Applications of the bispectrum in radar signature analysis and target identification

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Abstract

This paper considers the classification of radar targets using features extracted from the bispectrum of the backscattered signals. The classification performance of the bispectrum-derived features is compared with that of features extracted from the impulse response and the frequency response of the unknown target. In each case, the classification performance is evaluated using compact-range radar signal measurements of a set of five commercial aircraft. A number of scenarios of noise environment and azimuth ambiguity are consider. The case where extraneous scatterers are present in the vicinity of the measured target is also examined and the effects of the extraneous scatterers are discussed.

I. Introduction

Radio frequency signals scattered from a radar target contain significant information about the physical target geometry. Radar backscatter signals are often measured as amplitude and phase at uniformly spaced frequencies. These measurements, as a function of frequency, characterize the transfer function of the target as seen by the radar. The inverse Fourier transform of the measured finite data sequence is a time-domain profile of the illuminated target. This profile is known as the "impulse-response" of the target (or "transient response" for finite bandwidths)[1]. A peak in the impulse-response appearing at time t_0 is an indication that a fraction of the illuminating signal propagating along the target is scattered at time t_0. Therefore, a sequence of responses in the time-domain profile can be attributed to a sequence of scattered signals along the target. The impulse-response can, of course, be displayed as a function of range r using $t = 2r/c$, where c is the speed of light.

The impulse response displays all scattering mechanisms in a one-dimensional profile. In this form of representation, responses due to signals scattered directly from target sub-structures are superimposed with responses due to multiple interactions and other scattering mechanisms. The result is an apparent lack of correspondence between peaks in the impulse-response and physical locations and characteristics of the scattering components. Thus, it is difficult to deduce accurate information about the target geometry using the impulse-response alone.

[1] In what follows, the term "impulse response" generally refers to any type of time-domain, or range-domain profile.

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When employing Fourier-based processing of the impulse response, it is also possible to overlook some of the unresolved details concerning the target scattering features, due to limited resolution. Thus, responses that are closely spaced in time but correspond to different types of interactions are inseparable in the Fourier transformed impulse-response.

This paper focuses on the interpretation of the responses of bispectral processing of radar signals and the use of the resulting bispectral features to identify unknown targets. The purpose of this investigation is to identify some of the advantages and limitations of the application of bispectrum in radar signature analysis.

Interpretations of the bispectral features of radar signals that identify certain multiple interactions are given in this paper. Examples of real aircraft models such as the Boeing 707, DC10, and Concord are considered. Results on the classification of these targets based on the bispectrum of noisy measured returns are presented. These include classification in additive Gaussian and non-Gaussian noise.

The classification results obtained using bispectral features are compared with those obtained using impulse response features. A classification scheme using both the impulse response and the bispectrum is also discussed and evaluated.

This study is motivated by important properties of the bispectrum such as the detection of quadratic phase coupling and the suppression of zero mean additive Gaussian noise. We point out that the bispectral response of a radar target, while not a substitute for the impulse-response, may be viewed as an additional display of signatures that can be used to achieve a more complete description of the various scattering mechanisms and enhance the target classification process.

II. Time-Domain Bispectral Features

The bispectrum is defined as the two-dimensional Fourier transform of the third order cumulant of the data. Cumulants represent the triple correlation of the data sequence and are usually a function of time so that the bispectrum is typically a function of frequency. Consider, for example, the data ensemble $\{x(k)\}_{k=1}^{N}$. The third order cumulant of $\{x(k)\}$ is defined as

$$R(m,n) = E\{x(k)x(k+m)x(k+n)\} \tag{1}$$

assuming that the sequence $\{x(k)\}$ is third order stationary. The bispectrum of $\{x(k)\}$ can be obtained from $R(m,n)$ using Fourier transform [1].

$$B(\omega_1,\omega_2) = \sum_m \sum_n R(m,n)W(m,n)\exp\{-j(\omega_1 m + \omega_2 n)\} \tag{2}$$

where $W(m,n)$ is a two-dimensional window that has the same symmetry properties as $R(m,n)$. Details concerning the third order cumulants and bispectra processing are found in [1].

