A Parametric Model for Synthetic Aperture Radar Measurements

Michael J. Gerry, Lee C. Potter, Senior Member, IEEE, Indr J. Gupta, Senior Member, IEEE, and Andria van der Merve

Abstract—We present a parametric model for radar scattering as a function of frequency and aspect angle. The model is used for analysis of synthetic aperture radar measurements. The estimated parameters provide a concise, physically relevant description of measured scattering for use in target recognition, data compression and scattering studies. The scattering model and an image doqaut estimation algorithm are applied to two measured data examples.

Index Terms—Image resolution, inverse scattering, radar imaging.

I. INTRODUCTION

At high frequencies, the scattering response of an object is well approximated as a sum of responses from individual scattering centers (1). These scatterers provide a physically relevant, yet concise, description of the object and are thus good candidates for use in target recognition, radar data compression, and scattering phenomenology. In this paper we consider the analysis of radar data 

The model is motivated by both the physical optics and the geometric theory of diffraction (GTD) non-stationary scattering solutions and extends the one-dimensional GTD-based parametric model presented in [2] to include aspect angle. Our model provides a physical description of radar scattering centers, each of which is described by a set of parameters characterizing position, shape, orientation (pose), and relative amplitude. This is a richer description of target scattering than is available either from conventional Fourier-based imaging techniques [3] or from less physically accurate point-scattering parametric models.

Recent developments in mechanism extraction from 2-D radar data [4]-[9] are based on the assumption that scattering centers are localized to isolated points. While this description is valid for many scattering centers at many aspect angles, some common scattering mechanisms behave as distributed elements, and point scattering models fail to accurately model the scattering. The aspect dependence in our 2-D model allows description of both localized and distributed scattering centers, providing a higher fidelity description of scattered fields. The model provides the potential both for improved data compression and for the discrimination of localized versus distributed scattering mechanisms.

The paper is organized as follows. In Section II, we develop a simple parametric model of far-field scattering as a function of frequency and aspect angle. In Section III, we transform the frequency-aspect angle-domain model into the image domain for the purpose of parameter estimation; image segmentation provides the advantages of clutter suppression, model-order reduction, and computational savings. In Section IV, we present an algorithm for extraction of the unknown parameters of the model from an image-domain representation of the measured data. In Section V, we present experimental results obtained by applying our estimation algorithm to data measured in a compact-range anechoic chamber. In Section VI, we use the Cramér-Rao lower bound (CRLB) to predict uncertainty in the estimated model parameters.

II. MODEL DEVELOPMENT

We develop a parametric model for the backscatter from objects measured as a function of frequency and aspect angle. We seek a model that maintains high fidelity to the scattering physics for many objects, yet is sufficiently simple in its functional form to permit robust inference from estimated parameters.

For this development, we assume a data collection scenario consistent with synthetic aperture radar (SAR) imaging. A reference point is defined, and we require that the radar trajectory and reference point are coplanar. We label this imaging plane using an x-y-Cartesian coordinate system with origin at the reference point. The radar position is then described by an angle φ defined counterclockwise from the x axis. We assume far-field backscatter and, therefore, obtain plane wave incidence on objects.

From the GTD [1] and its uniform version [10], if the wavelength of the incident excitation is small relative to the target extent, then the backscattered field from an object consists of contributions from electrically isolated scattering centers. In developing our model, we characterize the frequency and aspect angle dependence of individual scattering centers. Each scattering center is described by a small number of parameters. The total scattered field from a target is then modeled as the sum of these individual scatterers.
We make three assumptions about the far zone backscattered field and each assumption leads to the functional form for a portion of our scattering model. First, phase dependence is linear and defined by the position of the scattering center. Second, amplitude dependence on frequency is defined by the high-frequency approximation derived from the GTD. Third, amplitude dependence on aspect angle is defined by characterizing the scattering center as either spatially localized or distributed. We consider these three dependencies, each in turn, to arrive at a parametric scattering model.

