HIGH RESOLUTION EXPONENTIAL MODELING OF FULLY POLARIZED RADAR RETURNS †

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Abstract

This paper considers a new method for modeling radar target scattering data for the purposes of automatic target recognition. The approach is to estimate a time domain feature vector which describes the target. The target is characterized by a set of scattering centers. Using the notion of a Transient Polarization Response, scattering centers are estimated along with their polarization. An exponential model for the fully polarized radar return and estimation algorithm are presented.

I. Introduction

This paper is concerned with processing the fully polarized radar return of a target in order to gain more information about the target. This work has application in the area of radar target identification (RTI). It is of interest to gain more information about the physical structure of an entire aircraft including its size, layout, and armaments. To this end many ways of processing stepped frequency radar measurements have been developed [1]–[5] in order to determine physical information about the aircraft such as overall length.

To arrive at the goal of determining such information, this paper considers a method for modeling of radar target signatures from a set of full polarization, stepped frequency measurements of the target. The target signature is modeled using an exponential model, which characterizes the target as a set of scattering centers, each characterized by its range and polarization ellipse.

In this paper, the approach taken is to estimate a time (range) domain feature vector which describes the target. This idea has been used for some time for single polarization measurements [2,4]. The advantages of time domain characterizations are that the target can be modeled as a relatively small number of scattering centers and that these scattering centers have a direct physical interpretation.

†This research was supported in part by the Office of Naval Research under Contract N00014-87-K-2011 and by the AFOSR, Bolling AFB, DC

Others have considered the importance of polarization effects of a target [5,6]. The work by Chamberlain et. al. [5] presents a new way to examine full polarization data which provides a more complete description of a target's interaction with an incident radar wave. This work introduced the idea of a Transient Polarization Response (TPR), the backscattered response of a target illuminated with an impulse of circularly polarized radiation. The idea is that as the circularly polarized electromagnetic pulse impinges on each scatterer, it interacts with the pulse and reflects a wave back with a polarization which is determined by the configuration of that scatterer. This type of analysis provides a more complete and descriptive representation of the target than can be obtained from a single polarization signature. Chamberlain has developed nonparametric techniques for extracting scattering centers and their corresponding elliptical polarization returns [5]. This paper extends his work to parametric modeling, which provides a higher resolution and a reduced set of data with which to identify a target.

II. The Exponential Parametric Model of Target Signatures

Radar systems are typically designed to transmit horizontally and vertically polarized radiation and receive the horizontally and vertically polarized radar scattering coefficients at stepped frequencies in a certain bandwidth: $s_{hh}(f)$, $s_{vh}(f) = s_{hv}(f)$, and $s_{vv}(f)$. It is well known that the inverse Fourier Transform of these data gives a time or down range impulse response of the target [2]. These down range profiles can be arrived at via various FFT and parametric techniques.

In this paper, the horizontal and vertical radar return from a target are modeled as a sum of complex exponential terms:

$$\begin{bmatrix} s_{hl}(f_i) \\ s_{vl}(f_i) \end{bmatrix} = \sum_{k=1}^{M} \begin{bmatrix} a_{hk} \\ a_{vk} \end{bmatrix} p_k^{f_i} \qquad (1)$$

where $\{f_i = f_0 + i\Delta f, i = 0, \ldots, N-1\}$ is a set of N stepped frequencies. Here, s_{hl} and s_{vl} are horizontal and vertical received data from a left circularly transmitted signal; this data can be obtained from the horizontally and vertically transmitted data by use of a simple transformation [7]. Left circular has been arbitrarily chosen since, on a macroscopic level, a target's features appear the same to both left and right circularly polarized transmit fields.