In radar signal processing, the data sequence that represents the backscattered signal is often recorded as a function of frequency. The third order cumulant is then defined as

$$R(f_1, f_2) = E\{H^*(f)H(f+f_1)H(f+f_2)\} \tag{3}$$

where $H(f)$ is the complex-valued coherent backscatter response of the radar target at frequency f, and $E\{\cdot\}$ denotes statistical expectation. The bispectrum is then obtained as a function of time as

$$B(t_1, t_2) = \sum_{f_1}\sum_{f_2} R(f_1, f_2) \exp\{-j(t_1 f_1 + t_2 f_2)\}. \tag{4}$$

The term "Bispectrum" is somewhat misleading in this application, as it is a function of time, not frequency. However, we use this terminology because it has become standard. The bispectrum can also be expressed as a function of range (r_1, r_2). Finally, if the data $H(f)$ is deterministic then the expectation in the third order cumulant is replaced by a summation over frequency.

The bispectrum can be explicitly defined in terms of the time (or range) components of the data thus providing some intuition into the character of bispectral processing. For the application to radar signal processing, the spectral components simply denote the impulse response of the target as seen by the radar, so that the bispectrum is defined as

$$B(t_1, t_2) = \langle h(t_1)h(t_2)h(t_1+t_2)\rangle \tag{5}$$

where $\langle\cdot\rangle$ denotes the ensemble average.

Bispectra derived from the above definition have been used to detect implicit dependencies between different responses in the target impulse response [2]. These dependencies can be related to multiple interactions between scattering subcomponents along the target. Therefore, a peak in the bispectrum at (t_i, t_j) indicates that an implicit coupling is detected between the time response at t_i and the time response at time t_j [2].

This paper is concerned with the classification performance of bispectral features extracted from radar target backscatter measurements. In order to further motivate this investigation, we note that the information made available through bispectral processing is not apparent in either the spectrum or the impulse response. Therefore, a pattern recognition machine that uses both the impulse response and the bispectrum may, in certain scenarios, have a significant advantage over a classifier designed to employ only one of these sets of features. The focus in this paper, however, is on the feasibility and performance of classification using bispectral features as compared to using other target features rather than the design of the optimal classifier.

III. Specification of Classification Algorithms

Each of the M targets employed in this investigation is represented by a vector $H_i(f)$ of a set of K stepped frequency measurements of the radar backscatter of that target. These M sets of K data points form a set of catalog measurements representing the set of aircraft of interest. An unknown target measurement $H_u(f_k)$ is simulated by disturbing one of the M catalog vectors with noise or some form of extraneous scattering signals as

$$H_u(f_k) = H_i(f_k) + n(f_k) \quad 0 \leq k \leq K-1 \tag{6}$$

for $i \in \{1, \ldots, M\}$, where $n(f_k)$ represents the disturbance term.

Using the spectra of the unknown and catalog targets, we can compute the impulse responses, $h_u(k)$ and $h_i(k)$ (where k denotes the time index) using the Discrete Fourier Transform. We can also compute the bispectra $B_u(t_1, t_2)$ and $B_i(t_1, t_2)$ using the method described in [1]. It is these latter terms which we use for classification.

The classifiers employed in this study base their decisions on either the Euclidean distance between the unknown and the catalog representations or the cross-correlation between the two representations. As such, they do not require or incorporate any prior information concerning the statistical properties of the measured data. The classification algorithms implemented here are also based on the assumption that the zero-time response of the unknown and catalog targets is fixed and known.

This study considers the three types of classification algorithms described below. The goal of each algorithm is to identify the catalog measurement j whose bispectral response $B_j(\cdot,\cdot)$ "matches" (in the senses defined below) the bispectral response of the unknown target $B_u(\cdot,\cdot)$.

