the decorrelating detectors.

1 Introduction

With the exploding growth of cellular communication systems, there has been much interest in advanced Personal Communication Systems (PCS). PCS promises to offer wireless transport for a variety of data sources such as images (facsimiles), video and data as well as voice. This is in contrast to current systems, which are focused on providing speech services. Different information sources will have inherently different data rates and bandwidths. Therefore, in order to have a wireless system that will serve these diverse sources, it is desirable to develop multiple data rate systems. We propose to explore the feasibility of rate separation via frequency-based techniques. In this work, we shall develop multi-rate multi-user receivers which perform user separation by rate, followed by single-rate multi-user detection, and we shall analyze their performance.

Recently, Direct-Sequence/Code-Division Multiple-Access (DS/CDMA) systems have received much attention for single-rate communications as they enable efficient and dynamic use of the radio spectrum; they also provide natural implementations for multi-rate communications. With DS/CDMA, several access methods for multi rate systems are possible [4]. In the current work, we will assume a fixed processing gain for each user, but varying chipping rates (also seen in [5]).

Several receivers for multi-rate CDMA systems have been proposed, including the conventional receiver [1], the optimum receiver, decorrelator-based receivers [3], multi-stage

receivers and successive interference cancellation-based receivers. Most of these receivers operate on the complete received signal which consists of the multi-rate signals without any pre-processing. This causes the receiver structure to be complex and in some cases, dependent on the particular choice of spreading codes.

In this paper, we propose a class of receivers that pre-process the received multi-rate signal into several single-rate signals. In this way, a multi-rate problem is broken into multiple single-rate problems. Due to the assumption of different chipping rates for users of different data rates, the users will have different bandwidths. These bandwidth differences can be exploited for frequency-based rate separation. The notions of frequency-based narrowband interference suppression of [6] will be extended to multi-rate systems. Three rate-separating pre-processors will be considered: Wiener filters, high and low pass filters and some new filters based on minimizing interference. Once rate separation has been accomplished, previously proposed multi-user receivers for single-rate communications can be applied (the matched filter and the decorrelating detector [2] are considered here). The probability of error for these different schemes will be calculated for a synchronous dual-rate scenario.

This paper is organized as follows. Section 2 describes the received signal and the dual-rate communication scenario. The proposed receiver structures and the derivations for the probability of error are given in Section 3. Other filter designs and ongoing research are given in Section 4. Numerical results are presented in Section 5 and final conclusions are drawn in Section 6.

2 Preliminaries

For clarity, we shall consider a synchronous dual-rate DS-CDMA system with K_0 low rate users and K_1 high rate users. The receiver systems proposed herein are easily applicable to multi-rate, asynchronous systems. In this work, we shall also assume coherent reception and non-multipath channels. The subscript g will refer to low rate users ($g = 0$) or high rate users ($g = 1$). The bit duration of a user of type g is denoted by T_g . The rate ratio M is defined as T_0/T_1 and is assumed to be an integer. The received signal over the low-rate interval

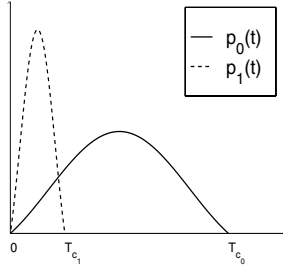


Figure 1: Pulse shapes for low rate and high rate users for $\alpha = 0.2$ and $M = 4$.

$[0, T_0]$ can be expressed as,

$$r(t) = \sum_{k=1}^{K_0} a_{k,0} b_{k,0}(t) s_{k,0}(t) + \sum_{k=1}^{K_1} \sum_{m=1}^M a_{k,1} b_{k,1}^m(t) s_{k,1}(t) + n(t), \quad (1)$$

where $a_{k,0}$ and $b_{k,0}(t)$ denote the received amplitude and received bit at time t for the k th low rate user. Similarly, $a_{k,1}$ and $b_{k,1}^m(t)$ denote the received amplitude and received bit during the m th bit at time t for the k th high rate user. The signature sequences are denoted by $s_{k,0}(t)$ and $s_{k,1}(t)$ for the k th low rate and high rate users, respectively. In this dual-rate system, we use a constant processing gain N for all users and let the chip rate vary according to the bit rate [5]. If T_{c_g} is the chip duration of the g th rate user, then $T_{c_0}/T_{c_1} = M$. The signature waveform can be expressed as

$$s_{k,g}(t) = \sum_{n=1}^N \frac{1}{\sqrt{N}} c_{k,g}(n) p_g(t - nT_{c_g}), \quad t \in [0, T_g], \quad (2)$$

where $c_{k,g}(n) \in \{-1, 1\}$ denotes the signature sequence of user k of group g .

