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Statistical properties of linear correlators for image pattern classification with application to SAR imagery

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ABSTRACT

In this paper we consider linear correlation filters for image pattern recognition, with particular application to Synthetic Aperture Radar (SAR). We investigate the statistical properties of several popular Synthetic Discriminate Function (SDF) based linear correlation filters, including SDF, MVSDF, and MACE filters. We compare these statistical properties both qualitatively and analytically for SAR applications. We also develop modifications to these SDF-type filters which have particular utility for Synthetic Aperture Radar (SAR) image classification. We compare the performance of the modified filters to the standard filters using X-patch generated SAR images with both white and colored noise. We also investigate effects of performance degradation caused by mis-estimated noise statistics, and the effects of image normalization on the target detection rates.

Keywords: linear correlators, SDF, pattern classification, pattern recognition, image classification, SAR images

1 INTRODUCTION

An Automatic Target Recognition (ATR) system typically consists of a number of stages which perform the tasks of initial detection, clutter rejection, target orientation estimation, and finally target classification.[1] For many ATR systems, the final stage is often implemented using some type of linear correlator. A number of correlator filters have been described in the literature, including Synthetic Discriminant Function (SDF), Minimum Variance Synthetic Discriminant Function (MVSDF), and Minimum Average Correlation Energy (MACE) filters, and their variants (see, *e.g.*,[2–6]). We refer to these methods as SDF-type linear correlators. They are popular for use in both optical and in SAR image classification applications.

Object classification in ATR systems must be robust to the presence of noise. Most of the popular SDF-type correlators are designed to optimize a performance criterion that provides immunity to this noise. In optimizing performance criteria, the noise on the image is frequently assumed to be both additive and wide-sense stationary. However, in most SAR classification problems, image noise is not additive. While it is often valid to assume additive complex noise for the complex SAR image, classification often is done using only the magnitude of the image. The reason for this is that the phase information of SAR pixels are quite sensitive to errors in sensor range, object orientations, etc. Thus, even though the *complex* SAR images are corrupted with additive noise, the noise in the magnitude image is multiplicative noise and is non-stationary. Consequently, SDF-type filters that are designed assuming additive noise models may not perform as expected.

In this paper, we address the issue of non-additive noise in two ways. First, we propose a modification to the performance optimization criteria of SDF-type linear correlation filters which is more appropriate for non-additive noise models such as that encountered in SAR image classification. Second, we analyze the statistical properties of both the original and modified SDF-type linear correlators under a variety of conditions, including additive, wide-sense stationary noise with different amounts of spatial correlation, and the corresponding multiplicative noise induced by using the magnitude of a complex image. We analyze the similarities and differences among some basic SDF-type linear correlation filters and draw some general qualitative conclusions on their performance.

We also provide several numerical experiments to compare the various SDF-type correlators for SAR classification. We use X-Patch synthesized SAR images corrupted by noise to investigate the classification detection rates of several filters. We also investigate how badly performance degrades as a result of mis-estimates of the second order statistics of the noise. Finally, we discuss the effect of image normalization on the performance of linear correlators for pattern recognition.

2 NOTATION AND PROBLEM FORMULATION

In this paper, we use lower-case variables such as $v(m,n)$ or $h(m,n)$ to indicate images or filters in the spatial domain. Upper-case variables such as $V(k,l)$ or $H(k,l)$ are the corresponding two dimensional Discrete Fourier Transforms of the images. All input images and all correlators are assumed to have the same size with $n_1 \times n_2$ pixels. Variables such as v and h are column vectors formed from stacking the pixel values of $v(m,n)$ and $h(m,n)$. In this paper, we refer to $(.)^*$ as the complex conjugate operator, $(.)^H$ as the complex conjugate transpose operator, and we refer to correlation outputs as the outputs of the correlators at the origin.