A. *Cross-Correlation of Bispectral Responses:*

For classification with the cross-correlation algorithm, target j is chosen whenever

$$\begin{aligned} j &= \arg\min_i \left\{ \int_{t_1} \int_{t_2} (B_i(t_1,t_2) - B_u(t_1,t_2))^2 \, dt_1 dt_2 \right\} && (7) \\ &= \arg\min_i \left\{ \int_{t_1} \int_{t_2} B_i^2(t_1,t_2) dt_1 dt_2 + \int_{t_1} \int_{t_2} B_u^2(t_1,t_2) dt_1 dt_2 \right. && (8) \\ & \qquad \left. - \int_{t_1} \int_{t_2} B_i(t_1,t_2) B_u(t_1,t_2) dt_1 dt_2 \right\} \end{aligned}$$

Since the first two terms in (9) are fixed, this entails maximizing

$$\int_{t_1} \int_{t_2} B_i(t_1,t_2) B_u(t_1,t_2) dt_1 dt_2 \tag{9}$$

and since the target zero-phase reference is assumed to be known, this is equivalent to maximizing

$$\Gamma_{i,u}(0,0) = \frac{\int_{t_1} \int_{t_2} B_i(t_1,t_2) B_u(t_1,t_2) \;\; dt_1 \;\; dt_2}{\left[\int_{t_1} \int_{t_2} |B_u(t_1,t_2)|^2 dt_1 dt_2\right]^{\frac{1}{2}} \left[\int_{t_1} \int_{t_2} |B_i(t_1,t_2)|^2 dt_1 dt_2\right]^{\frac{1}{2}}} \tag{10}$$

where $\Gamma_{i,u}(0,0)$ is the normalized cross correlation of the catalog target bispectral response $i = 1, 2, \ldots, M$ and the test target response u. Using two-dimensional Fourier transform identities and Parseval's theorem, we find that this cross-correlation can be written (for the discrete frequency case) as

$$\Gamma_{i,u}(0,0) = \frac{\mathrm{FFT}^{-1}\{R_i(f_1,f_2)R_u^*(f_1,f_2)\}}{\left[\sum_{f_1}\sum_{f_2}|R_i(f_1,f_2)|^2\right]^{\frac{1}{2}}\left[\sum_{f_1}\sum_{f_2}|R_u(f_1,f_2)|^2\right]^{\frac{1}{2}}} \tag{11}$$

The test target is classified as target j if

$$j = \arg\max_i\{\Gamma_{i,u}(0,0)\} \quad i = 1,\ldots,M. \tag{12}$$

This classifier is reasonably computationally efficient and makes use of much of the available bispectral information.

B. Nearest Neighbor Rule:

This classifier is used to identify an unknown target based on the backscatter data without employing any signal processing. Given that the measured backscatter is $H_u = [H_u(f_0),\ldots,H_u(f_{K-1}]$ (where K is the number of frequencies used) then choose target j whenever

$$j = \arg\min_i\{(H_u - H_i)^T(H_u - H_i)\} \quad i = 1,\ldots,M \tag{13}$$

C. Cross-Correlation of Impulse Responses:

This classifier identifies an unknown target based on its time-domain response $h_u(k)$ where k is the time index. For this algorithm, target j is chosen whenever

$$j = \arg\max_i\left[\left|\frac{\sum_k h_u(k)h_i(k)}{\sqrt{\sum_m |h_u(m)|^2\sum_n |h_i(n)|^2}}\right|\right], i = 1,\ldots,M \tag{14}$$

This is equivalent to maximizing the cross-correlation between the unknown target impulse response and the catalog impulse response.

IV. Classification Performance of Noisy Radar Signals

A comparison between the performance of the cross-correlation classifier using the bispectrum and the performance of other classifiers is given below. The comparison includes classification in additive white Gaussian noise, additive colored Gaussian noise, and additive non-Gaussian noise. Classification with azimuth ambiguity and azimuth estimation error of 10° is also presented. The probabilities of target misclassification at different signal-to-noise ratios are estimated using Monte-Carlo simulations. It is assumed that the targets have equal *a priori* probabilities of occurrence. Thus, N test samples are drawn randomly and then used to determine whether the classifier gives the correct decisions for these samples or not.