First, we consider only far-field scattering with a linear phase dependence on frequency. The phase of a scattering center, at a given aspect angle, is determined by the down range position of the scatterer. Accordingly, the backscattered field of the nth scattering center is expressed

$$E^b_k(k, \phi) = S_n(k, \phi) \exp[-i(k\cdot r_n)] \tag{1}$$

where \( k = 2\pi f/c \) is the wave number, \( f \) is frequency in hertz, \( c \) is the propagation velocity, \( \phi \) is the aspect angle, \( k \) is the unit vector in the direction of the scattered field, and \( r_n = [x_n, y_n, z_n] \) is the position vector of the nth scattering center projected to the plane. Note that the restriction to linear phase scatterers excludes phase dispersive scattering mechanisms such as resonant cavities and creeping waves. The c/2f^2 time convention is assumed and suppressed. Here we consider only the localized field, as such, all field quantities are written as scalars. The development is easily extendible to multiple polarizations. In summary, the phase dependence of our model describes the location of each scattering center in the plane of the radar measurement.

Second, we consider the amplitude dependence on frequency. In presenting the GTD, Keller (1) uses a conservation of energy argument to propose that the field diffracted from a point on an edge is proportional to \((jk)^{-1}\), and the field diffracted from a vertex is proportional to \((jk)^{-1}\). The simplicity of the GTD is that many practical object geometries give rise to a sum of two of these two scattering mechanisms. In [11] and [12], it is shown that in addition to the edge and vertex diffraction, a larger class of scattering geometries also fits the \((jk)^{-1}\) power dependence on frequency, where the parameter \( \alpha \) has a half integer value (see Table I).

Third, we consider aspect dependence of scattering amplitude. As aspect angle is varied, we assume that a scattering center behaves in one of two ways: either a scatterer is localized and appears to exist at a single point in space, or it is distributed in the imaging plane and appears as a finite, nonzero-length current distribution. The amplitude dependence on aspect angle is different for each of these scenarios, and we seek a model that accounts for both scattering behaviors in a physically accurate, yet simple, functional form.

Examples of localized scattering mechanisms are trihedral reflection, corner diffraction, and edge diffraction. All of these mechanisms have slowly varying amplitude as \( \phi \) function of aspect angle. We exploit the commonality of localized mechanisms by modeling this slowly varying function with a damped exponential

$$S_n(f, \phi) = A_n \exp(-2\pi f \gamma_n \sin \phi). \tag{2}$$

The exponential function provides a mathematically convenient approximation containing only a single parameter. Although physical insight is used to arrive at the exponential model, the parameter \( \gamma_n \) has no direct physical interpretation. On the other hand, examples of distributed scattering mechanisms are flat plate reflection, dihedral reflection, and cylinder reflection. Each of these scattering mechanisms has an amplitude dependence on aspect angle that contains \( \sin(\pi \phi) = \sin(\phi) \) function. In all cases, this \( \sin(\phi) \) function is the dominant term in the physical optics far-zone scattering solution and we adopt the \( \sin(\phi) \) function to characterize angle dependence in the scattering model for scattering centers that are distributed.

$$S_n(f, \phi) = A_n \sin \left( \frac{2\pi f}{c} L_n \sin \phi(\phi - \phi_n) \right) \tag{3}$$

where \( L_n \) is the length and \( \phi_n \) is the orientation angle of the distributed scatterer.

We combine the different model terms from the localized and the distributed scattering mechanisms to write our 2-D scattering model in a single expression

$$E^b_n(f, \phi) = A_n \left( \frac{2\pi f}{c} \right)^{\alpha_n} \sin \left( \frac{2\pi f}{c} L_n \sin \phi(\phi - \phi_n) \right) \times \exp(-2\pi f \gamma_n \sin \phi) \times \exp \left( \frac{2\pi f}{c} (x_n \cos \phi + y_n \sin \phi) \right) \tag{4}$$

where \( L_n = 0 \) if the scattering center is localized, and \( \gamma_n = 0 \) if the scatterer is distributed. The parameter \( A_n \) is a relative amplitude for each scattering center. The total scattered field is a sum of \( p \) individual scattering terms

$$E^b(f, \phi) = \sum_{n=1}^{p} E^b_n(f, \phi). \tag{5}$$

The scattering model in (4) is a function of frequency and aspect angle and is described by the parameter set \((A_n, x_n, y_n, \alpha_n, \gamma_n, L_n, \phi_n)\) for \( n = 1, \ldots, p \). The parameters provide a rich physical description of the scatterers that are present in the data set. Each parameter, with the exception of \( \gamma_n \), has a direct physical interpretation. Example scattering geometries distinguishable by their \((\alpha, L)\) parameters are presented in Table II. The model is based on scattering physics and is developed to describe a large class of scatterers while still maintaining a relatively simple form.
III. TRANSFORMATION OF MODEL INTO IMAGE DOMAIN

The model in (4) describes scattering in the frequency-aspect domain. For most SAR data collection geometries, imaging in a frequency-aspect space is often done as a fast Fourier transform (FFT) of the measured frequency-aspect data. The imaging algorithm is widely used on SAR systems for which the center frequency of the radar is large compared to the bandwidth of the radar. According to the FFT, it is possible to perform a 2-D FFT on the measured frequency-aspect domain scattering model.