With the above notation, the problem we address is as follows. We assume the training patterns $\{v_{ij}(m,n)\}_{j=1}^{M_i}$ are given for each i in one of L classes. Each $v_{ij}(m,n)$ can be either a real-valued or a complex-valued image. Assume the test pattern to classified is given as $\mathbf{y} = f(v_{pq}, \mathbf{n})$, where p is the class index, $1 \leq q \leq M_p$, and $\mathbf{n}$ is the background clutter. Given a test pattern $\mathbf{y}$, our task is to recognize it to be in the pth class. For additive noise, $f(v_{pq}, \mathbf{n}) = v_{pq} + \mathbf{n}$; for SAR imagery, we assume v_{pq} and $\mathbf{n}$ are complex-valued and $f(v_{pq}, \mathbf{n}) = |v_{pq} + \mathbf{n}|$, *i.e.*, only the magnitude images are used for classification.

3 REVIEW OF SDF-TYPE LINEAR CORRELATORS

SDF-type linear correlators can be used to solve the above problem by synthesizing linear correlation filters, one for each class of objects, namely h_i for the ith class. A test pattern $\mathbf{y}$ is assigned to the class whose correlation output is maximum. Each correlator h_i is designed to satisfy the following constraints:

C1: h_i is linear and circular-shift invariant.

C2: h_i minimizes $h_i^H A h_i$ or $H_i^H A H_i$ for some user-specified positive definite matrix A.

C3: The output at the origin for filter h_i is 1 for input training vectors from the ith class and 0 for input vectors from all other classes. That is, $v_{kj}^H h_i = \delta_{i-k}$, for all $k = 1, \ldots, L$, and all $j = 1, \ldots, M_k$.

By employing Lagrange Multipliers, one can solve the above problem[4]; the solution is given by one of

$$h_i = A^{-1} x (x^H A^{-1} x)^{-1} u_i \tag{1}$$
$$H_i = d \cdot A^{-1} X (X^H A^{-1} X)^{-1} u_i \tag{2}$$

depending on whether the second constraint is imposed in the spatial or frequency domain. Here, x is a matrix of training vectors, X is its corresponding discrete Fourier transform, d is a constant equal to the dimension of the training patterns, and each element of u_i is a 0 or 1 corresponding to the third constraint above. Particular choices of domain and A result in some common filters. If A is the identity matrix, I, the corresponding correlator is an SDF filter for both the spatial and frequency domain. It can be shown[2] that for $A = I$, the second design constraint is equivalent limiting the filter to be a linear combination of the training patterns. If A is the noise covariance matrix, $R_{\mathbf{nn}}$, the corresponding correlator is an MVSDF filter in the spatial domain,[2] which is the best SDF-type correlator for wide-sense stationary (W.S.S.) additive, zero-mean noise, since it minimizes the output noise variance. If A is a diagonal matrix whose diagonal elements are the samples of the average magnitude spectra of the training patterns, D, the corresponding correlator is a MACE filter in the frequency domain, which minimizes the average correlation energy corresponding to training patterns, and produces sharp correlation peaks.

SDF, MVSDF, and MACE filters are the most basic correlators among the SDF-type correlators. There are many variations of these filters developed in the literature, such as a frequency-domain version of the MVSDF filters, a spatial-domain version of MACE filters, Improved SDF (ISDF)-type composite filters which linearly combine the optimization constraints of MACE and MVSDF (or MACE and SDF) filters, etc.[4–7]

4 MODIFICATIONS FOR NON-ZERO-MEAN NOISE

The filter designs above are based on the assumption of zero mean noise. However, for SAR applications, the noise term in the magnitude image is not zero mean. Below we address ways of modifying the SDF-type correlators when the noise is not zero mean.

4.1 Specifying means of correlation outputs

The goal of the filter design is to distinguish a noisy image as belonging to one of several classes, so we want the correlator outputs to be different for input images from different classes. For zero mean noise and linear correlator filters, this goal is implemented using criterion C3 above. When the noise is nonzero mean, criterion C3 requires modification.