The data base used in the classification examples has been frequently used in radar target identification studies [3, 4]. The data base consists of experimental measurements in the frequency band from $1 - 12$ GHz of scale models of commercial aircraft. The scaled data corresponds to measurements of the radar cross section (RCS) of full scale aircraft in the HF/VHF frequency band (8-58 MHz). Details of these measurements can be found in [5].

Decision statistics for each target are computed at a fixed noise level, and total statistics of classification error for all targets are obtained. One hundred experiments were performed for each test target for a total of 500 experiments. (For this experiment, a 95% confidence interval for a misclassification probability of 30% is 4%.) The entire test is repeated at different noise levels. Finally the misclassification (error) percentage is plotted versus the signal to noise ratio.

The performance of the cross-correlation classifier using bispectral features is dependent on the bispectral estimation procedure (or the estimation of the triple correlation), see [1]. The amount of segmentation used in computing the triple correlation has a significant effect on the performance of the classifier.

The triple correlation lag used and the number of data points also influence the performance of the classifier. In addition, removing the average from both the unknown and the catalog improves the classifier performance. It was experimentally found that segmenting the data into 5 records of 21 samples each with a correlation lag of 10 (see [1]) gave nearly the best classification performance over the cases considered, so these values were used in the examples shown below.

Figure 1 shows the classification performance for five commercial aircraft, with complete azimuth information, using additive white Gaussian noise. In this case, the catalog consists of scattering data for five commercial aircraft at $0°, 10°$, and $20°$ azimuth, and $0°$ elevation. We note that the performance of the nearest neighbor (NN) algorithm is optimal for this case.

From these results, we see that the bispectrum classifier is outperformed by the impulse response classifier by a small margin. This figure indicates that bispectral features can be used effectively in radar target identification. Increasing the number of data samples and employing an optimized classification scheme may improve the performance of the bispectrum classifier and reduces its sensitivity to the triple correlation estimation procedure.

Figure 2 shows the classification performance for the case of additive colored noise, generated by passing white Gaussian noise through a moving-average (MA) filter with coefficients $[1.0, 0.8]$. In this case, the target azimuth is assumed to be completely known. The NN rule applied to the frequency-domain data outperforms the time-domain classifiers. Also in this case, the performance of the bispectrum classifier compares favorably with the performance of the impulse response classifier. The degradation of the time-domain classifiers compared to the nearest neighbor rule is slightly lower than the additive white noise case, which may suggest a comparable performance of all classifiers under other colored noise conditions.

Figure 3 shows the classification performance when additive non-Gaussian noise is used (the square root of a Weibull distributed random variable added to both the in-phase and quadrature components of the data). The azimuth is assumed to be completely known. The performance of the bispectrum classifier is improved and is comparable to that of the other classifiers, which may