In contrast to much research on CDMA, we shall assume essentially band-limited chip shapes. In particular, $p_g(t)$ shown in Figure 1 is assumed to have a raised cosine spectral pulse, and is defined by,

$$p_g(t) = \begin{cases} \text{sinc}((2t - T_{c_g})/T_{c_g}) \frac{\cos(\pi\alpha(2t - T_{c_g})/T_{c_g})}{1 - 4\alpha^2(2t - T_{c_g})^2/T_{c_g}^2}, & 0 \leq t \leq T_{c_g} \\ 0, & \text{elsewhere,} \end{cases}$$

where α is the *rolloff* factor, which takes values in the range $0 \leq \alpha \leq 1$. These pulses are essentially band-limited, in contrast to rectangular pulses which have much higher spectral leakage. The energy of the pulse $p_g(t)$ is normalized so that

$$\int_0^{T_g} (s_{k,g}(t))^2 dt = 1, \quad g = 0, 1; \quad k = 1, \dots, K_g. \quad (3)$$

Finally, the additive white Gaussian noise is denoted by $n(t)$, with variance σ^2 .

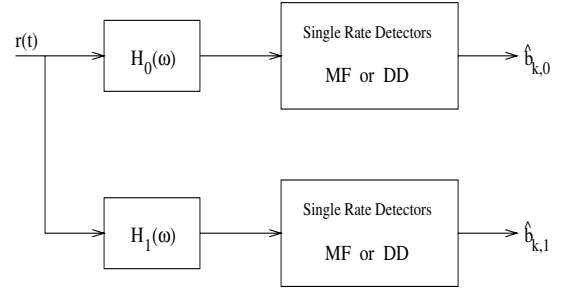


Figure 2: Receiver structure for dual-rate DS-CDMA system

3 Receiver Structures

In this dual chip-rate system, users with different rates have different bandwidths and therefore we can exploit this difference to separate users of different rates. Simple filters like low pass, high pass and Wiener filters can achieve frequency-based separation. Thus the dual-rate problem is decomposed into two single rate problems to which we can apply existing detection schemes. The proposed receiver system can be described by the block diagram in Figure 2. We begin by describing the filter choices and then review the single-rate multi-user receivers, modified to include other-rate interference and filter effects. In the sequel, new filter structures are explored.

3.1 Filter Choices

3.1.1 High and Low Pass Filters

The difference in the bandwidth between users of different rates suggests the use of simple filters, such as high pass and low pass filters, for detecting the high rate and low rate users, respectively. High and low pass FIR digital filters are designed using the Kaiser window with $\beta = 5$. The chosen cut-off frequencies of the filters depend on the rate ratio, M .

The filtering operation introduces inter symbol interference (ISI) which poses a constraint on the length of the filter. To limit the ISI to only adjacent bits, the filter length should be less than the bit duration of the high rate user.

3.1.2 Wiener Filter

It has been shown in [7], that suppression of multiple access interference in a single-rate DS-CDMA system using Wiener filters gives better performance than a single stage of parallel cancellation. In this dual-rate system, to detect users of one rate, Wiener filters can be used to suppress the multiple access interference due to the users of the other rate. The Wiener filter can be designed to be dependent or independent of the amplitude of the users. If g is the desired group of users, then the Wiener equation to suppress other rate users, independent of the amplitudes of the users can be written as,

$$H_g(\omega) = \frac{S_g(\omega)}{S_0(\omega) + S_1(\omega) + S_N(\omega)}, \quad (4)$$

where $S_N(\omega)$ and $S_g(\omega)$ refer to the spectrum of noise and of the users, respectively and are defined as follows.

$$S_N(\omega) = \sigma^2, \quad \forall \omega \quad (5)$$

$$S_g(\omega) = |\mathcal{F}(p_g(t))|^2, \quad (6)$$

where \mathcal{F} denotes the Fourier transform operator. The raised cosine frequency characteristics for a pulse of infinite length is given as

$$X_{rc}(\omega) = \begin{cases} \frac{T_{c_g}}{2}, & 0 \leq |\omega| \leq (1-\alpha)\pi/T_{c_g} \\ \frac{T_{c_g}}{2} [1 + \cos \frac{T_{c_g}}{2\alpha} (|\omega| - \frac{(1-\alpha)\pi}{T_{c_g}})], & (1-\alpha)\pi/T_{c_g} \leq |\omega| \leq (1+\alpha)\pi/T_{c_g} \\ 0, & (1+\alpha)\pi/T_{c_g} \leq |\omega| \leq \pi \end{cases}$$