Let $s(m,n)$ be any input pattern to a linear circular-shift invariant correlator with frequency response $H^*(k,l)$, and let the associated output pattern be $y(m,n)$. The corresponding correlation output, r, is given by:

$$r = y(0,0) = \frac{1}{n_1 \cdot n_2} \sum_{k=0}^{n_1-1} \sum_{l=0}^{n_2-1} H^*(k,l)S(k,l) = \frac{1}{n1 \cdot n2} H^H S \tag{3}$$

Also, by Parseval's Theorem:

$$r = \sum_{m=0}^{n_1-1} \sum_{n=0}^{n_2-1} h^H(m,n)s(m,n) = h^H s \tag{4}$$

Let $\mathbf{s}_{pq}$ be the noisy patterns $f(v_{pq}, \mathbf{n})$, h_i,i=1,...,L, be correlators corresponding to L classes of objects, and $\mathbf{r}_{pqi}, i = 1, \ldots, L$ be the associated correlation outputs of h_i's. We propose the following modification to criterion C3:

$$\textbf{C3':} \quad E\{\mathbf{r}_{pqi}\} = \delta_{i-p}, \qquad \forall i, p, q \tag{5}$$

Let $\hat{m}$ be the matrix of vectors $E\{\mathbf{s}_{pq}\}$, and $\hat{M}$ be the matrix of vectors $E\{\mathbf{S}_{pq}\}$. By equations (3), (4) the constraints are equivalent to:

$$E\{\hat{M}\}^H H_i = u_i(n1 \cdot n2), \qquad \text{and} \qquad E\{\hat{m}\}^H h_i = u_i \tag{6}$$

where u_i is a column vector with each element equal to a 0 or 1 corresponding to the constraint in equation (5).

We note that criterion C3 in the previous section is a special case of the more general constraint we show in equation (5). Specifically, previous designs have used versions of equation (5) which are appropriate for additive, zero-mean noise, instead of the more general form used here.

4.2 Optimization criteria

To accommodate non-additive noise, some of the optimization criteria must be suitably modified. As examples, we provide the (modified) optimization criterion for some basic SDF-type correlators, including SDF, MVSDF, MACE, and ISDF correlators.

a. SDF :
There are two modified SDF filters, depending on whether we limit the correlator to be a linear combination of $E\{\mathbf{s}_{pq}\}$ or of v_{pq} ($|v_{pq}|$ on SAR imagery). We refer to these two SDF-type filters as SDF_1 and SDF_2, respectively. It is easy to show that SDF_1 is equivalently obtained by minimizing the filter's energy under the constraint in equation (6). There is no simple optimization criterion corresponding to SDF_2. We note that the original motivation for limiting the correlator to a linear combination of v_{pq} is that it allows optical correlators to be implemented by multiple-time-exposure techniques[2]; for electronic (hardware or software) correlators there is little reason for this limitation. Thus, the SDF_1 filter is recommended for SAR applications.

b. MVSDF:
We can set the optimization criterion of MVSDF such that the result is to minimize the average variances of correlation outputs corresponding to $\mathbf{s}_{pq}$'s, which is given by:

$$Vave_i \;=\; \frac{1}{N}\sum_{p,q} h_i^H R_{\mathbf{s}_{pq}\mathbf{s}_{pq}} h_i \;=\; h_i^H \hat{R}_{\mathbf{ss}} h_i \tag{7}$$

Where $R_{\mathbf{s}_{pq}\mathbf{s}_{pq}}$ is the covariance matrix of $\mathbf{s}_{pq}$, and $\hat{R}_{\mathbf{ss}} = \frac{1}{N}\sum_{p,q} R_{\mathbf{s}_{pq}\mathbf{s}_{pq}}$.