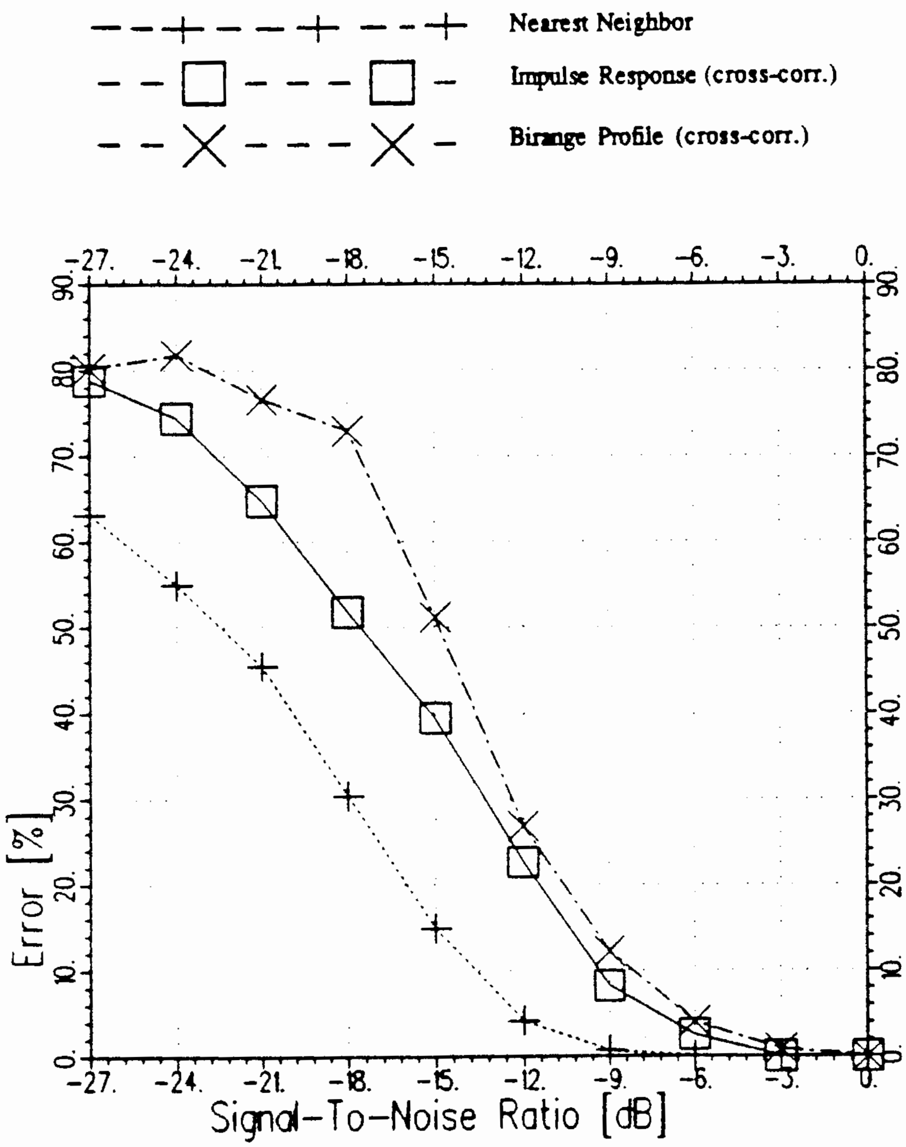

Figure 1: Classification performance of five commercial aircraft with known azimuth and additive white Gaussian noise.

indicate a significant role for the bispectrum in classification of unknown targets in a non-Gaussian noise environment.

Figure 4 shows the classification performance when the azimuth is known only to be within $\pm 20°$. Although the nearest neighbor rule is not optimal in this case, it outperforms the time-domain classifiers. Further, the performance of the bispectrum classifier degrades compared to the impulse response classifier when the azimuth ambiguity range increases. In fact, if the target is assumed to be known within $\pm 30°$ (not shown in the Figures) then the performance of the bispectrum classifier degrades significantly compared to the impulse response classifier.

These figures show that the bispectrum is sensitive to changes in target azimuth position. This