We begin with the model in (4) and arrive at an image model by two straightforward, first, the power dependence of amplitude on frequency with an exponential angle measurements would pass during image formation. There are many methods for image formation [13], but we limit the discussion in this work to a 2-D inverse Fourier transform (IFT) of the measured frequency-aspect data. This imaging algorithm is widely used in spotlight SAR systems for which the center frequency of the radar is large compared to the bandwidth of the radar. Accordingly, we analytically perform a 2-D IFT on the measured frequency-aspect domain scattering model.

In order to accomplish the image domain parameter estimation, we analytically transform the scattering model from the frequency-aspect domain into the image domain. We process the parameter model using the same series of operations through which the motion-compensated frequency-aspect angle measurements would pass during image formation. There are many methods for image formation [13], but we limit the discussion in this work to a 2-D inverse Fourier transform (IFT) of the measured frequency-aspect data. This imaging algorithm is widely used in spotlight SAR systems for which the center frequency of the radar is large compared to the bandwidth of the radar. Accordingly, we analytically perform a 2-D IFT on the measured frequency-aspect domain scattering model.

We begin with the model in (4) and arrive at an image model by two steps. First, we replace the power dependence of amplitude on frequency with an exponential in (13):

\[
(2\pi f) e^{i\phi} = \exp(-2\pi f) \quad (6)
\]

where \(f_0\) is a damping factor. We let the \(f_0\) term be absorbed into the complex amplitude \(A_0\). We adopt the following affine map from \(f_0\) to \(f\) as proposed in (13):

\[
\alpha_0 = \frac{f}{f_0} (1 - i \Delta f) \quad (7)
\]

where \(f_0\) is the center frequency, and \(\Delta f\) is the frequency increment. The expression in (7), while analytically convenient, is nonetheless extremely accurate for small relative bandwidths [13]. For example, at 10% relative bandwidth, the approximation has less than 0.0001% relative error. As the bandwidth increases this error increases. Using this approximation, we first estimate \(f_0\) and then map \(f_0\) to \(f\).

By making this coordinate transformation to the Cartesian frequency plane, we assume that the measured data is sufficiently narrow in bandwidth so as to allow simple, approximate interpolation in [14], [15] to a rectangular grid. We further approximate

\[
2f_0 \sim 2f, \quad (9)
\]

in the frequency-dependent exponential of (6); this approximation is valid for small angle spans.

Third, frequency and angle domain window functions are often used in SAR imaging for sidelobe suppression. We assume that the window functions are separable in their Cartesian components and can be written as

\[
W(f_x, f_y) = W(f_x) W(f_y) \quad (10)
\]

Many commonly used window functions such as rectangular, Hamming, and Taylor windows can be expressed exactly as in (10).

Fourth, we transform \(E_x(f_x, f_y)\) to the image domain with a 2-D IFT. Note that, in practice, measured data exists at a finite number of discrete frequencies and aspect angles. As a result, the IFT performs to generate radar imagery is typically an Inverse Discrete Fourier Transform (IDFT). Here we analytically perform a continuous IFT for simplicity. In fact, the alternative image domain model using the IDFT is not available in closed form. The IDFT is approximately equal to the continuous IFT when the image-domain signal is essentially support limited. Since most radar imagery contains a small number of high-energy regions that are limited in extent, the sampling-induced aliasing is negligible. Thus, we assume that the sampled IFT is well-approximated by a continuous IFT for radar imagery.

The image-domain model \(e_x(t_x, t_y)\) for a single scattering center is then written as

\[
e_x(t_x, t_y) = \int_{f_y} \int_{f_x} \sum_{k=1}^{P} \sum_{l=1}^{Q} A_k B_l e^{i\theta_k} e^{i\theta_l} \quad (11)
\]

This completes our imaging model for synthetic aperture radar measurements.
where $f_{x1}$, $f_{y1}$ are the first and last $f_x$ frequencies, and $f_{y1}$, $f_{y2}$ are the first and last $f_y$ frequencies.