c. MACE:
For the modified MACE filters we can minimize either the mean of the average correlation energy, corresponding to the $\mathbf{s}_{pq}$'s or the average correlation energy corresponding to the v_{pq}'s. We refer to the associated correlators as $MACE_1$ and $MACE_2$, respectively. The mean of the average correlation energy corresponding to $\mathbf{s}_{pq}$'s is:

$$E1_i \;=\; \frac{1}{N}\sum_{p,q} E\{\frac{1}{n_1 n_2}\sum_{k,l} |H_i(k,l)^H \mathbf{S}_{pq}(k,l)|^2\} \;=\; H_i^H \hat{D} H_i \tag{8}$$

where $\hat{D}$ is a $n_1 n_2 \times n_1 n_2$ diagonal matrix with diagonal elements equal to $\frac{1}{n_1 n_2 N}\sum_{pq} E\{|\mathbf{S}_{pq}(k,l)|^2\}$.
The average correlation energy corresponding to v_{pq}'s is[3]:

$$E2_i \;=\; \frac{1}{N}\sum_{p,q} \frac{1}{n_1 n_2}\sum_{k,l} |H_i(k,l)^H v_{pq}(k,l)|^2 \;=\; H_i^H D H_i \tag{9}$$

Where D is a $n_1 n_2 \times n_1 n_2$ diagonal matrix with diagonal elements equal to $\frac{1}{n_1 n_2 N}\sum_{pq} |V_{pq}(k,l)|^2$.

d. ISDF:
The criterion to minimize, for the modified ISDF filters, are chosen as linear combinations of the criteria to minimize for ($MACE_1$ or $MACE_2$) and ($MVSDF_1$ or SDF_1 or SDF_2).

4.3 Structures of modified correlators

Similar to the standard SDF-type correlators, each of the corresponding modified correlators (except SDF_2) can be formulated as minimizing a quadratic function of either the form $h_i^H A h_i$ or $H_i^H A H_i$, subject to constraints on the correlation outputs. The modified filters thus have the following structure:

$$h_i = A^{-1}\hat{m}(\hat{m}^H A^{-1}\hat{m})^{-1}u_i \quad (H_i = A^{-1}\hat{M}(\hat{M}^H A^{-1}\hat{M})^{-1}u_i(n_1 n_2)) \tag{10}$$

For SDF_2, h_i (H_i) is limited to be a linear combination of v_{pq} (V_{pq}) subject to the constraint on equation (6). The solution can be readily obtained, and is:

$$h_i = x(\hat{m}^H x)^{-1}u_i \quad (H_i = X(\hat{M}^H X)^{-1}u_i(n_1 n_2)) \tag{11}$$

Where x is a matrix of training vectors v_{pq} ($|v_{pq}|$ for SAR images).

For SAR images, if $\mathbf{n}$ is complex white Gaussian noise, each element in $\mathbf{s}_{pq}$ is Rician distributed[7] and elements in $\hat{m}$, $\hat{D}$, and $\hat{R}_{\mathbf{ss}}$ can be computed from first order numerical integration of the pdf of the Rician densities. For colored or non-Gaussian noise, it is hard to obtain a closed form of the pdf of $\mathbf{s}_{pq}$ and higher-order numerical integration is needed to compute the ensemble statistics. For details on the impact on filter synthesis, refer to.[7]

5 STATISTICAL PROPERTIES

In this section we develop the second-order statistical properties and the relative performance of these linear correlators. We assume the noise $\mathbf{n}$ is W.S.S. (restricted to the image size).

5.1 Relationships among linear correlators

It well known that nearly all SDF-type correlators can be related to the basic SDF correlator by decomposing A^{-1} into $A^{-\frac{1}{2}}A^{-\frac{T}{2}}$, and by rearranging the filter structure in equations (1) (equation (2)) as[4]

$$h_i = A^{-\frac{1}{2}}h_i^{SDF^y} \quad (H_i = A^{-\frac{1}{2}}H_i^{SDF^y}) \tag{12}$$

Where y is the transformed matrix of training patterns $A^{-\frac{T}{2}}x$, and $h_i^{SDF^y}$ is the SDF designed from the transformed training patterns y. The matrix $A^{-\frac{1}{2}}$ can be regarded as a whitening filter. For example, for a MVSDF correlator, $R_{\mathbf{nn}}^{-\frac{1}{2}}$ is a noise whitening filter[2]; for a MACE correlator, $D^{-\frac{1}{2}}$ is a whitening filter to whiten the average spectra of the training patterns.[3]