Since $p_g(t)$ has finite duration T_{c_g} and is set to zero outside this interval, the Fourier transform of $p_g(t)$ can be obtained by convolving $\sqrt{X_{rc}}$ with the Fourier transform $W(\omega)$ of a rectangular window. Thus, $S_g(\omega) = |\sqrt{X_{rc}} * W(\omega)|^2$.

If one wishes to include the amplitude of the users, the corresponding Wiener equation is given by,

$$H_g(\omega) = \frac{a_g^2 S_g(\omega)}{a_0^2 S_0(\omega) + a_1^2 S_1(\omega) + S_N(\omega)}, \quad (7)$$

where $a_g^2 = \sum_{k=1}^{K_g} a_{k,g}^2$ is the effective amplitude squared of all users in rate g ($g = 0, 1$).

The rate separation filters are implemented as linear-phase, finite impulse response (FIR) approximations of the Wiener filter. We use FIR filters that are the best least-squares approximation to the desired frequency response $H_g(\omega)$.

3.2 Performance Analysis

We first describe the filtered signal and then consider two single-rate receivers. The signal at the output of the filter for the g th rate over the i th low rate bit can be written as,

$$\begin{aligned} \tilde{r}_g(t) = & \sum_{k=1}^{K_0} a_{k,0} \tilde{s}_{k,0}(t) b_{k,0}^i(t) + \sum_{k=1}^{K_0} a_{k,0} \tilde{s}_{k,0}^l(t) b_{k,0}^{i+1}(t) \\ & + \sum_{k=1}^{K_0} a_{k,0} \tilde{s}_{k,0}^r(t) b_{k,0}^{i-1}(t) \\ & + \sum_{k=1}^{K_1} \sum_{m=(i-1)M+1}^{iM} a_{k,1} \tilde{s}_{k,1}(t) b_{k,1}^m(t) \\ & + \sum_{k=1}^{K_1} \sum_{m=iM+1}^{(i+1)M} a_{k,1} \tilde{s}_{k,1}^l(t) b_{k,1}^m(t) \\ & + \sum_{k=1}^{K_1} \sum_{m=(i-2)M+1}^{(i-1)M} a_{k,1} \tilde{s}_{k,1}^r(t) b_{k,1}^m(t) + \tilde{n}(t), \end{aligned} \quad (8)$$

where $\tilde{s}_{k,0}(t)$ and $\tilde{s}_{k,1}(t)$ are the filtered signature sequences modified by $H_g(\omega)$ and $\tilde{n}(t)$ is colored Gaussian noise with zero mean and variance $\frac{\sigma^2}{2\pi} \int_{-\infty}^{\infty} |H_g(\omega)|^2 d\omega$. The ISI due to the bits on the left and right are denoted by $\tilde{s}_{k,g}^l(t)$ and $\tilde{s}_{k,g}^r(t)$, respectively.

3.2.1 Matched Filter

We first consider the conventional matched filter. This receiver is simply a filter which correlates the received signal with the spreading waveform of the user of interest. The output of the matched filters for all low rate users for the i th bit can be written in matrix form as,

$$\begin{aligned} \underline{y}_{MF(0)}^i = & \tilde{R}_{00} A_0 \underline{b}_0^i + \tilde{R}_{00}^l A_0 \underline{b}_0^{i-1} + \tilde{R}_{00}^r A_0 \underline{b}_0^{i+1} \\ & + \sum_{m=(i-1)M+1}^{iM} \tilde{R}_{01}^m A_1 \underline{b}_1^m + \tilde{R}_{01}^l A_1 \underline{b}_1^{(i-1)M} \\ & + \tilde{R}_{01}^r A_1 \underline{b}_1^{iM+1} + \tilde{S}_0^T \tilde{\mathbf{u}}, \end{aligned} \quad (9)$$

where A_g is a diagonal matrix whose diagonal elements are the amplitudes of users at rate g , and $\tilde{R}_{gg} = \tilde{S}_g^T \tilde{S}_g$ is the cross-correlation matrix between the users at rate g , where \tilde{S}_g is the filtered spreading code matrix of all the users at rate g .