As discussed in Section II, either $L_x = 0$ or $\gamma_x = 0$ and we consider each case separately. If $L_x = 0$, evaluation of the integrals in (11) yields

$$
c^*_b (t_x, t_y) = A_x \sum_{p=1}^{P} \sum_{q=1}^{Q} 2d^p \beta^{q} F_x F_y \left( \exp \left\{ 2\pi f_{x1} \left[ -t_x + \frac{j2\beta}{c} \right] + j f_{x2} t_x \right\} ight)
- \exp \left\{ 2\pi f_{y1} \left[ -t_y + \frac{j2\beta}{c} \right] + f_{y2} t_y \right\}
- \sinh \left( \pi F_x \right) \left[ -t_x + \frac{j2\beta}{c} \right] + j f_{x2} t_x 
- \sinh \left( \pi F_y \right) \left[ -t_y + \frac{j2\beta}{c} \right] + j f_{y2} t_y \right\} 
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chip, while $\nu_x$ and $\nu_y$ are initialized at zero (isotropic point scattering). For a fixed parameter set $\theta$, the least-squares cost function $J$ is quadratic in the complex amplitude parameter $A$; therefore, the least-squares estimate of $A$ is computed noniteratively using a matrix pseudo-inverse.

At convergence, the simplex downhill optimization yields estimates of scattering parameters that describe the position, shape, and orientation of the scattering centers that comprise the measured target. Automation of model order selection and parameter initialization is a topic of continuing development, both for our proposed scattering model and for simpler point scattering models [21], [22].

V. EXAMPLES

We present two examples to illustrate the fidelity of the scattering model and the accuracy of the estimation procedure. The estimation algorithm extracts parameters that describe the position, shape, and orientation of the scattering centers on the target. The estimation results show that the physically based scattering model provides an excellent means for compressing a large, measured data record into a small set of physically relevant parameters. Measurements were collected at The Ohio State University Electr0Science Laboratory (ESL) Compact Range [23].

First, we consider the scattering from a square flat plate. We analyze stepped frequency measurements of the plate for frequencies 9.5–10.5 GHz in 20-MHz steps and for angles $\pm 3^\circ$ (in 0.5° steps) from broadside to one of the edges. The plate is a two foot square and lies in the plane of rotation. The measurement polarization is horizontal.

Fig. 1 shows an image of the plate. The image contains three scattering centers. The broadside response of the edge of the plate appears as a line in the image. The two corners on the back of the plate appear as localized scattering mechanisms. These three mechanisms are seen in the image and the algorithm of Section IV is used to estimate the parameters. Table III shows the estimated parameters and their actual values. The actual values are based on the assumption that the plate is exactly two foot square and is perfectly aligned during radar measurements so that $\theta_0$ corresponds to broadside to an edge. The estimated tilt angle is approximately $-0.6^\circ$, which is an indication that the plate was not exactly aligned with $0^\circ$ broadside to the radar. Fig. 2 shows the amplitude of the scattering from the plate as a function of angle at the center frequency 10 GHz. Note that the peak is not at $0^\circ$ as we would expect for a perfectly aligned target. The misalignment of the target also contributes to a small amount of error in the expected locations of the three scattering centers.

The image generated with the estimated parameters has less than 3% mean-square error (MSE) with the measured image. The error in the estimated location of the individual scattering centers is small and in each case is less than one tenth the Fourier resolution. The geometric type (iv) estimates correctly identify the edge specular and corner diffraction scattering behaviors. The algorithm compresses the measured, complex-valued $51 \times 13$ point data array into a small table of seven numbers describing the edge mechanism and six numbers describing each of the two corner mechanisms. The model, therefore, provides a 69:1 lossy compression of the original measured frequency-azimuth data.