The inverse Fourier transform of equation (1) leads to the following range domain model for the signals:

$$\begin{bmatrix} s_{hl}(r) \\ s_{vl}(r) \end{bmatrix} = \sum_{k=1}^{M} \begin{bmatrix} a_{hk} \\ a_{vk} \end{bmatrix} \frac{1}{e^{-j2\pi r/R} - p_k} \qquad 0 \le r \le R \tag{2}$$

where R is the maximum unambiguous range. From equations (1) and (2) we see that there are M scattering centers. Each p_k is a scattering center pole; its angle corresponds to the range of the scattering center, and its magnitude corresponds to a frequency dependent return from the scatterer. Each a_{hk} and a_{vk} are the horizontal and vertical amplitudes associated with that pole. These scattering centers cause peaks in the TPR (a time domain response) which correspond to the ranges (times) at which the incident wave is reflected. The horizontal and vertical amplitudes associated with each pole contain the information about polarization characteristics of each scattering center. From a_{hi} and a_{vi} the response polarization in terms of tilt τ_i, ellipticity ϵ_i, and major axis A_i can be determined (see [5,7]). The set of parameters $\{r_i, \tau_i, \epsilon_i, A_i; i = 1, \ldots, M\}$ provides a concise description of the target as a set of M scattering centers, each one described by its range and a polarization ellipse of the scattered energy.

III. Estimating The Exponential Parametric Model From Data

This section presents an algorithm for estimating the model parameters from scattering center data. First, the poles are estimated using backward linear prediction coupled with least squares [8,9]. The backward linear prediction equations can be written as:

$$\begin{bmatrix} s_{hl}(1) & s_{hl}(2) & \cdots & s_{hl}(L+1) \\ s_{hl}(2) & s_{hl}(3) & & \vdots \\ \vdots & \vdots & & \vdots \\ s_{hl}(N-L) & s_{hl}(N-L+1) & \cdots & s_{hl}(N) \\ s_{vl}(1) & s_{vl}(2) & \cdots & s_{vl}(L+1) \\ s_{vl}(2) & s_{vl}(3) & & \vdots \\ \vdots & \vdots & & \vdots \\ s_{vl}(N-L) & s_{vl}(N-L+1) & \cdots & s_{vl}(N) \end{bmatrix} \begin{bmatrix} 1 \\ b_1 \\ \vdots \\ b_L \end{bmatrix} = 0 \tag{3}$$

$$\text{or} \quad Sb = -s \tag{4}$$

where L is the order of prediction, and b is the coefficient vector of the polynomial $B(z)$ given by

$$B(z) = 1 + b_1 z^{-1} + \cdots + b_L z^{-L}. \tag{5}$$

Ideally, L can be any integer greater than or equal to the model order M; in practice, choosing $L > M$ results in more accurate parameter estimates. Note that both the $s_{hl}(f)$ and $s_{vl}(f)$ sets of data are used simultaneously to estimate a single set of prediction coefficients.

The solution of equation (3) involves performing a singular value decomposition (SVD) on the matrix in (3), then truncating all but the first M singular values, to arrive at a noise cleaned estimate $[\hat{s} : \hat{S}]$ [9]. Next, the linear prediction coefficient vector is found as $\hat{b} = -\hat{S}^{+}\hat{s}$, where the $^{+}$ denotes pseudoinverse. Finally, the estimated poles are given by inverting the zeros of $\hat{B}(z)$.

Once these L poles have been determined, the amplitude equations for both the horizontal and vertical components can be formed. From equation (1),

$$\begin{bmatrix} \hat{p}_1^1 & \cdots & \hat{p}_L^1 \\ \vdots & & \vdots \\ \hat{p}_1^N & \cdots & \hat{p}_L^N \end{bmatrix} \begin{bmatrix} \hat{a}_{h1} & \hat{a}_{v1} \\ \vdots & \vdots \\ \hat{a}_{hL} & \hat{a}_{vL} \end{bmatrix} = \begin{bmatrix} s_{hl}(1) & s_{vl}(1) \\ \vdots & \vdots \\ s_{hl}(N) & s_{vl}(N) \end{bmatrix} \tag{6}$$

$$\text{or} \quad \hat{P}\hat{A} = S_a. \tag{7}$$

The amplitudes can be found from a least squares solution to equation (7), $\hat{A} = (\hat{P}^H \hat{P})^{-1} \hat{P}^H S_a$. And, since only M singular values were kept, no more than M of these scatterers can be anything but noise. Therefore the $L' = \min\{M, L\}$ scatterers with the largest energy should be kept.