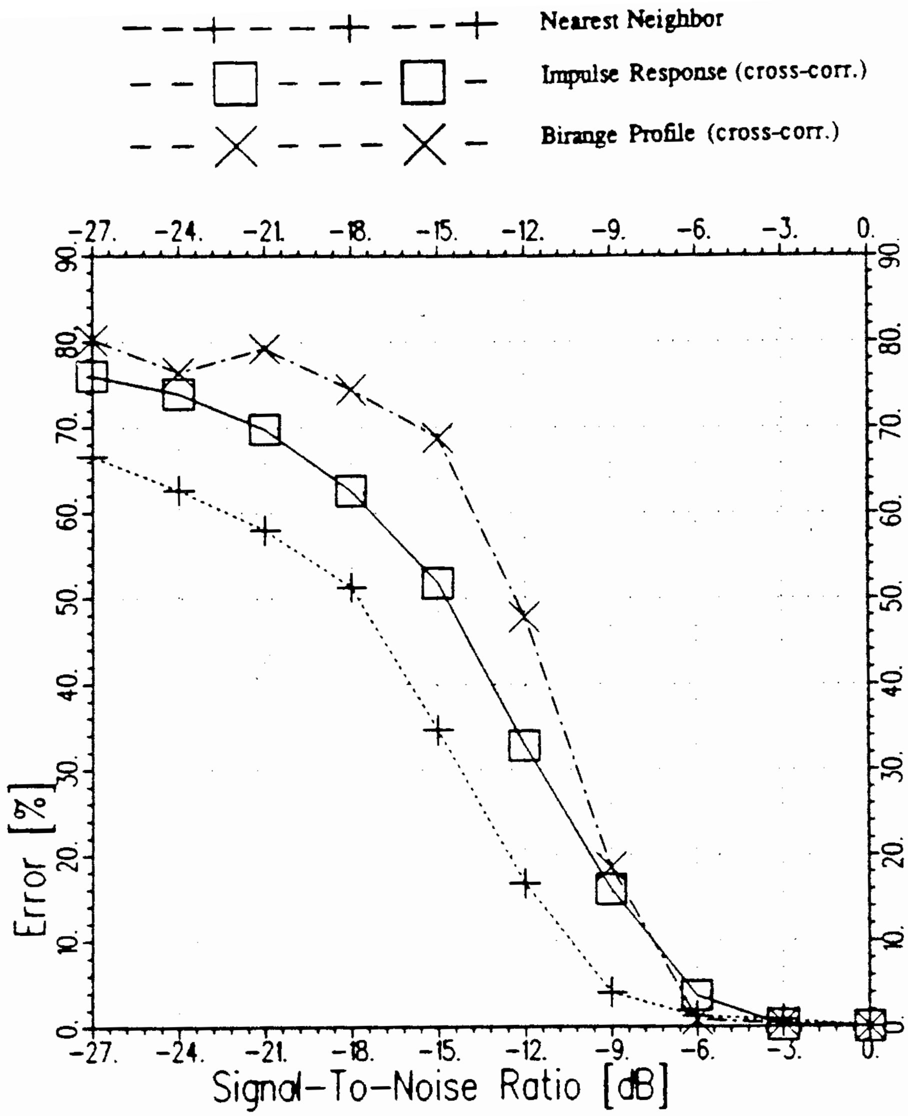

Figure 2: Classification performance of five commercial aircraft with known azimuth and additive colored Gaussian noise generated by an MA filter.

sensitivity may be explained by the fact that changing the azimuth may introduce additional multiple interactions and delete others. Although these interactions do appear in the impulse response, they appear more strongly in the bispectrum [2].

V. Classification of Signals with Extraneous Scatterers

In this section, we discuss the effect of extraneous scatterers in the unknown target frequency response (scatterers not included in the catalog) on the performance of the classifiers. This type