Second, we consider a scale model of an F117 aircraft. The model is constructed from flat aluminum plates. We analyze data from 9.5 to 10.5 GHz in 10-MHz steps and $\pm 3^\circ$ from normally incident on the leading edge of one of the wings in 0.1° steps. Fig. 3 shows the image of the aircraft with an overlay showing the true location of the target in the image plane. The alpha estimate for the wing edge is zero, which is consistent with the target geometry. Also, we fit two localized mechanisms to the tail region of the aircraft. The estimated locations of these scattering are indicated by the small circles in the image. Table IV shows the parameter estimates for this example. The thick solid line that is nearly coincident with the leading edge of the wing shows the estimated location, tilt, and length of the scattering center. When considering all three scattering centers, the overall MSE in the image is less than 7%. The model provides a compression ratio of 64:1.
To illustrate the advantages of incorporating aspect dependence in our scattering model, we compare with a point scattering model. We use the localized scattering center model in (15), (with frequency and angle dispersion parameters $\gamma$ and $\phi$ and with $L = 0$) to fit the scattering from the leading edge of the aircraft wing. Model order is varied from one to six. Fig. 4 shows the results for a model order of four. The localized scattering center model requires a much larger set of parameters to achieve comparable MSE than the distributed scattering center ($L \neq 0$) model. Although a lower MSE is achieved for increasing model order (see Table V), using six or fewer points, the localized scattering center model does not achieve the MSE of the proposed distributed scattering model. For example, a model order of six corresponds to a table of 30 numbers describing the scattering and, in this example, yields a MSE greater than 1%. A further disadvantage of the localized scattering center model for describing distributed scattering is that the estimated locations are not related to any physical quantity,
TABLE IV
ESTIMATED SCATTERING PARAMETERS FOR F117 EXAMPLE (EMITTED SCATTERING MODEL)

<table>
<thead>
<tr>
<th>Scatterer</th>
<th>Attribute</th>
<th>Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing Edge</td>
<td>Length</td>
<td>1.469 m</td>
</tr>
<tr>
<td></td>
<td>Tip</td>
<td>-0.304 m</td>
</tr>
<tr>
<td></td>
<td>Down Range</td>
<td>7.3127 m</td>
</tr>
<tr>
<td></td>
<td>Cross Range</td>
<td>1.2644 m</td>
</tr>
<tr>
<td></td>
<td>Alpha</td>
<td>0</td>
</tr>
<tr>
<td>Tail Region #1</td>
<td>Down Range</td>
<td>7.8753 m</td>
</tr>
<tr>
<td></td>
<td>Cross Range</td>
<td>4.7674 m</td>
</tr>
<tr>
<td></td>
<td>Alpha</td>
<td>-1</td>
</tr>
<tr>
<td>Tail Region #2</td>
<td>Down Range</td>
<td>7.8735 m</td>
</tr>
<tr>
<td></td>
<td>Cross Range</td>
<td>1.9986 m</td>
</tr>
<tr>
<td></td>
<td>Alpha</td>
<td>-2</td>
</tr>
</tbody>
</table>

TABLE V
MEAN SQUARE ERROR VS. K.IN Model, TABLE FOR F117 Example (EMITTED SCATTERING Model)

<table>
<thead>
<tr>
<th>Model Order</th>
<th>MSE Square Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>99.90%</td>
</tr>
<tr>
<td>2</td>
<td>0.64%</td>
</tr>
<tr>
<td>3</td>
<td>50.27%</td>
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<tr>
<td>4</td>
<td>34.16%</td>
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<tr>
<td>5</td>
<td>21.82%</td>
</tr>
<tr>
<td>6</td>
<td>11.55%</td>
</tr>
</tbody>
</table>

VI. STATISTICAL ANALYSIS

In this section, we investigate the noise sensitivity of estimated parameters for the scattering model requested in (4). We present theoretical predictions of estimation performance and compare theory to measurements of a flat plate. Specifically, we use the CRB to address two practical issues: the resolution limit for closely spaced scattering centers and the role of relative bandwidth in estimating the frequency dependence parameter, $\alpha$.

The CRB for the model is derived in [16] and provides an algorithm-independent lower bound on the error variance for unbiased estimates of the model parameters. The derivation assumes the scattering model of (4) with an additive perturbation

$$E(f, \phi) = \sum_{i=1}^{p} E_i^f(f, \phi) + \eta(f, \phi).$$

Here, $\eta(f, \phi)$ represents the modeling error (background clutter, sensor noise, model mismatch, incoherent motion compensation, antenna calibration errors, etc.) and is assumed to be a white Gaussian noise process. For any choice of model parameters, the bound is computed by inversion of the information matrix [24]. We report signal-to-noise (SNR) values using the ratio of signal energy to noise energy computed for the frequency-aspect data set samples; interpretation of SNR in the spatial domain as a difference between peak signal level and clutter floor (i.e., after pulse compression) requires a shift of $10\log_{10} N M$ dB for a point scatterer (less for other types of scattering), where $N$ and $M$ are the number of frequency and aspect samples.