IV. Simulation Results

This section presents results of the algorithm as applied to compact range measurements of a simplified aircraft model and a scale model of a real aircraft.

Simplified Aircraft Measurements

The exponential modeling procedure was applied to compact range measurements of a simplified, configurable aircraft target 6 inches (15.24 cm) in length. This target consists of a cylindrical fuselage (F) with removable wings (W), horizontal stabilizers (S), and tail (T). Compact range measurements of this model were taken with various parts removed. Each measurement set consists of full polarization measurements at frequencies between 2 and 18 GHz in 50 MHz steps from a nose-on aspect angle with no roll. In order to keep the unambiguous range near the target size, the frequency spacing between measurements should be about 500 MHz, not 50 MHz. To obtain this frequency spacing, the data was decimated by a factor of

10 before processing (see [7]). The model order was chosen as 10 and the number of singular which were kept varied from 3 to 7 depending on the complexity of the target.

Figures 1–3 show the estimated scattering responses for FT, FWT, and FWST configurations of the aircraft. By comparing each target silhouette with its corresponding response, one can note that the estimated scattering centers correspond well to target geometry. From these figures it can be seen that the nose scattering is accurately estimated as a nearly circularly polarized response. The two scattering responses at the end of the fuselage-tail configuration in Figure 1 correspond to leading and trailing edges of the tail; note the strong vertical polarization of the leading edge. The more circularly polarized trailing edge response is probably due to a combination of the tail trailing edge and the cylindrical trailing edge. Similar conclusions can be drawn from the fuselage-wing-tail estimate in Figure 2. The more complicated configurations in general correspond less to the target geometry towards the rear of the target. However, the primary scattering centers for the more complicated target configurations are still well-estimated.

Scale Model Aircraft Measurements

In this section, the results of modeling a scale model (the Boeing 707) are shown. Eighty measurements of the scale model were taken for frequencies between 6.45 and 10.4 GHz in 50 MHz steps from a nose-on aspect angle with no roll. This model is scaled at 150 so these results correspond to measurements from 43 to 69 MHz for the full sized aircraft. To achieve an unambiguous range of 75 cm, the data was decimated by four in the estimation procedure. A model order of 10 was chosen with 5 singular values kept. One can note that the estimated scattering centers correspond well to target geometry. Specifically, the nose, cockpit cavity, leading edge of wings, and engine inlets are all located.

To examine the effects of noise on the estimate, Figures 5-6 show results of the 707 in 10 and 0 dB SNR. In each case, independent white noise was added to each term of the scattering matrix (in left circular coordinates). Five different estimates obtained from five different noise realizations are shown overlapped in these figures. Two plots of each estimate is shown. The first is the polarization ellipse versus range plot as shown earlier. The second plot shows the hypotenuse length between the major and minor axes versus range; this plot can be viewed as a projection of the first plot. From these figures it can be seen that at 10 dB SNR the estimates show little deviation from the noiseless estimate. As the SNR is reduced to 0 dB, the major scattering centers are still estimated; however, the accuracy of the range, amplitude, ellipticity, and tilt decreases somewhat (as is expected). Some spurious estimates are seen but these have small magnitude compared to the actual scattering center magnitudes. This experiment was also conducted for four other commercial aircraft with similar results (see [7]).

V. Conclusions

This paper has presented a method of processing full polarization, stepped frequency measurements of a target. A parametric model which describes the target as a set of scattering centers is developed. Each scattering center is characterized by a polarization ellipse, which corresponds to the backscattered polarization ellipse from a circularly polarized incident wave. An estimation procedure which directly estimates the parameters of this model is then developed.