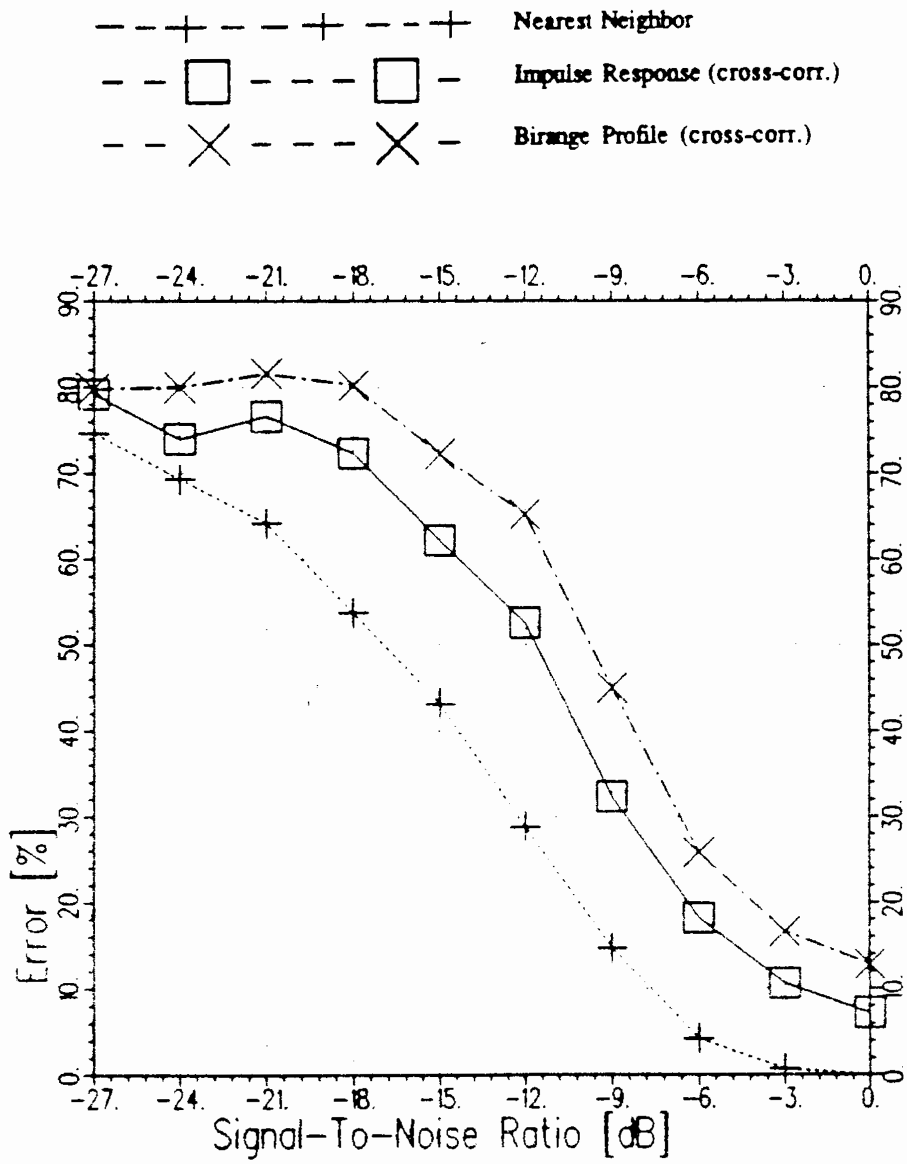

Figure 4: Classification performance of five commercial aircraft with azimuth known to within ±20° and additive white Gaussian noise.

age of misclassification when scattering from extraneous point scatterers (described in the captions) is added to the frequency response of all five targets. Scattering from the unknown target is assumed noise-free for the purpose of examining the effects of extraneous responses. The extraneous signatures used in these figures are three point scatterers where the location of the response due to each scatterer is random and uniformly distributed over $[-\frac{T}{2}, \frac{T}{2}]$, where $T = \frac{1}{\Delta f}$, (Δf is the frequency increment of the measured data).

The following conclusions can be drawn from these experimental results. First, it appears that classification with nearest neighbor rule is the most sensitive to the presence of extraneous uncataloged scatterers. In addition, we see that classification with impulse response is less sensitive