For the modified linear correlators, the same technique can also be applied to relate all the other correlators (except SDF_2) to a basic SDF_1correlator. That is, we rewrite equation (10) to:

$$h_i = A^{-\frac{1}{2}}h_i^{SDF_1^y} \quad (H_i = A^{-\frac{1}{2}}H_i^{SDF_1^y}) \tag{13}$$

where y is the transformed matrix of mean vectors $A^{-\frac{T}{2}}\hat{m}$. For an $MVSDF_1$, $\hat{R}_{\mathbf{ss}}^{-\frac{1}{2}}$ is a whitening filter to whiten the average covariance matrix of s_{pq}'s. For $MACE_1$ and $MACE_2$, $\hat{D}^{-\frac{1}{2}}$ and $D^{-\frac{1}{2}}$ are whitening filters which whiten the average means of the spectra of $\mathbf{s}_{pq}$ and v_{pq}, respectively. For a modified ISDF, its' whitening filter whitens a linear combination of average covariance matrix of s_{pq} and average mean spectra of training patterns.

5.2 Second-order statistical properties and relative performance

Below we discuss the qualitative statistical properties and relative classification performance of the different SDF-type correlators when applied to image classification. Our discussion assumes that the training patterns employed in designing the filters are narrow-band low-pass images. This assumption is true in most practical SAR applications which employ a magnitude image. As we discuss below, this low-pass property can be employed to develop the statistical properties of the linear correlators in question.

5.2.1 Spectral density of the linear correlators

SDF, SDF_1, and SDF_2 are linear combinations of low-pass images v_{pq} or $E\{|v_{pq} + \mathbf{n}|\}$. Thus, we observe that the above correlators are all low-pass, since a linear combination of low-pass images is low-pass with high probability. Thus, in the frequency domain these linear filters weight the input patterns more where it is strong and less where it is weak.

On the contrary, $MACE$, $MACE_1$ and $MACE_2$ are high-pass filters since in their alternate expressions (equations (12) and (13)) the whitening filter $A^{-\frac{1}{2}}$ effectively forms the inverse spectra of the training patterns. It follows that both SDF^y and SDF_1^y are approximately white since y is the matrix of the whitened training or mean training patterns. That is, in the frequency domain, $MACE$ filters weight the input patterns more where they are weak and less where they are strong. Therefore, in the frequency domain, if distortion or noise occurs where v_{pq} or $|v_{pq}|$ has high energy (*i.e.* it is low-pass), SDF, SDF_1 and SDF_2 are less capable of suppressing it than $MACE$, $MACE_1$, and $MACE_2$, and vice versa.

The spectral density of $MVSDF$ and $MVSDF_1$ can be either low-pass or high-pass depending on the (average) covariance matrices of $\mathbf{s}_{pq}$'s. If the noise is highly correlated, and thus the $\mathbf{s}_{pq}$'s are narrow-band low-pass images, the spectral density of $MVSDF$ and $MVSDF_1$ are high-pass just like those of $MACE$'s. If the noise is relatively-uncorrelated, the whitening filters in $MVSDF$ and $MVSDF_1$ have little effect on the (mean) input patterns, and thus they will look like the SDF's which are low-pass.

5.2.2 Relative performance with respect to W.S.S. noise

For additive noise, analytic performance of these linear correlators is much easier to develop since we can treat the training patterns, v_{pq}, and noise $\mathbf{n}$ separately. Accordingly, we first develop our arguments for additive noise, and then extend the arguments to applications which include SAR imagery, where magnitude images are used and the noise is not additive.