First, we consider resolution. For a given SNR of a single-point scatterer (SNR model), we define resolution as the minimum distance between two equal-amplitude scattering centers resulting in nonoverlapping 95% confidence regions for the estimated locations [25]. Our definition is illustrated in Fig. 5. The figure is computed for 500-MHz bandwidth with a 1-d$^2$ aperture and $f = 10$ GHz, consistent with the existing SAR sensor used for the MSTAR [26] data set. This bandwidth yields a Fourier resolution of 30 cm, windowing for sidelobe suppression results, in coarser resolution. Further, we assume 64 equally spaced samples in both frequency and aspect. The SNR is $-20$ dB for the figure. The ellipses show the 95%
confidence regions for the location estimates of four localized scatterers. The pair in the lower portion of the figure are not resolved since the ellipses overlap. The pair in the upper portion are, by definition, resolved since the confidence regions are disjoint.

Adopting this definition, resolution versus SNR/\text{mode} is shown in Fig. 6. The resolution depends on the orientation of the two point scatterers. The dashed line shows resolution for point scatterers separated an equal distance in both down range and cross range (i.e., aligned 45\degree to the aperture). The solid line and the dash-dot line show resolution for two-point scatterers aligned parallel and orthogonal to the aperture, respectively. For an SNR/\text{mode} of \(-4\) dB, the limit of resolution achievable by model-based scattering analysis is below one-half the Fresnel resolution; model-based resolution is limited by sensor bandwidth and SNR, which includes mismatch from the model in (15).

Second, we consider the effect of relative bandwidth and SNR in accurately detecting the frequency dependence parameter \(\alpha\) for a single scattering mechanism. We characterize performance limits by assuming a parameter estimator that is unbiased, statistically efficient [24] and normally distributed (as is asymptotically true for the least-squares estimator). Fig. 7(a) shows the probability of correct detection of the discrete-valued \(\alpha\) parameter versus SNR for 1\% resolution \(X\)-band and \(K\)-band SAR systems. The \(X\)-band data are as specified above; the \(K\)-band data are for 500-MHz bandwidth, \(\pm0.242\)\degree aperture, and \(f_c = 55\) GHz, consistent with a Lincoln Laboratory sensor [27]. The analytically derived detection results are averaged over live scattering types \((\alpha \in \{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\})\). Notably, uncertainty in estimating \(\alpha\) decreases logarithmically with an increase in relative bandwidth. This finding reconfirms the one-dimensional results in [2] and [13] that accurate estimation of \(\alpha\) in scattering amplitude versus frequency requires either high bandwidth or low noise power. In Fig. 7(b) the detection of \(\alpha\) is restricted to the binary hypotheses of \(\alpha = \frac{1}{2}\) or \(\alpha = 1\); this represents, for \(L \neq 0\), the scenario of distinguishing a cylinder from a dihedral.
SAR data compression, and scattering studies. The model is developed in the frequency-aspect domain and is motivated by GTD-based and physical optics scattering principles. We present an image-domain algorithm for estimating model parameters and thereby gain both clutter suppression and computational savings. We use the CRB as a tool for predicting uncertainty in estimated parameters. The scattering model and the image-domain estimation algorithm are demonstrated in three measured data examples.

APPENDIX

\[
I_2(k) = -j \cot \phi_0 \exp \left( -K \cot \phi_0 \left( r_0 + j \left( \frac{2r}{c} \right. \right. \right. \\
\left. \left. \left. - \beta^2 \right) - t_0 \right) + \tan \phi_0 \left( \frac{2r}{c} - \beta^2 \right) \right) \\
\times \left( j2\pi \sin(k) \sin(\theta) \right)^2 \times \left( \tan \phi_0 \left( t_0 - \frac{2r}{c} + \frac{\beta^2}{c} \right) \right)^2 \\
+ E_0 \left( -K \cos \phi_0 - j K \cos \phi_0 \left( \frac{2r}{c} - \beta^2 \right) - t_0 \right) \\
+ L \cos \phi_0 \left( \cos \theta - t_0 \right) - E_1 \left( -K \cos \phi_0 - j K \cos \phi_0 \left( \frac{2r}{c} - \beta^2 \right) - t_0 \right) \\
+ j \pi \sin(k) \left( \cos \phi_0 \left( \frac{2r}{c} - \beta^2 \right) + \beta^2 \right) \\
+ j \pi \sin(k) \left( \cos \phi_0 \left( \frac{2r}{c} - \beta^2 \right) + \beta^2 \right) \\
- E_1 \left( -K \cos \phi_0 - j K \cos \phi_0 \left( \frac{2r}{c} - \beta^2 \right) - t_0 \right) \\
- \frac{L \cos \phi_0}{\cos \theta - t_0} \\
\]