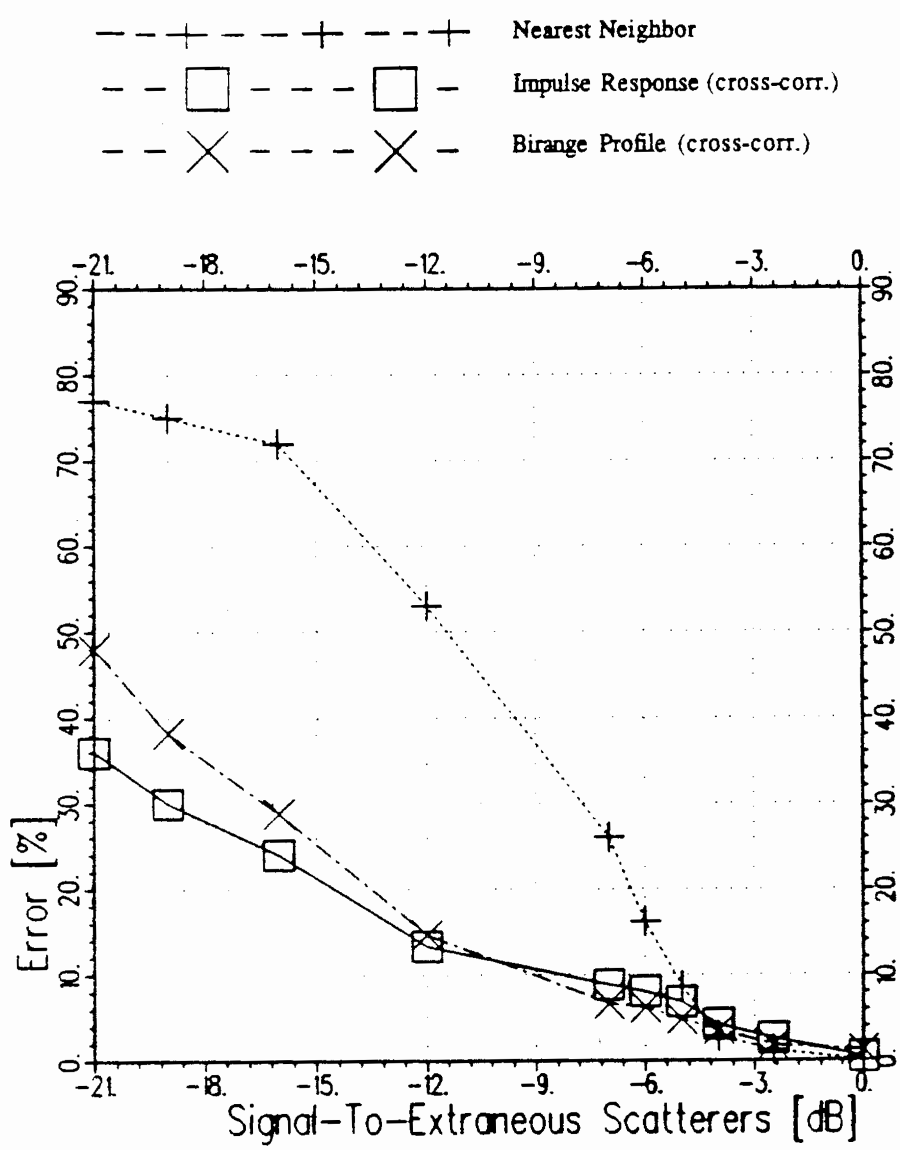

Figure 5: Comparison between classification using frequency-domain data, impulse response, and bispectral response as a function of SER for complete azimuth information (three extraneous scatterers used).

to extraneous scatterers than the NN classifier.

The results presented above also indicate that classification with the bispectrum is less sensitive to the presence of extraneous scatterers than the nearest neighbor classifiers. Our experience has shown that the bispectrum classifier is even less sensitive to uncataloged scatterers if the responses from such extraneous scatterers do not happen to coincide with a valid response of the unknown target.

VI. Conclusions

In this paper, target classification using bispectral features was evaluated and compared to classification using spectral responses and time-domain responses. The results in his paper show that bispectral processing of radar signatures may enhance the target identification process particularly under conditions characterized by additive colored noise, additive non-Gaussian noise and scattering from extraneous scatterers. It is also evident that bispectral features of unknown radar targets may be less sensitive to scattering from extraneous scatterers.

VII. References

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