For the class of filters under consideration, the output noise is approximately W.S.S. if the input noise is W.S.S., where "approximately" arises from the fact that finitely-sized circular convolution instead of infinitely sized linear convolution is employed by the correlators. In this case the variance of the correlation output is equal to the variance of correlation noise, so is approximately equal to the average power of the output noise. Therefore, the average output noise power dominates the performance (*i.e.*, the detection rate) of a correlator for a given W.S.S. noise power. The average output noise power is:

$$\frac{1}{n_1 n_2} \sum_{k=0}^{n_1-1} \sum_{l=0}^{n_2-1} |H_i(k,l)|^2 P_{\mathbf{nn}}(k,l) \tag{14}$$

where $P_{\mathbf{nn}}(k,l)$ is the power spectral density of noise $\mathbf{n}$. Thus, if the high-energy band of the power spectra of the noise is coincident with the pass-band of the energy spectra of a particular filter, then the output noise power is large, and thus the performance of the correlator degrades, and vice versa.

For non-additive noise, such as is the case for SAR imagery, same characteristics are likely, but not guaranteed. For SAR imagery, the equivalent additive noise presented to the correlators is:

$$\hat{\mathbf{n}} = |v_{pq} + \mathbf{n}| - E\{|v_{pq} + \mathbf{n}|\}, \tag{15}$$

which depends on v_{pq} and is not W.S.S. in general. Thus, the power spectral density of $\hat{\mathbf{n}}$ is not defined. Furthermore, because of the nonlinear transformation above, there is no longer a linear relationship between the average power of the noise $\mathbf{n}$ and the noise variance at the origin of the correlator output. Thus, while it is likely that an increase in noise power results in higher correlator output variance, it is not certain to be the case.

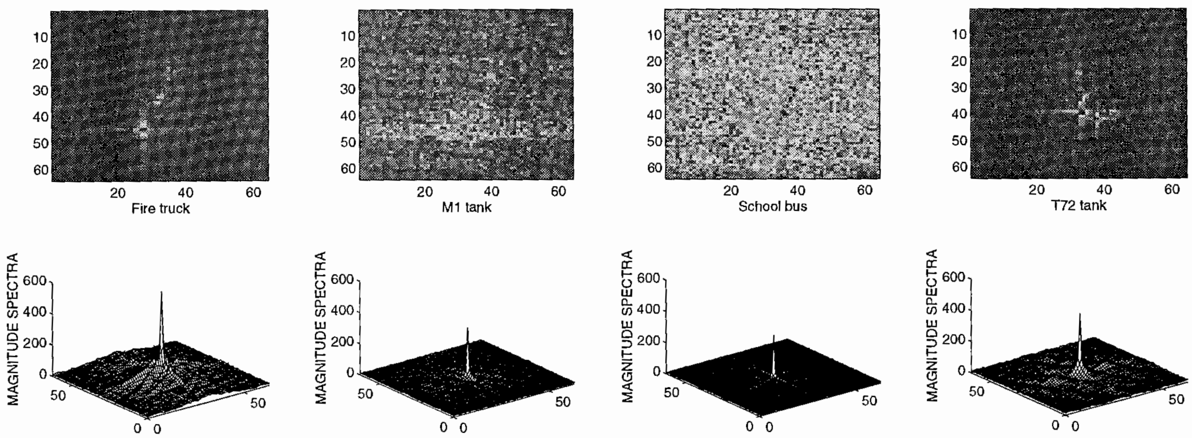

Figure 1: Training images at one azimuth angle: (a) magnitude images (b) the corresponding magnitude spectra