The correlation matrices are defined as follows:

$$[\tilde{R}_{01}]_{i,j}^m = [\tilde{s}_{i,0}^{m-1} \tilde{s}_{i,0}^m \tilde{s}_{i,0}^{m+1}] [\tilde{s}_{j,1}^r \tilde{s}_{j,1} \tilde{s}_{j,1}^l]^T$$

$$[\tilde{R}_{01}]_{i,j}^l = (\tilde{s}_{i,0}^l)^T (\tilde{s}_{j,1}^l); [\tilde{R}_{01}]_{i,j}^r = (\tilde{s}_{i,0}^r)^T (\tilde{s}_{j,1}^r),$$

where $\tilde{s}_{i,0}^0 = \tilde{s}_{i,0}^M = 0$ and $\tilde{s}_{i,0}^m$ denotes the m th interval of the filtered spreading code of the i th low rate user.

Similarly, the output of the matched filter for the $(p = (i-1)M + m)$ th bit of the high rate users is given by,

$$\begin{aligned} \underline{y}_{MF(1)}^p = & \tilde{R}_{11} A_1 \underline{b}_1^p + \tilde{R}_{11}^l A_1 \underline{b}_1^{p-1} + \tilde{R}_{11}^r A_1 \underline{b}_1^{p+1} \\ & + \tilde{R}_{10}^m A_0 \underline{b}_0^i + \tilde{R}_{10}^l A_0 \underline{b}_0^{i-1} + \tilde{R}_{10}^r A_0 \underline{b}_0^{i+1} \\ & + \tilde{S}_1^T \tilde{\mathbf{u}}, \end{aligned} \quad (10)$$

where $[\tilde{R}_{10}]_{i,j}^x = (\tilde{s}_{i,1}^x)^T (\tilde{s}_{j,0}^x)$ for $x = m, l, r$. Because the filter lengths are restricted to one bit duration of the high rate user, there is no ISI from the low rate users for most of the high rate bits. Therefore, \tilde{R}_{10}^l and \tilde{R}_{10}^r are non-zero matrices only for $m = 1$ and $m = M$, respectively.

The probability of error for the k th low rate user can be written as,¹

$$P_{k,0}^{MF} = \frac{1}{2^{3K_0+(M+2)K_1-1}} \sum_{(\underline{b})_k=1} Q \left(\frac{(\underline{y}_{MF(0)}(\underline{b}))_k}{\sigma \sqrt{\tilde{s}_{k,0}^T \Sigma_N \tilde{s}_{k,0}}} \right), \quad (11)$$

where $(\underline{y}_{MF(0)}(\underline{b}))_k$ is as defined in (9) without the noise term and the elements of \underline{b} refer to the elements of all the \underline{b} vectors in (9). The summation is over all possible combinations of \underline{b} , however with the desired user's bit, $(\underline{b})_k$ as 1. The noise covariance matrix is denoted by Σ_N . The probability of error for the k th high rate user is

$$P_{k,1}^{MF} = \frac{1}{M} \sum_{m=1}^M \frac{1}{2^{3(K_0+K_1)-1}} \sum_{(\underline{b})_k=1} Q \left(\frac{(\underline{y}_{MF(1)}(\underline{b}))_k}{\sigma \sqrt{\tilde{s}_{k,1}^T \Sigma_N \tilde{s}_{k,1}}} \right), \quad (12)$$

¹ $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp -\frac{v^2}{2} dv$.

where $\left(\underline{y}_{MF(1)}(\mathbf{b})\right)_k$ is defined in (10).

3.2.2 Decorrelating detector

Using equations (9) and (10), the output of the decorrelating detector [2] is

$$\underline{y}_{DD(g)} = \tilde{R}_{gg}^{-1} \underline{y}_g \quad (13)$$

The output of the decorrelator for the low rate and high rate users can be expanded as,

$$\begin{aligned} \underline{y}_{DD(0)}^i &= A_0 \underline{b}_0^i + \tilde{R}_{00}^{-1} (\tilde{R}_{00}^l A_0 \underline{b}_0^{i-1} + \tilde{R}_{00}^r A_0 \underline{b}_0^{i+1} \\ &\quad + \sum_{m=(i-1)M+1}^{iM} \tilde{R}_{01}^m A_1 \underline{b}_1^m + \tilde{R}_{01}^l A_1 \underline{b}_1^{(i-1)M} \\ &\quad + \tilde{R}_{01}^r A_1 \underline{b}_1^{iM+1} + \tilde{S}_0^T \tilde{\mathbf{u}}) \end{aligned} \quad (14)$$