where

\[
K_1 = 2\pi \left( f_2 - f_1 \tan \phi_0 \right) \\
K_2 = 2\pi \left( f_1 \tan \phi_0 - f_2 \tan \phi_0 \right) \\
K_3 = 2\pi \left( f_1 \tan \phi_0 - f_2 \tan \phi_0 \right) \\
K_4 = 2\pi \left( f_2 - f_1 \tan \phi_0 \right) \\
\nu = r_0 + \frac{2r}{c} - \beta^2 - t_0 \\
- \tan \phi_0 \left( t_0 - \frac{2r}{c} + \beta^2 \right) \right) \\
\text{rect}(x) = \begin{cases} 1, & -T/2 \leq x \leq T/2 \\
0, & \text{otherwise} \end{cases} \\
E_1(t) = \int_{-\infty}^{\infty} \exp(-t) \frac{d}{dt} \text{dt.}
\]

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REFERENCES


Michael J. Greer was born in Burlington, VT. He received the B.S. degree in electrical engineering from the University of Delaware, Newark, in 1989, and the M.S. and Ph.D. degrees in electrical engineering from The Ohio State University, Columbus, in 1993 and 1997, respectively. From 1990 to 1991, he was with W. J. G. Greer Associates, Inc., Newark, DE, working on the Electronic Products Division. From 1993 to 1997 he was with a Graduate Research Assistant at the ElectroScience Laboratory, Department of Electrical Engineering, The Ohio State University. Since September 1997, he has been with Synoptx/Corporation, Dublin, OH, designing and developing satellite surveillance radar systems for traffic and census applications.

Lee C. Potter (S’81-M’90-A’88) received the B.E. degree from Vanderbilt University, Nashville, TN, and the M.S. and Ph.D. degrees from the University of Illinois at Urbana-Champaign, all in electrical engineering. Since 1992, he has been with the Department of Electrical Engineering at The Ohio State University, Columbus, where he is currently an Associate Professor. His research interests include statistical signal processing, radar problems, detection, and estimation, with applications to target identification and ultrawideband systems. Dr. Potter is a 1993 recipient of the Ohio State College of Engineering MacQube Award for Outstanding Teaching.

Jean R. Crump (S’83-M’88) received the B.S. degree in electronics engineering from Purdue University, Champaign, IL in 1975, the M.S. degree in electrical engineering from the Indian Institute of Technology, Kharagpur, India, in 1977, and the Ph.D. degree in electrical engineering from "The Ohio State University, Columbus, in 1982. Since 1979, he has been working at The Ohio State University, Electrical Engineering Laboratory where currently he is a Senior Research Scientist. His predoctoral work experience included Research Engineer at HIT Aerospace, India, and Indoor/Outdoor Research Associate at the University of Texas at Space Flight Laboratory, Texas. He has done extensive research on space antenna arrays and system and antenna range measurement systems and has written several journal articles in these areas. His current research interests include adaptive antenna arrays, electromagnetics scattering, compact target technology, radar imaging, and target recognition. Dr. Crump was the President of Antenna Measurement Techniques Association in 1997. He received IEEE Antennas and Propagation Society S. R. & A. Waveshot Applications Post Ph.D. Award in 1991. He is also the recipient of The Ohio State University College of Engineering, Loring Research Award 1991 and 1998.

Andrej van der Merwe was born in South Africa. She received the B.B. (with honors) and M.S. degrees in electronic engineering from the University of Pretoria, South Africa. She is currently working toward the Ph.D. degree in electrical engineering at The Ohio State University, Columbus. Her areas of interest include image processing, optimization, detection and estimation, and electromagnetic scattering.