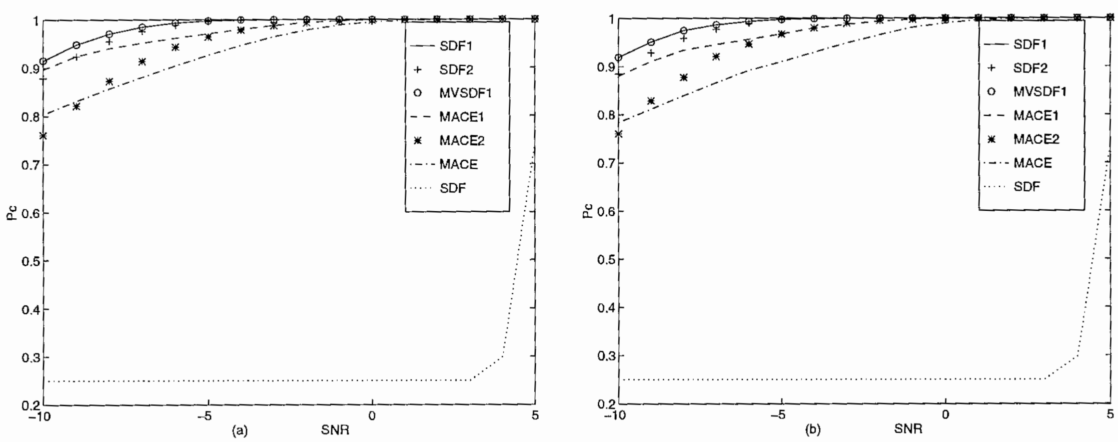

Figure 2: Performance of the linear correlators for white Gaussian noise: (a) theoretical performance computed by approximating the pdf of correlation outputs with Gaussian densities (b) Monte-Carlo simulation results where ensemble estimates of means and variances of noisy input images are computed from numerically integrating Rician densities

It is well known that an SDF filter is the best SDF-type linear correlator for additive white noise.[2] However, with respect to SAR imagery in which magnitude images are used, the standard SDF filters perform poorly. In fact, in this simulation, when the SNR < 5, the mean of correlation output for the School bus class is always higher than those for the other three classes, given any test patterns from the four classes. Therefore, the unmodified SDF filter decision rule always chooses the School bus class.

Recall that the magnitude spectra of a $MACE$ filter is high-pass, while the magnitude spectra of an SDF filter is low-pass. The difference between the original training patterns $|v_{pq}|$ (which are used to synthesize the SDF and $MACE$ filters) and the means of the noisy patterns $|v_{pq} + \mathbf{n}|$ are low-pass for general low-pass $|v_{pq}|$. Thus, if we employ the standard SDF and $MACE$ filters with this SAR data, and if we treat the difference between the means of $|v_{pq} + \mathbf{n}|$ and $|v_{pq}|$ as "noise", the corresponding correlator output "noise" energy for the SDF is higher than that for the $MACE$. Thus, output "noise" tends to produces serious effects on the SDF filter and substantially alters the correlator output means.

As discussed in Section (5.2.2) and verified in Figure 2, $MVSDF_1$, SDF_1, and SDF_2 are more robust with respect to white $\mathbf{n}$, while $MACE_1$ and $MACE_2$ are more sensitive to white $\mathbf{n}$. $MVSDF_1$ should be the optimal standard MACE increases significantly as ρ increases.[7]

The performance of $MVSDF_1$, although not simulated for colored noise, should be bounded below by the performance of $SDF_{1,2}$ and $MACE_{1,2}$. Since $MACE_1$ minimizes the mean of the average correlation energy corresponding to $|v_{pq} + \mathbf{n}|$, its performance on reducing the correlation energy should be better than that of $MACE_2$, which minimizes the average correlation energy corresponding to $|v_{pq}|$. However, it is difficult to predict which will have the better detection rate. Also $MACE_1$ needs more statistical information ($\hat{D}$) than does $MACE_2$. Figures 3-(b) suggests that for more highly correlated noise, the performance of $MACE_2$ is better than that of $MACE_1$ in this scenario.

6.3 Noise model mismatch

We present a simple example to evaluate the sensitivity of SDF-type correlators to the assumed noise model. In this example, we use the true noise variance information to synthesize the correlators, but we incorrectly assume that the noise is white. We then test the linear correlators with the colored noise model we used in the above examples. Recall that if we set the noise variance to zero, then the modified correlators reduce to previously published correlators.