$$\begin{aligned} \underline{y}_{DD(1)}^p &= A_1 \underline{b}_1^p + \tilde{R}_{11}^{-1} (\tilde{R}_{11}^l A_1 \underline{b}_1^{p-1} + \tilde{R}_{11}^r A_1 \underline{b}_1^{p+1} \\ &\quad + \tilde{R}_{10}^m A_0 \underline{b}_0^i + \tilde{R}_{10}^l A_0 \underline{b}_0^{i-1} + \tilde{R}_{10}^r A_0 \underline{b}_0^{i+1} \\ &\quad + \tilde{S}_1^T \tilde{\mathbf{u}}). \end{aligned} \quad (15)$$

Thus, the probabilities of the k th low rate and high rate users are given as,

$$\begin{aligned} P_{k,0}^{DD} &= \frac{1}{2^{2K_0+(M+2)K_1}} \sum_{\mathbf{b}, (\mathbf{b})_k=1} Q \left(\frac{\left(\underline{y}_{DD(0)}(\mathbf{b})\right)_k}{\sigma \sqrt{(\tilde{\Sigma}_0)_{k,k}}} \right) \\ P_{k,1}^{DD} &= \frac{1}{M} \sum_{m=1}^M \frac{1}{2^{2K_0+3K_1}} \sum_{\mathbf{b}, (\mathbf{b})_k=1} Q \left(\frac{\left(\underline{y}_{DD(1)}(\mathbf{b})\right)_k}{\sigma \sqrt{(\tilde{\Sigma}_1)_{k,k}}} \right), \end{aligned}$$

where $\tilde{\Sigma}_g = \tilde{R}_{gg}^{-1} \tilde{S}_g^T \Sigma_N \tilde{S}_g \tilde{R}_{gg}^{-1}$.

In the absence of a filter, the expressions for the probabilities of error simplify to the standard case as there are no terms due to ISI:

$$\begin{aligned} P_{k,0}^{MF} &= \frac{1}{2^{K_0+MK_1-1}} \sum Q \left(\frac{R_{00} A_0 \underline{b}_0 + \sum_{m=1}^M R_{01}^m A_1 \underline{b}_1^m}{\sigma \sqrt{s_{k,0}^T s_{k,0}}} \right) \\ P_{k,1}^{MF} &= \frac{1}{M} \sum_{m=1}^M \frac{1}{2^{K_0+K_1-1}} \sum Q \left(\frac{R_{11} A_1 \underline{b}_1 + R_{10}^m A_0 \underline{b}_0}{\sigma \sqrt{s_{k,1}^T s_{k,1}}} \right) \\ P_{k,0}^{DD} &= \frac{1}{2^{MK_1}} \sum Q \left(\frac{A_0 \underline{b}_0 + R_{00}^{-1} \left(\sum_{m=1}^M R_{01}^m A_1 \underline{b}_1^m \right)}{\sigma \sqrt{(R_{00}^{-1})_{k,k}}} \right) \\ P_{k,1}^{DD} &= \frac{1}{M} \sum_{m=1}^M \frac{1}{2^{K_1}} \sum Q \left(\frac{A_1 \underline{b}_1 + R_{11}^{-1} (R_{10}^m A_0 \underline{b}_0)}{\sigma \sqrt{(R_{11}^{-1})_{k,k}}} \right), \end{aligned}$$

where all the summations are over all possible combinations of \underline{b}_0 and \underline{b}_1 with $(\underline{b}_g)_k = 1$.

4 Other Filter Designs

It will be observed in Section 5 that the Wiener and high-pass filters work well for high-rate users, but the performance

gains for low-rate users are not as significant. We provide the following intuitive explanation. We focus on filters for the matched filter detector, but similar comments and filter designs apply for rate-separating prefilters for the decorrelating detector receiver. In an effort to suppress the MAI due to out-of-rate users, the Wiener filter reduces the desired user's signal energy and increases correlation amongst users at the same rate. Furthermore, as noted earlier, ISI is introduced and the overall noise variance is increased. Assessment of the causes of the limited performance gains for low rate users showed that there are five quantities to be considered when designing filters that would achieve higher performance gains: preserving the desired user's energy, reducing the correlation between in-rate users, reducing the correlation between users of different rates, reducing the ISI, and reducing the filter output noise variance. We can see that all of these constraints are present in the argument of the Q function in (11).