Simulation results are presented for compact range measurements of aircraft. Results of this algorithm applied to these data verify that it is capable of identifying scattering mechanisms of the target, and that the estimated polarization ellipse of each scattering center correlates well with the geometry of the target. The signatures of various aircraft are seen to show such features as wings, engine inlets, cockpit cavity, and tail. Tests using noisy data shows that dominant scattering is well estimated even at 0 dB SNR.

References

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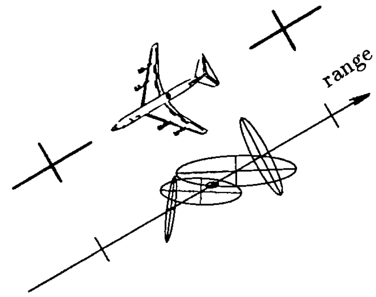

Figure 4: 707 Scatterers and Polarizations

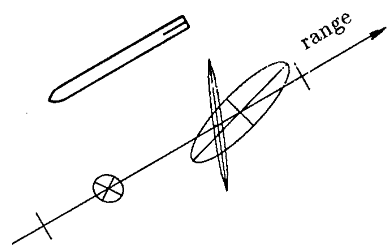

Figure 1: FT Scatterers and Polarizations

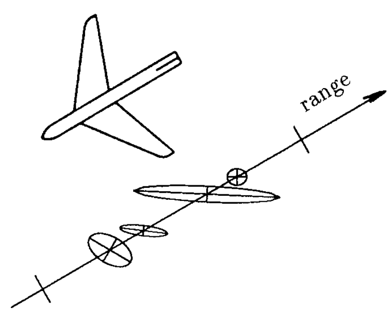

Figure 2: FWT Scatterers and Polarizations

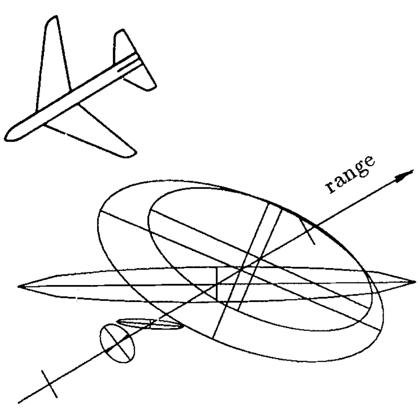

Figure 3: FWST Scatterers and Polarizations

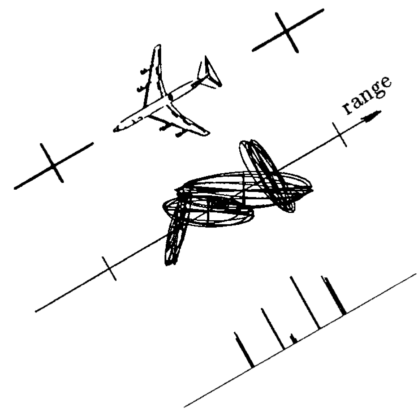

Figure 5: 707 Scatterers and Polarizations, SNR of 10dB

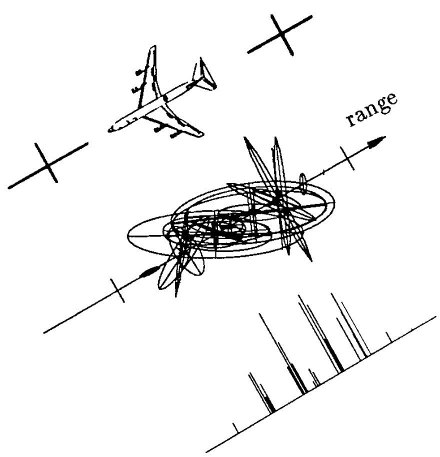

Figure 6: 707 Scatterers and Polarizations, SNR of 0dB