Figures 4-(a) and 4-(b) show the results of this simulation for SNR = -10 dB and SNR = -5 dB. respectively. We see that the performance degradation is minimal if the mismatch on noise model is not dramatic. We also see that $MACE_1$ and $MACE_2$ are more robust to the noise model mismatch than either SDF_1 or SDF_2. It is interesting that in this scenario, the detection rates of $MACE_2$ increase as the mismatch on noise model increases. Recall that $MACE$-type correlators are designed to minimize average correlation energy, and this criterion is not directly related to the detection rate.

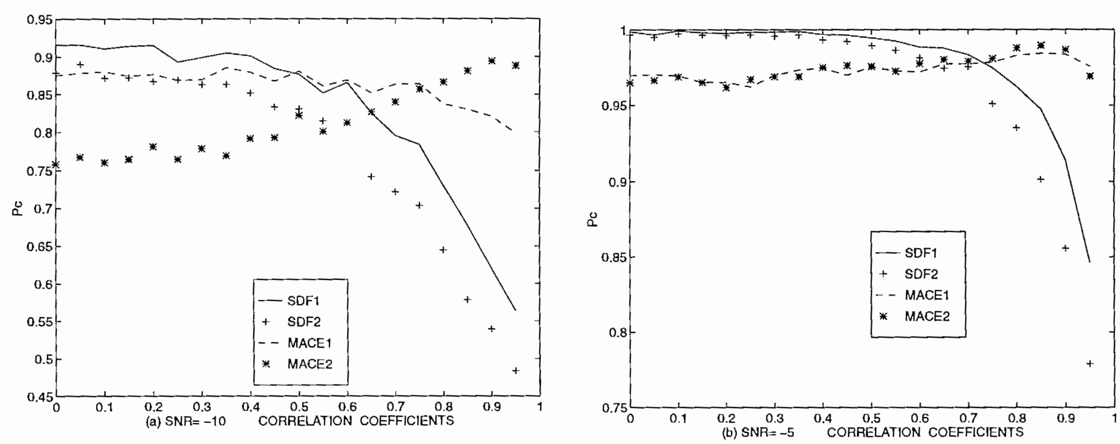

Figure 4: Simulations using white Gaussian noise model for correlator synthesis, and tested with colored Gaussian noise: (a) SNR=-10 (b) SNR=-5

6.4 Image normalization

From Figure 1 we see that the energy of the different images from different classes may significantly differ. Moreover, even for images from the same class, power may also differ as the viewing angle changes. Novak, Owirka and Netishen[1] found that normalization can improve the inter-class performance on SAR image classification. Here, we normalize the Frobenius norm of images used in the simulations, and discuss why normalization improves the performance of these linear correlators.

Among the modified correlators, $MVSDF_1$, SDF_1 and SDF_2 are found to be more robust with respect to white or relatively uncorrelated noise than $MACE_1$, and $MACE_2$. We also find that $MACE_1$, and $MACE_2$ are more robust with respect to highly correlated noise. These results are similar to the results obtained for additive noise using previously published correlators.

The theoretical performance of these correlators can be evaluated approximately by approximating the pdf of the correlator outputs as Gaussian densities. This approximation is usually good if the spatial correlation of the noise is low for pixels far apart in the images. We provide theoretical performance in the example for white Gaussian noise which shows good agreement with the results obtained by Monte-Carlo simulations. For colored noise, it is problematic to compute the theoretical performance because of the need to construct large sized covariance matrices which require extensive memory.

We also employed sample statistics in order to synthesize correlators. The results show modest degradation for white noise case when compared to those results obtained by employing the ensemble statistics. However, this degradation can be made small by increasing the sample size. For colored noise, this appears to be a good alternative to employing numerical integration to solve for ensemble statistics. The effect of noise model mismatch was also evaluated. Simulation results show that the detection rates of these linear correlation filters degrades only slightly if the mismatch with the noise model is not significant. Finally, the effect of image normalization was discussed and simulated. We determined that normalization of images improves the performance of these linear correlators by making the variances of the correlator outputs closer to each other and thus makes the decision criterion of the linear correlators closer to the Bayes criterion.

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