In the high SNR region, the probability of error is dominated by the largest Q function. Therefore, considering the high SNR region, one option is to find a filter that minimizes the dominant Q function in order to reduce the probability of error for the matched filter. In other words, for the k th low rate user, maximize the following expression:

$$\begin{aligned} f(\mathbf{h}) &= \frac{1}{\sigma \sqrt{s_{k,0}^T \Sigma_N s_{k,0}}} \left(|\tilde{R}_{00}| A_0 \underline{b}_0^d + |\tilde{R}_{00}^l| A_0 \underline{b}_0 + |\tilde{R}_{00}^r| A_0 \underline{b}_0 \right. \\ &\quad \left. + \sum_{m=1}^M |\tilde{R}_{01}^m| A_1 \underline{b}_1 + |\tilde{R}_{01}^l| A_1 \underline{b}_1 + |\tilde{R}_{01}^r| A_1 \underline{b}_1 \right)_k, \end{aligned} \quad (16)$$

where \underline{b}_0 and \underline{b}_1 are of length K_0 and K_1 , respectively, with all elements equal to -1. The vector \underline{b}_0^d is of length K_0 with the k th element as 1 and all other elements equal to -1.

The filtered sequence is a linear transformation of shifted versions of the unfiltered sequence; that is, $\tilde{\mathbf{x}}_{k,0} = B_{k,0} \mathbf{h}$ where \mathbf{h} is the filter of length L . The matrix $B_{k,0}$ is defined as,

$$B_{k,0} = \begin{bmatrix} s_{k,0}(\frac{P}{2} + 1) & \cdot & s_{k,0}(N) & \cdot & 0 \\ s_{k,0}(\frac{P}{2}) & \cdot & s_{k,0}(N-1) & \cdot & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{k,0}(1) & \cdot & s_{k,0}(N - \frac{P}{2}) & \cdot & s_{k,0}(N) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdot & s_{k,0}(N - (P)) & \cdot & s_{k,0}(N - \frac{P}{2}) \end{bmatrix},$$

where $P = L - 1$. The elements of the cross-correlation matrix $\tilde{R}_{0,0}$ can be re-written as $[\tilde{R}_{0,0}]_{k,j} = \mathbf{h}^T B_{0,k}^T B_{0,j} \mathbf{h} = \mathbf{h}^T \Gamma_{0,k,0_j} \mathbf{h}$. Thus, the function to be maximized can be expressed explicitly in terms of the filter as,

$$\begin{aligned} f(\mathbf{h}) &= \frac{1}{\sigma \sqrt{\mathbf{h}^T B_{0,k}^T \Sigma_N B_{0,j} \mathbf{h}}} \left(a_{k,0} \left| \mathbf{h}^T \Gamma_{0,k,0_k} \mathbf{h} \right| \right. \\ &\quad - \sum_{j=1; j \neq k}^{K_0} a_{j,0} \left| \mathbf{h}^T \Gamma_{0,k,0_j} \mathbf{h} \right| - \sum_{j=1}^{K_0} a_{j,0} \left| \mathbf{h}^T \Gamma_{0,k,0_j}^l \mathbf{h} \right| \\ &\quad \left. - \sum_{j=1}^{K_0} a_{j,0} \left| \mathbf{h}^T \Gamma_{0,k,0_j}^r \mathbf{h} \right| - \sum_{j=1}^{K_1} \sum_{m=1}^M a_{j,1} \left| \mathbf{h}^T \Gamma_{0,k,1_j}^m \mathbf{h} \right| \right) \end{aligned}$$

$$-\sum_{j=1}^{K_1} \left(a_{j,1} \left| \mathbf{h}^T \Gamma_{0_k,1,j}^l \mathbf{h} \right| + a_{j,1} \left| \mathbf{h}^T \Gamma_{0_k,1,j}^r \mathbf{h} \right| \right)$$

We call the maximizing \mathbf{h} the *Q-filter design*. Similar cost functions can be derived for the decorrelator. This is a non-linear maximization problem. Our experience is that the iterative algorithm seeking the desired filter often converges to a local maxima. At present, research is ongoing to address this issue. Simpler cost functions that are easier to maximize, that depend on less multi-rate multi-user information and give good bit-error rate performance are also under consideration. Lower bit-error rate can be achieved by weighting the five constraints in the cost function appropriately. For example, reduction of the noise variance can be emphasized by using the function: $f_1(\mathbf{h}) = f(\mathbf{h}) - \left(\sigma \sqrt{\mathbf{h}^T B_{0,k}^T \Sigma_N B_{0,k} \mathbf{h}} \right) / \left(\left| \mathbf{h}^T \Gamma_{0_k,0_k} \mathbf{h} \right| \right)$. It is shown in Figure 6 that for low values of SNR the filter which uses this latter function performs better than the one which uses $f(\mathbf{h})$.

5 Numerical Results

In this section, the performance of the single-rate receivers with filtering will be compared to that without any filtering. Let us consider a synchronous CDMA system which supports four users where $K_0 = 2$ and $K_1 = 2$ with rate ratio² $M = 4$. The processing gain for all the users is 63.

Figure 3 provides the normalized spectra of the low rate and high rate users, along with the filters $H_0(\omega)$ and $H_1(\omega)$ used for rate separation. Figure 4 shows the bit error rate for the first high rate user as a function of SNR. The amplitudes of the high rate users have been fixed to 1 and that of the low rate users have been fixed to 8. The performance of the receiver with the Wiener filter and with a high pass filter is shown. The filter lengths have been chosen to be 31 as the performance did not improve for higher values of the lengths. With the high pass filter, the performance of the receiver for various cut-off frequencies was observed and it was found that a cut-off of $.15\pi$ yielded the best result. This cut-off frequency is specific for a rate ratio of 4. It is clear that the suppression of interference due to the low rate users using filters improves the performance for the high rate user.

The Wiener filter which depends on the amplitudes of the users reduces the energy of the desired rate users in an effort to suppress the interference due to the other rate users. This causes the performance of the receiver to be poor when compared to the performance for the amplitude-independent Wiener filter. Figure 4 also shows that the performance gain in using the *Q*-filter instead of the Wiener filter is negligible.

Figure 5 shows the BER of the high rate user as a function of the amplitude ratio with the SNR fixed at 12 dB. When the interference due to the low rate users is low, the receiver without any filter performs better than those with filters. This is

²Modest rate ratios and user populations are considered due to the exponential complexity of the probability of error calculations. Gaussian approximations which would enable larger user sets are under consideration.

due to the fact that the filters reduce the energy of the desired user, introduce ISI and increase the variance of the noise, in addition to suppressing MAI due to other rate users. Here, we can see that the amplitude-dependent Wiener filter performs better than the amplitude-independent filters for low values of $a_{k,0}/a_{k,1}$.

The performance of the low rate users is presented in Figures 6 and 7. It can be seen that the low rate performance has the same trend as that of the high rate users. However, the improvement of the receiver with Wiener filter over that without any filter is much lower in this case. This is due to the high interference caused by the high rate users over the major portion of the low rate spectrum. For low rate users, there is thus a trade-off between suppressing the interference due to other rate users and preserving the desired user energy.

6 Conclusion

In this paper, we have proposed receivers which use filters to achieve user separation by rate, followed by single-rate multi-user detection. The proposed receivers offer improved performance in the detection of high data rate users when compared to single-rate detectors such as matched filter or decorrelator which ignore the out-of-rate users. However, the performance gains for the low rate users are not significant. Although the performance of this receiver is lower than that of the decorrelators proposed in [3], the proposed receiver has the advantage of low complexity. Simple filters like the Wiener filter and high and low pass filters have been used to achieve frequency-based rate separation. The modest performance gain for low rate users motivates our investigation into more complex filter designs, such as the *Q*-filter. Additionally, we note that since the BER of the high rate users is low, improvement in performance for the low rate users can be achieved by using multi-stage detectors which include high rate user detection and cancellation.

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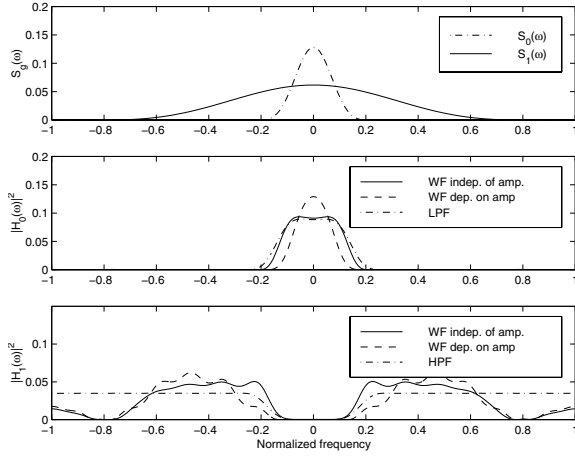


Figure 3: Normalized spectra of low rate and high rate users (top), and rate separation filters $H_0(\omega)$ (middle) and $H_1(\omega)$ (bottom) for the examples considered.

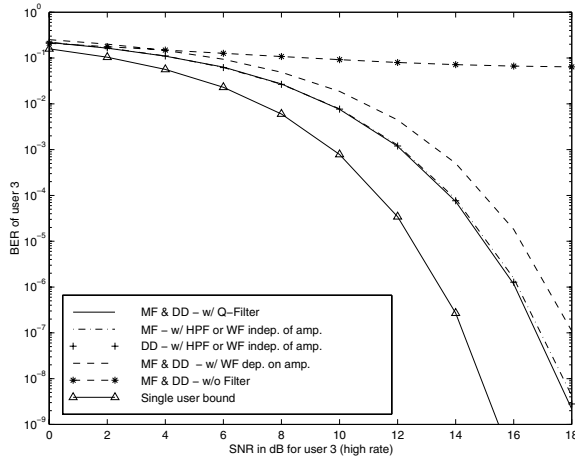


Figure 4: High rate user performance as a function of SNR; $K_0 = 2, K_1 = 2; a_{k,1} = 1, a_{k,0} = 8; M = 4$.

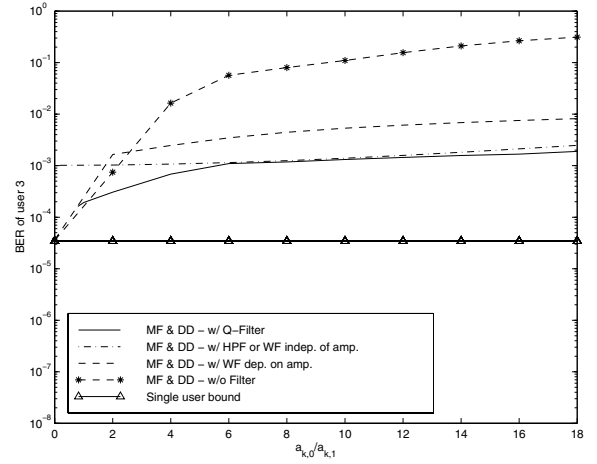


Figure 5: High rate user performance as a function of near-far ratio; $K_0 = 2, K_1 = 2; SNR = 12dB; M = 4$.

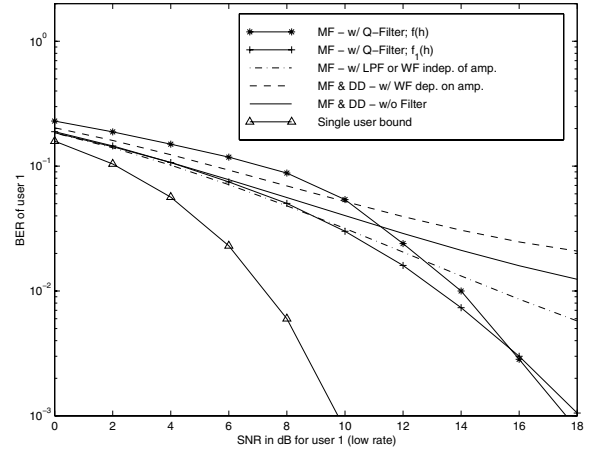


Figure 6: Low rate user performance as a function of SNR; $K_0 = 2, K_1 = 2; a_{k,0} = 1, a_{k,1} = 4; M = 4$.

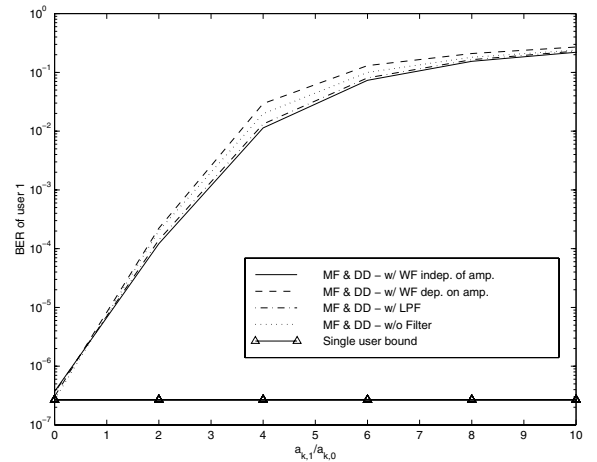


Figure 7: Low rate user performance as a function of near-far ratio; $K_0 = 2, K_1 = 2; SNR = 14dB; M = 4$.